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Increasing the Bending Strength of Steel I-Beams Under Load by Rational Strengthening of the Cross-Section in the Tension Zone

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***Abstract*— An analytical method is presented for calculating the strength (within the elastic range) of metal beams with symmetrical cross-sections, witch are strengthened by horizontal steel elements without prestressing. The strength calculation of metal beams is performed taking into account the history of their loading. An example of the strength calculation of a metal I-beam is presented, which is strengthened under partial loading in the tension zone by horizontal reinforcement bars, which are attached to its lower flange using support elements or electric arc welding.**

***Keywords*— steel beams, cross section, strengthening, reinforcing bars**

I. INTRODUCTION

Statement of the problem. Connection with scientific and practical tasks. A significant role in industrial and civil construction is played by the reconstruction of industrial, civil, and residential buildings and structures. Reinforcing steel beams with symmetrical cross-sections using the traditional method of increasing the cross-sectional area in the tension zone through additional longitudinal steel elements allows for repair work to be carried out with minimal economic costs.

When reinforcing the tension zones of metal beam cross-sections with additional steel elements made of rolled sections or reinforcement bars in the form of horizontal or stressed rods of sprengel construction, a number of factors arise that directly affect their strength and are not considered together in their calculation, namely:

- load level of metal beams during reinforcement;
- strength characteristics of the material of metal beams and additional lengthwise reinforcement elements;
- the uneven distribution of forces in the reinforced beam cross-sections and in the additional longitudinal elements of its reinforcement at different stages of their stressed-deformed state (during reinforcement and failure).

At the same time, the determination of the optimal parameters of additional steel elements or reinforcement bars used for strengthening steel beams in the tension zone of their cross-section is carried out through a multi-variant calculation approach. The multi-variant calculation is associated with certain time costs and the absence of clear criteria for the limit state of their combined cross-sections, which would correspond to the stressed-deformed state at the moment of their failure or loss of stability under bending.

Therefore, at this stage, there is a need to improve the method of calculating the bending strength of steel beams with symmetrical cross-sections by introducing additional longitudinal reinforcement bars or other elements in their tension zone, which would account for the aforementioned factors.

Analysis of publications. Identification of unresolved issues. The analysis of scientific research has shown that there is a wide range of methods for strengthening the bending strength of metal beams by increasing (augmenting) their cross-sections along the length. In recent years, in addition to traditional methods of reinforcing metal beams by augmenting their cross-sections with additional elements made of shaped and sheet rolled sections or reinforcement bars, methods of reinforcement through cross-sectional increase using adaptive wire manufacturing have also been introduced (WAAM) and the installation (fixing, bonding) of additional elements made of carbon fiber (CFRP) to the lower tension zone of metal beams. In the studies [1], [2], researchers J. Yang, M.A. Wadee, and L. Gardner conducted experimental investigations to evaluate the bending strength of hot-rolled steel beams, which were reinforced along their length by increasing the I-shaped cross-section through arc-based additive wire manufacturing (WAAM). The beams were tested under four-point or three-point bending conditions. The results showed that as the mass of the specimens' cross-sections increased from 2,63% to 12,26%, the bending strength of the beams increased by 11,5%–33,2%.

In the work [3], researchers Ghafoori E. and Motavalli M. experimentally and numerically investigated the transverse-torsional bending of steel beams reinforced with a layered carbon fiber reinforced polymer (CFRP) material of normal modulus of elasticity (NM). The test results showed that reinforcing steel beams with layered carbon fiber materials almost equally increases the elastic stiffness of the specimens, both for bonded and unbonded systems, compared to the reference beam. Specimens reinforced with unbonded carbon fiber reinforced polymer (CFRP) laminates have higher strength than those reinforced with bonded carbon fiber reinforced polymer laminates. Pre-stressing of the carbon fiber reinforced polymer layered materials does not affect the elastic stiffness of the reinforced beams, but it significantly influences their bending strength. Higher levels of prestressing do not necessarily lead to an increase in bending strength in the retrofitted beams. [3]. The article by Omnia R. AbouEl-Hamd [4] presents the results of experimental and numerical studies on steel beams, which were reinforced along their length with hybrid fiber-reinforced polymers (HFRP) using steel bolts. Tests on 22 steel beams demonstrated their plastic behavior upon failure, with an increase in yield and ultimate bending strength by 15,1% and 22,2%, respectively. [4]. The study of a structural solution for strengthening steel beams using adhesive strips made of shape memory alloy steel (also known as iron-based shape memory alloy, Fe-SMA) was conducted by Sizhe Wang and other researchers in work [5].

In study [6] Liu, Y. and Gannon, L. conducted experimental investigations on steel I-beams with cross-section W 310×100×28,3 and effective lengths of L_u = 2,4 m and L_u= 1,2 m. The beams were strengthened using conventional methods: by welding under load steel vertical cover plates with dimensions t×h = 9.5×320 mm along the length of the specimens symmetrically to the top and bottom flanges; and by welding a horizontal cover plate with dimensions t×b = 9,5×137 mm along the length of the tensioned bottom flange of the I-section beam. The experimental results indicated that the degree of preloading influences the strength of the reinforced beams, which fail as a result of torsional bending of the vertical plane. Specifically, when the beams were strengthened under a preload level of 50–53% of the ultimate strength of the unstrengthen beam, the ultimate strength of the reinforced beam under torsional buckling decreased by 13–14,2%. For the specimens that predominantly failed due to flexural bending about the horizontal plane, the level of preloading showed a negligible effect on the ultimate strength of the strengthened specimens. In article [7] and research work [8], researchers Mohammadzadeh, M. and Bhowmick, A. conducted finite element (FE) analyses to investigate the stress–strain behavior and flexural strength of steel I-beams with cross-sections strengthened under load by additional steel cover plates. A series of simply supported steel I-beams, reinforced with cover plates welded to the bottom tension flanges, was thoroughly analyzed. The finite element analysis shows that as the level of preloading increases, the load-bearing capacity of the I-beam strengthened under load decreases when the failure mode is lateral-torsional buckling (loss of stability from the vertical plane). On the other hand, variations in the level of preloading have a minimal effect on the behavior and ultimate strength of the strengthened beam when failure occurs in pure bending. In studies [9] and [10], Assaad Taoum, together with Jiao, H. and Holloway, D., investigated the effectiveness of the Local Prestressing Technique (LPT) for improving the repair of damaged steel beams. The studies demonstrated that the level of prestressing, the type of LPT method (internal or external), and the diameter of the reinforcing bars used had a significant impact on the stiffness of the beams and their ultimate load-bearing capacity [9],[10].

In article [11], Piotr Szewczyk and Maciej Szumigała presented the results of experimental investigations and numerical analysis on the strengthening of steel–concrete composite beams. The steel section of the IPE200 I-beam profile was strengthened along its length by welding additional steel plates with a thickness ranging from 6 to 22 mm to its bottom flange. One of the objectives was to find an optimal solution that minimizes costs while maximizing bending strength. To achieve this objective, the energy parameters available in numerical simulations were reviewed and analyzed. The proposed numerical models were experimentally validated using three specimens. The maximum deviation between the experimental and the average numerical bending strength of the reinforced composite beams is approximately 2,5%.

The strengthening of composite steel–concrete beams using externally prestressed reinforcement bars has been the focus of studies by researchers such as El-Zohairy A. and Salim H. [12], M.Yu. Izbash [13], R.M. Shemet [14], as well as T.A. Halinska, V.F. Pentz, and Yu.O. Kushnir [15]. In article [12], El-Zohairy A. and Salim H. presented a numerical model for investigating the nonlinear behavior of composite beams reinforced with externally prestressed reinforcement bars (tie rods). The reliability of the developed numerical model was confirmed by comparing its results with available experimental data. Study [12] investigated the influence of various strengthening parameters, including inclined (triangular and trapezoidal profiles) and straight bars (tie elements); the length of the bars (ties); the effect of additional prestressing on restoring the bending performance of an overloaded beam; the eccentricity of stress application; and the degree of shear connection. A good agreement was obtained between the proposed model and the experimental data. The results indicate that, at the same stress application eccentricity, the trapezoidal profile demonstrates superior load-bearing capacity for reinforced beams compared to the horizontal strengthening element. In article [15], the authors present a classification of steel–concrete composite beams strengthened with prestressed horizontal elements (tie rods), based on the composite properties of their components and the cases of the ultimate stress–strain state.

Thus, the conducted analysis of scientific studies has shown that an unresolved aspect of the broader issue concerning the development and improvement of strength calculation methods for strengthened structural elements is the lack of a clear (engineering) methodology for calculating the bending strength of metal beam structures (elements) strengthened by the addition of supplementary components in the tensile zone of their cross-section such a methodology should take into account the loading history, the strength characteristics of the materials, and the stages of their stress–strain state at the moment of failure.

The overall **aim of the research** is to develop an analytical method for calculating the bending strength of metal beams with symmetrical cross-sections that are strengthened under load by installing additional longitudinal reinforcement bars or other elements in their tensile zone, without prior prestressing.

II. Presentation of the Main Material

Presentation of the Main Material. The theoretical studies consider simply supported single-span metal beams with symmetrical cross-sections, which are strengthened under partial loading (q_0, *kN/m*) by installing horizontal elements—either reinforcement bars or rolled steel strips—in the tensile zone (see the calculation scheme in Fig. 1). The criterion for the rational selection of the cross-sectional area of the additional reinforcement bars or steel elements (A_{sd}), while ensuring the bending strength of a metal beam strengthened without full or partial unloading, is the stress–strain state stage (see Fig. 2), at which the ultimate stress values are simultaneously reached in the design cross-section, specifically: the stress in the extreme fiber of the compressed zone of the metal beam σ_a reaches the design yield strength of the beam material f_a , and the stress in the additional reinforcement bars (σ_{sd}) reaches the design tensile strength of the reinforcement steel f_s .

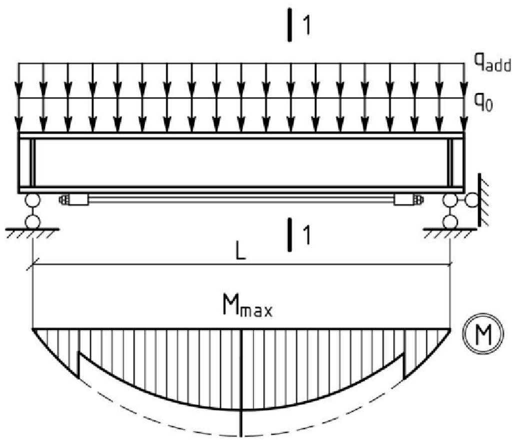

Fig. 1. Calculation scheme of a metal beam strengthened under partial loading (q_0, kN/m) by installing horizontal elements—either reinforcement bars or rolled steel strips—in the tensile zone.

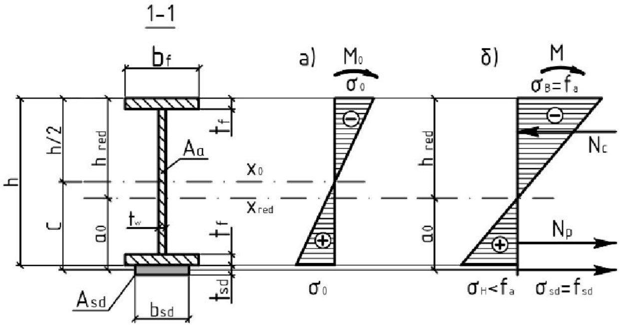

Fig. 2. Stress–strain state of the composite cross-section in the bending zone of a metal beam strengthened by adding longitudinal reinforcement bars or rolled steel elements in the tensile region: a) under the service load at which the beam is being strengthened; b) under the full design load at the ultimate elastic stage.

The cross-sectional area of the additional longitudinal reinforcement bars (A_{sd}) or steel strengthening elements (A_{ad}) is determined using Eqs. (1) – (6), in accordance with the above-mentioned criterion, under which the resulting composite design cross-section of the beam operates within the elastic stage, that is: when $\sigma_a = \sigma_в = f_a$ and $\sigma_{sd} = f_{sd}$ or when $\sigma_a = \sigma_в = f_a$ and $\sigma_{ad} = f_{ad}$:

$$A_{sd} = (M - 2/3 \times [h_{red} \times (N_c - N_p) + N_p h]) / (f_{sd} a_0) ; \quad (1)$$

$$N_c = 0{,}5 \times \Delta\sigma \times h_{red} ; \; N_p = 0{,}5 \times \sigma_н \times (h - h_{red}); \quad (2)$$

$$a_0 = h / 2 + C - h_{red} ; \quad (3)$$

$$\Delta\sigma = f_a - \sigma_0 ;$$

$$\sigma_0 = M_0 / W_x ; \quad (4)$$

$$\sigma_н = \Delta\sigma \times (h - h_{red}) / h_{red} ;$$

$$A_{sd} = \frac{M - \Delta\sigma \times h \times [h_{red}^2 - h \times h_{red} + 1/3 \times h^2]}{f_{sd} a_0} ; \quad (5)$$

$$h_{red} = [A_a \times h / 2 + A_{sd} \times (h / 2 + C)] / (A_a + A_{sd}) , \quad (6)$$

where M_0, M – are the initial and design bending moments, respectively, in the cross-sections of the metal beam due to the external load at the time of strengthening and under the full design load in the bending region; h – working height of the section of the reinforced metal beam; C – the distance between the geometric horizontal axes of the additional longitudinal reinforcement or the horizontal rolled steel element and the cross-section of the strengthened metal beam; h_{red} – the height from the top compressed edge of the resulting composite cross-section of the strengthened beam to its geometric horizontal axis. a_0 – the distance between the geometric horizontal axes of the additional longitudinal reinforcement or horizontal rolled element and the composite reduced cross-section of the metal beam to be reinforced; σ_0 – stresses in the outermost fiber of the compressed zone of the beam cross-section under the action of the initial load at which its reinforcement is carried out; $\Delta\sigma = \sigma_в, \sigma_н$ – the stresses, respectively, in the upper and lower fibers of the composite section of a reinforced metal beam at the limit stress-strain state in the elastic stage; A_a, W_x – cross-sectional area and moment of resistance relative to the horizontal axis of the reinforced metal beam; f_a – design resistance to tension or compression of a metal beam in accordance with standards [16] depending on its steel grade; $f_{sd}(f_{ad})$ – calculated resistance to tension of additional reinforcing bars or horizontal rolled elements in accordance with standards [17] or [16].

The determination of the cross-sectional area of additional reinforcing bars or steel elements (A_{sd}), used for strengthening the tensile zone of steel beam sections, is complicated by the number of unknown variables ($x_i \geq 3$), which are interrelated. This process is only possible through a step-by-step assumption of one of the unknown components, leading to additional labor during calculations and resulting in numerous possible solutions, which are not always economically efficient. Therefore, to improve the methodology for calculating the required cross-sectional area of strengthening elements, additional coefficients *n* and *k* were proposed. These coefficients take into account the ratio of the area of the additional reinforcement A_{sd} to the cross-sectional area of the strengthened steel beam A_a :

$$n = A_{sd} / A_a; \quad k = n / (1+n). \tag{7}$$

The value of coefficient n is determined using Eqs. (8) – (10), taking into account the equilibrium in the design cross-section of the beam under the adopted ultimate criteria of its stress-strain state:

$$-2 f_{sd} n^2 (h + 2C) + 2n(4\Delta\sigma C - f_{sd} h) = 0; \tag{8}$$

$$D = [2 \times (4 \times \Delta\sigma \times C - f_{sd} \times h)]^2; \tag{9}$$

$$n_{1,2} = \frac{4 \times \Delta\sigma \times C - f_{sd} \times h \pm \sqrt{D}}{2 \times f_{sd} \times (h + 2 \times C)}. \tag{10}$$

The cross-sectional area of the longitudinal strengthening element made of reinforcing bars or rolled steel will be:

$$A_{sd} = n \times A_a; \quad n = k / (1-k). \tag{11}$$

The flexural strength verification in the design cross-sections of a steel beam strengthened under load with additional reinforcement in the tensile zone, in the form of longitudinal bars or rolled steel elements, is performed using Eqs. (12) – (14):

$$M \leq \Delta\sigma \times h \times [h_{red}^2 - h \times h_{red} + \frac{1}{3} h^2] + f_{sd} \times A_{sd} \times a_0; \tag{12}$$

$$h_{red} = 0{,}5 \times h + n \times C / (1+n) = 0{,}5 \times h + C \times k; \tag{13}$$

$$a_0 = C \times (1 - n / (1+n)) = C \times (1-k). \tag{14}$$

III. CALCULATION EXAMPLE

Example Calculation. It is required to strengthen a steel beam made of a rolled I-beam № 40 with a span of *L* = 6 m, which is subjected to a uniformly distributed load along its entire length. $q = 54 кN/m$ (see Figure 3). Geometric characteristics of the cross-section of an I-beam: $A_a = 72{,}7 cm^2$; $I_x = 19062 cm^4$; $W_x = 953 cm^3$; $f_a = 210 MPa$. Before strengthening, the steel beam was preliminarily unloaded; therefore, during the strengthening process, it was subjected only to a uniformly distributed load caused by its self-weight.: $q_0 = 0{,}57 кN/m$; $q_0 = 13{,}0 кN/m$; $q_0 = 20{,}0 кN/m$ and $q_0 = 26{,}0 кN/m$.

Solution: The strengthening of the cross-sections of the I-beam No. 40 is carried out by installing horizontal reinforcing bars in its tensile zone, the length of which is assumed to be equal to: $l_{ad} = 0{,}8 \times l = 0{,}8 \times 6 = 4{,}8 m$. The distance of the horizontal axis of the bars from the lower edge of the bottom flange of the beam section is initially assumed to be *a =1 cm* therefore, the distance from the horizontal geometric axis of the strengthened I-beam to the horizontal axis of the additional reinforcing bars will be: $C = h/2 + a = 40/2 + 1 = 21$ *cm*. The reinforcing bars are assumed to be made of reinforcement steel class A400C with a design tensile strength of f_{sd} = 365 *MPa*.

1. Verification of the bearing capacity of the beam:

$$\sigma = M / W_x = 243 \times 10^5 / 953 = 25498 H / см^2 = 255 MPa,$$

where $M = ql^2 / 8 = 54 \times 6^2 / 8 = 243 кN \times m$ – the value of the maximum bending moment due to the applied load in the cross-sections at the midspan of the beam.

Since $\sigma = 255 MPa > f_a = 210 MPa$, the bending strength of the beam is not sufficient, and strengthening is required.

2. We determine the stress in the beam cross-section at a distance $x = 0{,}5 \times l$ from the point of action of the uniformly distributed load $q_0 = 0{,}57 кN/m$ using Eq. (4):

$$\sigma_0 = q_0 l^2 / (8W_x) = 0{,}57 \times 6^2 \times 10^5 / (8 \times 953) =$$
$$= 267 N/cm^2 = 2{,}7 MPa,$$

what constitutes 1,28% of the ultimate stress f_a.

3. We determine the value of the ultimate stress in the upper fiber of the calculated composite cross-section of the beam:

$$\Delta\sigma = f_a - \sigma_0 = 210 - 2{,}7 = 207{,}3 MPa.$$

4. We determine the value of the coefficient *n* using Eqs. (9) and (10) for the beam load during its strengthening $q_0 = 0{,}57 кN/m$:

$$n_{1,2} = \frac{4 \times 20730 \times 21 - 36500 \times 40 \pm \sqrt{D}}{2 \times 36500 \times (40 + 2 \times 21)} = 0{,}14273; -0{,}04757,$$

$$\sqrt{D} = \sqrt{[2 \times (4 \times 20730 \times 21 - 36500 \times 40)]^2} = 562640 \ N/cm.$$

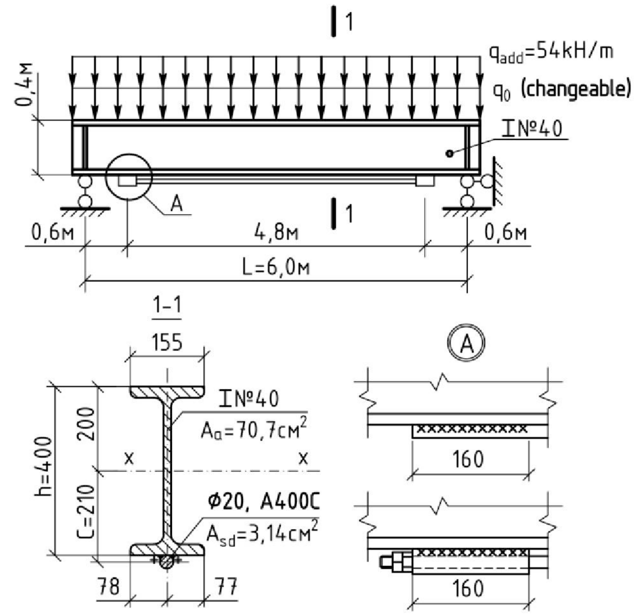

Fig. 3. Calculation scheme of the steel beam made of I-beam № 40, with a span *L = 6 m*, which is strengthened under partial loading. (q_0, кN/m.) by installing horizontal elements made of reinforcing bars or rolled steel strips of length l_{sd}= 4.8 m in the tensile zone, which are fixed to its bottom flange using support elements or arc welding.

5. We determine the value of the limiting area of the additional longitudinal reinforcing bars A_{sd} used for strengthening the steel beam in its tensile zone of the calculated cross-section, at the ultimate strength of the beam, using Eq. (11):

$$A_{sd} = n \times A_a = 0{,}14273 \times 72{,}7 = 10{,}37 cm^2.$$

6. According to Eqs. (12) – (14), the flexural strength (M_u) of a steel beam strengthened with longitudinal reinforcing bars is verified under a load of $q_0 = 0{,}57 \kappa N / m$:

$$M = 243 \kappa Nm < M_u = 20730 \times 40 \times (22{,}63^2 - 40 \times$$
$$\times 22{,}63 + 40^2 / 3) + 36500 \times 10{,}37 \times 18{,}377 =$$
$$= 123251086{,}3\, Ncm = 1232{,}5\, \kappa Nm,$$

where $h_{red} = 0{,}5 \times 40 + \dfrac{0{,}14273 \times 21}{1 + 0{,}14273} = 22{,}63 cm;$

$$a_0 = 21 \times \left(1 - \frac{0{,}14273}{1 + 0{,}14273}\right) = 18{,}37 cm.$$

The strength condition of the section of the reinforced beam under a load of $q_0 = 0{,}57 \kappa N / m$ is satisfied.

Next, following the same sequence, the calculation of the coefficient *n* and the flexural strength M_u is performed for the limiting area of additional longitudinal reinforcement bars A_{sd}, which are used to strengthen the steel beam in the tensile zone, under the following load levels: $q_0 = 13{,}0 \kappa N / m$ $(\sigma_0 / f_a = 29{,}24\%)$; $q_0 = 20{,}0 \kappa N / m$ $(\sigma_0 / f_a = 44{,}95\%)$; $q_0 = 26{,}0 \kappa N / m$ $(\sigma_0 / f_a = 58{,}46\%)$. The calculation results are presented in table 1.

TABLE 1 The calculation results for the coefficient n and the following values are presented: the stress in the design section of the beam under strengthening *σ₀, MPa*; the area of the additional longitudinal reinforcement bars A_{sd}, cm²; and the flexural strength of the strengthened steel beam M_u, *kNm*.

Quantities	Load level q_0, кN/m (as a % of σ_0/f_a)			
	0,57 (1,28%)	13,0 (29,24%)	20,0 (44,95%)	26,0 (58,46%)
σ_0, MPa	2,7	61,4	94,44	122,77
n	0,14273	0,035813	0,08275	0,123
A_a, cm²	72,7			
A_{sd}, cm²	10,37	2,6	6,02	2,6
M_u, кNm	1232,5	814,9	670,8	544,7
M, кNm	243,0			
M_u/M	5,07	3,35	2,76	2,24

Justification of the results obtained. The optimal reinforcement option for the steel beam corresponds to the load level $q_0 = 13{,}0 \kappa N / m$ $(\sigma_0 / f_a = 29{,}24\%)$, at which the area of the reinforcement elements is $A_{sd} = 2{,}6 cm^2$ and the flexural strength is $M_u = 814{,}9 \kappa Nm$, which ensures that the beam can carry a design load of $q = 54 \kappa N / m$.

The steel beam (I-beam No. 40) is strengthened (see Fig. 3) by welding symmetrically to its lower flange a reinforcing bar d=∅20 *mm*, (A400C) $A_{sd} = 3{,}14 cm^2$, length $l_{sd} = 4{,}8 m$; by fastening the ends of the reinforcing bar with bolted joints in additional support elements, which are also welded to the lower flange of the beam. In this case, the value $C = h / 2 + a = 40 / 2 + 1 = 21\ cm.$ is equal to the value previously accepted in the calculation.

IV. CONCLUSION

Conclusion and Prospects for Further Research. An analytical method is proposed for calculating the flexural strength (in the elastic stage) of steel beams with symmetric cross-sections, strengthened with horizontal steel elements without prestressing. The strength calculation of steel beams is performed with consideration of their loading history. An example is presented for the strength calculation of a steel I-beam strengthened in the tension zone under partial loading by horizontal reinforcement bars, which are connected to its lower flange through support elements or by arc welding.

The aim of further research is to develop a methodology for calculating the strength of steel beams strengthened under loading, subjected to torsional bending out of the vertical plane. Further scientific research will also focus on the development of typical structural solutions for strengthening steel beams by enlarging their cross-section in the tensile zone, the basis of which will be the established relationships between the coefficients *n* and *k* and the parameters quantities: the load level during strengthening (values M_0, σ_0); the cross-sectional area of the beam (A_a); the distance between the horizontal geometric axes of the strengthened steel beam and its longitudinal reinforcement elements (*C*); the maximum flexural strength (*M*); and the span length of the beam (*L*).

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Effect of Backfill Compaction on the Deformation Behavior of Metal Silo Foundations

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