https://doi.org./10.58225/mpmma.2026.*567*

NONLINEAR ANALYSIS OF CRACK RESISTANCE IN RC SECTIONS

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Abstract. This paper presents an analytical method for evaluating the crack resistance of rectangular reinforced concrete (RC) sections using a nonlinear deformation model. Concrete behavior in both tension and compression is described by tri-linear stress–strain diagrams, allowing consistent consideration of elastic and inelastic stages up to crack initiation. Based on the plane-sections hypothesis, three deformation cases are defined depending on the stress state in the compression zone. For each case, equilibrium conditions lead to quadratic equations for determining the neutral axis depth, from which closed-form expressions for the cracking moment are derived. Parametric studies show that reinforcement ratio and concrete strength influence the cracking moment in a nonlinear manner. The proposed approach provides a unified, transparent, and efficient tool for analyzing and designing RC members at the serviceability limit state.

Keywords: reinforced concrete; crack resistance; cracking moment; nonlinear deformation model; tri-linear stress–strain diagram

1. Introduction

The prediction of crack initiation in reinforced concrete (RC) members is essential for ensuring serviceability, durability, and structural reliability. However, conventional linear approaches often fail to accurately represent the nonlinear behavior of concrete in both tension and compression near the cracking stage. To address this limitation, this study develops a deformation-based analytical approach using tri-linear stress–strain diagrams for concrete. The model allows a consistent description of material behavior up to crack formation and considers multiple deformation cases within the compression zone. The proposed method provides a unified framework for determining the cracking moment of rectangular RC sections through equilibrium and strain compatibility conditions, improving the accuracy of crack resistance assessment in structural design.

2. Tri-Linear Stress–Strain Models for Concrete in Compression and Tension

In this section, the constitutive behavior of concrete in compression and tension is described using tri-linear (three-segment) stress–strain diagrams, which are suitable for analytical integration within the deformation (strain-based) framework. The conceptual form of these diagrams is illustrated in Figure 1. The tri-linear representation enables a consistent characterization of concrete behavior across elastic and inelastic stages. In compression, the diagram captures the transition from initial elastic response to nonlinear hardening and ultimate deformation. In tension, it reflects the progression from elastic behavior to microcracking and eventual crack initiation.

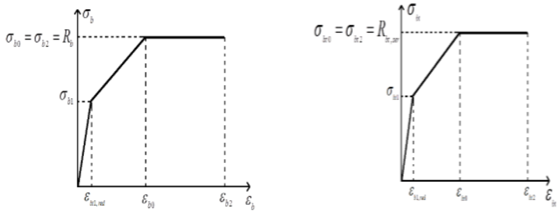

Fig. 1. Tri-linear stress–strain diagrams for concrete (compression, tension)

2. Elastic Behavior of Concrete in the Compression Zone

Traditional approaches distinguish between elastic and nonlinear behavior of concrete in the compression zone. However, such simplifications do not fully represent the actual deformation behavior within a nonlinear framework. In the proposed approach, the compression zone is described using a full tri-linear stress–strain diagram, allowing deformation along any of its segments. At the cracking stage, the tension zone is assumed to fully mobilize all segments of the tri-linear diagram, ensuring a consistent and realistic representation of material behavior. Deformation of compressible reinforced bars is expressed as the ultimate deformation of concrete under tension $\varepsilon_s' = \frac{x - a_s'}{h - x} \cdot \varepsilon_{bt2}$ (1) based on the hypothesis of flat sections. Then the value of the normal stress arising in this reinforced bars $\sigma_s' = E_s \cdot \varepsilon_s' = \frac{x - a_s'}{h - x} \cdot E_s \cdot \varepsilon_{bt2}$ (2), the normal force generated by these stresses $N_s' = A_s' \cdot \sigma_s' = \frac{x - a_s'}{h - x} \cdot A_s' \cdot E_s \cdot \varepsilon_{bt2}$ (3), and finally the moment of this force relative to the center of gravity of the concrete section will be determined as follows:

$$M_s' = A_s' \cdot \sigma_s' \cdot \left(\frac{h}{2} - a_s' \right) = \frac{x - a_s'}{h - x} \cdot A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot E_s \cdot \varepsilon_{bt2}$$

(4)

In this case, deformations develop along the first and second branches of the tri-linear stress–strain diagram, reflecting the initiation of nonlinear behavior in the compression zone. Only the parameters of the compressed concrete are affected, while all other sectional properties remain unchanged. Therefore, the analysis focuses on determining the modified characteristics of the compression zone.

For the case where only the first and second branches of the tri-linear compression diagram are activated, the following condition defines the cracking stage:

$$M_{crc} = M_{b1} + M_{b2} + M_s + M'_s + M_{bt1} + M_{bt2} + M_{bt3}$$
(5)

Conclusion

Analytical expressions for the cracking moment of reinforced concrete sections were developed using tri-linear stress–strain diagrams. The method reduces the determination of the compression zone depth to solving a quadratic equation, providing a unified approach.Results show that the compression zone remains mostly near-elastic at cracking. The cracking resistance depends nonlinearly on reinforcement ratio and concrete strength, highlighting the need for deformation-based models.

References

1. Kolmogorov, A. G., & Plevkov, V. S. (2011). Calculation of reinforced concrete structures according to Russian and foreign standards. Moscow: ASV Publishing House.
2. Gadzhieva, U. M. (2021). Calculation of compression reinforced concrete elements of circular cross-section using a nonlinear deformation model. The Expert: Theory and Practice, (5), 13–20.
3. Sanzharovsky, R. S., & Veselov, A. A. (2001). Theory of calculation of building structures for stability and modern standards. St. Petersburg–Moscow: ASV.
4. Beglov, A. D., & Sanzharovsky, R. S. (2006). Theory of calculation of reinforced concrete structures for strength and stability:

Modern standards and European standards. St. Petersburg–Moscow: ASV.
5. Trekin, N. N., & Kodysh, E. N. (2016). Calculation of the formation of normal cracks based on the deformation model. Industrial and Civil Engineering, (7), 74–78.
6. Karpenko, N. I., & Radaykin, O. V. (2012). Towards improving concrete deformation diagrams to determine the cracking moment and ultimate moment in bent reinforced concrete elements. Construction and Reconstruction, 3(41), 10–15.
7. Veselov, A. A., Hegai, A. O., Hegai, M. O., & Nikolaeva, S. P. (2016). Calculation of reinforced concrete elements for crack formation. Bulletin of Civil Engineers, 3(56), 52–55.
8. Kodysh, E. N., Nikitin, I. K., & Trekin, N. N. (2010). Calculation of reinforced concrete structures made of heavy concrete in terms of strength, crack resistance and deformation. Moscow: Association of Construction Universities Publishing House.
9. SP 63.13330.2018. (2018). Concrete and reinforced concrete structures: Main provisions. Moscow: A.O. SRC Construction, NIIZHB named after A. A. Gvozdev.
10. Hajiyev, M. A., Aeinehchi, S., Bashirzade, S. R., Asadov, E. Z., & Khalilov, H. A. (2024). Structural performance of compressive zone strengthened RC elements.

Scientific and Technical Journal on Engineering Mechanics, 8(2), 56–72.