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The Complete Concrete Strength Use Criterion of Biaxially Bent Columns of Frame Structural Systems

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Abstract The precast flat plate frame structural system of buildings is the most effective and demanded for the construction of residential and civil facilities. It requires improving the methods of calculating its carrying capacity to guarantee the reliability and safety of building operation. The article reflects the actual state of methods for determining the carrying capacity of reinforced concrete columns in a flat plate frame structural system and suggests directions for their improvement. It is demonstrated using finite element modeling of the spatial frame operation that its columns undergo biaxial bending. The problem of defining the calculated ultimate values of fiber concrete strain in the biaxially bent reinforced concrete elements is solved. These are the important values in calculating the carrying capacity of columns. The strength criterion for biaxially bent reinforced concrete columns is proposed, provided the reinforcement operates in the elastic stage. The diagram of the design values of the ultimate fiber strain of concrete in biaxially bent columns is constructed based on the formulated criterion. The diagram allows designing concrete structures with optimal reinforcement. For practical implementation, an approach has been proposed to evaluate the carrying capacity of reinforced concrete columns using the determined strain parameters. The application of the developed method is illustrated by an example.

Keywords Reinforced Concrete Member, Biaxial Bending, Ultimate Concrete Strain, Section Analysis, Bearing Capacity

1. Introduction

Reconstruction and rebuilding of the housing stock are among the priority tasks of construction in Ukraine today. One of the key indicators in this process is the high speed of building construction, the use of heat-efficient materials for enclosing structures, the free transformation of the internal space of premises, as well as low cost. It is possible to ensure that buildings comply with the mentioned modern requirements by using prefabricated frame structural systems of reinforced concrete or composite steel and concrete structures [1-3].

A large number of structural systems of prefabricated buildings are based on frames. At the same time, there is an obvious tendency to improve them in the direction of reducing the floor thickness. The most advanced frame structural systems with flat floors are currently the most effective and demanded for the construction of residential buildings.

The mentioned structural system is endowed with all the listed above advantages. This undoubtedly poses special challenges for the development of a thorough calculation of the carrying capacity of the elements of this structural system to ensure reliable operation.

The one-way slab-on-beams floor systems in the first reinforced concrete frames of buildings were arranged to ensure that the load was applied to the columns in one of the inertia planes of their sections. Such column loading was implemented by arranging the main beams of the floor so that the axes of the beams matched the inertia axes of the column section (Figure 1a). Therefore, the resultant external load was forced (theoretically) to lie in the plane of the inertia axis of the column section. In the precast version of the frame, such application of the load to the columns was ensured by the presence of symmetrical consoles in the precast columns.

Improvement of the design of the floors by converting one-way floors into two-way floors led to the application of the external load to the column in two planes (Figure 1b). In this case, the total resultant external load, as a rule, did not lie in the plane of any axes of inertia of the column section. This fact was usually ignored, and therefore, the calculation of the carrying capacity of the columns was made for plane loading in two orthogonal planes.

The creation of flat slab floors, and later their improvement by eliminating capitals, led to complex deformation of reinforced concrete columns, namely, their biaxial bending (Figure 1c). Such a transformation of the frame structural system contributed to its even greater functioning as a single whole, with the impossibility of separation into components – flat frames. Since such separation distorts the nature of the frame as a whole, as well as its individual elements under load, this leads to the calculation of columns for axial loading with bending moment in two separate planes. Since in such a calculation, compressed areas in orthogonal directions are superimposed and taken into account twice, this approach leads to an overestimation of the carrying capacity of biaxially bent columns.

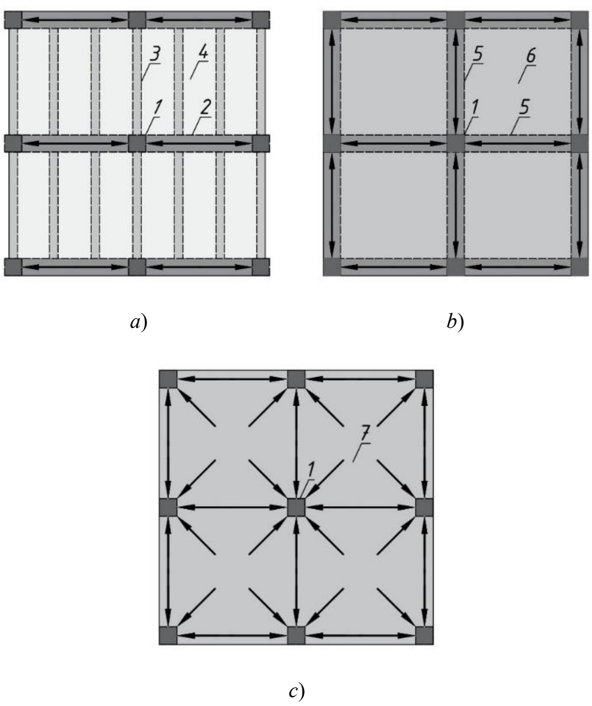

Figure 1. Directions of load transfer to columns: *a*) – from one-way slab-on-beams floor systems; *b*) – from two-way slab-on-beams floor systems; *c*) – from flat plate floor; 1 – column; 2 – main beam; 3 – secondary beam; 4 – one-way slab; 5 – beam; 6 – two-way slab; 7 – flat plate floor

Direct design of columns for biaxial bending was developed, for example, in [4]. Some simplifications related to the work of concrete were adopted, namely: uniform distribution of stresses in the compressed area and the deformation criterion of strength. This criterion sets the ultimate values of concrete strains at the moment of failure as fixed depending on the concrete grade. This approach allows calculations to be made with the necessary engineering accuracy. Meanwhile, it does not fully take into account the properties of concrete deformation in the area of the descending branch of its stress-strain diagram, which are realized precisely under the condition of biaxial bending in the composition of a reinforced concrete element.

Determination of the ultimate values of concrete strain in the composition of reinforced concrete elements, in particular columns, is also necessary for nonlinear analysis of their work using modern software packages operating on the basis of the finite element method [5,6].

For determining ultimate compressive strain, various methods are used, for example, the energy balance method [7,8], the ultimate strain criteria [9], the meta-heuristic algorithm [10] and others [11-13].

This issue is also given attention in an attempt to create a general theory of reinforced concrete, which can be used for calculations of various types of structures under the action of various internal factors [14,15]. The study of the ultimate strains of concrete in the composition of various types of structures is also devoted to works [16-18], which concern the study of composite steel and concrete structures and their self-stressing.

When designing biaxially bent columns of frame buildings, a situation is characteristic when, at the moment of destruction, the stresses in the most tensile reinforcement do not achieve the yield point, i.e. it works elastically. For such a case of calculation, determining the ultimate strains of concrete is an urgent task.

The problem was addressed in studies reported in works [19,20] using the maximum value of the product of the current stress σ_c and the current strain ε_c.

The disadvantage of this approach is that it is based on the identification of two different phenomena in physical essence: the concrete strain during axial compression of a concrete specimen and the concrete strain in the composition of a reinforced concrete member. As a result, the proposed criterion has inherent disadvantages of the deformation criterion, namely, the regulation of the ultimate strain values depending only on the concrete grade without taking into account other factors that have been proven to have an impact on the specified strains.

The solution to the existing problem of calculating the ultimate strain of concrete in the compressed area in reinforced concrete elements, where the reinforcement stress at the moment of failure is not a fixed value, is carried out in [21]. The criterion of the maximum strength of the compressed concrete area is formulated.

This criterion, together with the extreme criterion of concrete strength, can be used as the basis for the method of calculating the strength of reinforced concrete members. The dependencies derived in [21] allow finding the ultimate values of fiber strain of concrete for reinforced concrete members of rectangular cross-section at plain bending. There are no propositions yet in solving the current problem for elements under biaxial bending. Therefore, the calculation of the ultimate values of concrete strain in biaxially bent reinforced concrete columns at reinforcement operation in the elastic stage is a relevant problem. Its solution will allow performing calculations of the carrying capacity of such structures by direct analytical calculation with full consideration of the elastic-plastic properties of concrete.

2. Methods

The finite element modeling of the frame of a 16-story residential building is performed to study the operation of columns in the spatial frame with flat plate floors and stiffening diaphragms. The modeling is carried out using the RFEM6.04.0008 software package (Trial Version for teachers was used). Geometrically nonlinear calculation using the Picard iterative method is applied. Linear rod elements are used for modelling columns, and flat quadrangular elements with dimensions of 0.5×0.5 m were used to model slabs and diaphragms. The connection of slabs to columns is assumed to be rigid. To reduce the effect of stress singularities above columns, rigid inserts with the dimensions of the column cross-section 400×400 mm are created. A bilinear stress-strain law is assumed for concrete and reinforcement.

The results of the simulation showing the values of internal forces in the columns of the 16th floor are presented in Table 1.

Table 1. The internal forces values in the sections of the considered columns of the 16th floor of the spatial building frame

Column location (Figure 2)	N_{Ed}, kN	$M_{y,Ed}$, kN·m	$M_{z,Ed}$, kN·m
Corner (1)	197.58	53.31	-54.02
Extreme (2)	212.59	-5.94	-9.40
Average (3)	171.88	-72.14	73.10

A frame model was created with a 6×6 m column grid (Figure 2) with a floor height of 3 m. The study took into account the dead load from the self-weight of the 160 mm thick floor, which was 4 kN/m^2, and a live load on the floor of 5 kN/m^2 was set. The variable wind load in the horizontal plane with an intensity of W_0 = 1.5 kN/m was also taken into account, which was applied in the form of a linearly distributed load at the floor levels.

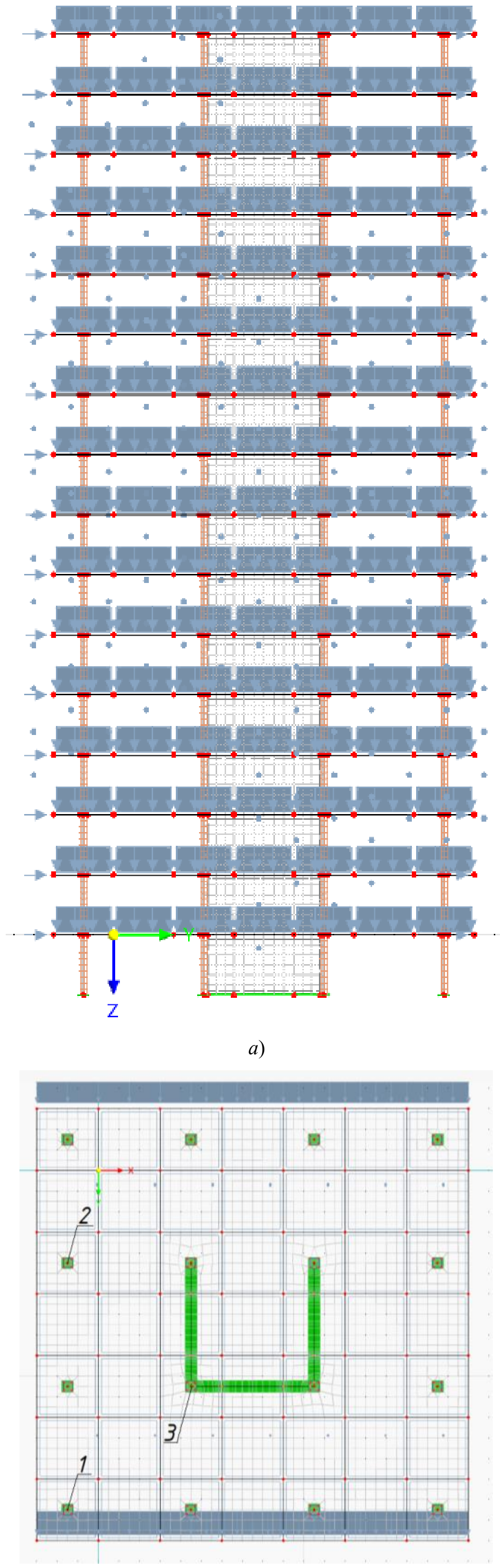

Figure 2. Model of a 16-story frame with flat plate floors: *a*) – view along the X axis; *b*) – top view; 1 – corner column under study; 2 – end column under study; 3 – middle column under study

When analyzing the obtained data, it was found that the columns of the 16th floor operate in biaxial bending under the condition of non-uniform deformation of their sections. The values of eccentricities $e_b = M_{y,Ed} / N_{Ed}$ and $e_h = M_{z,Ed} / N_{Ed}$ of the application of longitudinal force are out of the section core, determined for reinforced concrete elements considering the nonlinear properties of concrete. In this case, it is possible to implement a triangular shape of the compressed area for corner columns, trapezoidal or pentagonal for extreme columns and middle columns. Thus, it is shown that the columns of the precast frame structural system of buildings are characterized by biaxial bending with all possible shapes of the compressed zone in their rectangular cross-section.

On the basis of a stress-strain model for biaxially bent columns experiencing nonuniform deformation, a design scheme was formulated to represent the stresses, strains, and internal forces in the cross-section of a biaxially bent column characterized by a triangular compressed concrete region (Figure 3).

A method for evaluating the load-bearing capacity of a column with a triangular compressed concrete zone under biaxial bending has been developed for cases of external loading with small eccentricities, assuming that the tensile reinforcement does not reach its yield stress at the ultimate limit state.

In this case, a fractional-rational function according to [22] is adopted to describe the work of compressed concrete. The stress-strain state of the reinforcement is described by a bilinear diagram [22].

To derive the design formulas, the general equations of equilibrium in the coordinate plane YOZ (Y axis intersects the most compressed edge of the section and is orthogonal to the neutral axis) are presented (Figure 3):

$$\sum Z = 0: \sum_{i=1}^{n} N_{si} + N_{Ed} - N_c = 0; \tag{1}$$

$$\sum M_A = 0: \sum_{i=1}^{n} N_{si}\left(y_{Ed} - y_{si}\right) + N_c\left(y_{Ed} - y_{Nc}\right) = 0, \tag{2}$$

where N_c – resultant in the compressed concrete area;
N_{si} – force in the i-th rebar;
n – amount of rebars in the cross-section;
N_{Ed} – longitudinal force from external load;
y_{Ed}, y_{Nc}, y_{si} – coordinates of N_{Ed}, N_c, N_{si} application points, respectively.

Determination of the force N_c and the coordinate y_{Nc} for the triangular shape of the compressed zone is carried out:

$$N_c = \int_0^X \int_{x_1}^{x_2} \frac{f_{cd}\eta_{(1)} y\left(kx - \eta_{(1)} y\right)}{x\left(x + (k-2)\eta_{(1)} y\right)} dxdy = \frac{f_{cd} X^2}{\sin 2\theta}\omega_1; \tag{3}$$

$$y_{Nc} = \frac{S_c}{N_c} = \frac{\varphi_1}{\omega_1} X; \tag{4}$$

$$S_c = \int_0^X \int_{x_1}^{x_2} \frac{f_{cd}\eta_{(1)} y\left(kx - \eta_{(1)} y\right)}{x\left(x + (k-2)\eta_{(1)} y\right)} ydxdy = \frac{f_{cd} X^3}{\sin 2\theta}\varphi_1, \tag{5}$$

where $x_1 = \dfrac{y - X}{\operatorname{tg}\theta}$; $x_2 = (X - y)\operatorname{tg}\theta$;

θ – angle between the neutral axis and the horizontal axis of symmetry of the cross-section;

ω_1, φ_1 – coefficients of fullness of the stress diagram in the compressed concrete area and relative value of the coordinate of N_c application point, respectively

$$\left.\begin{aligned} \omega_1 &= \frac{\left(c(c - 2\ln c) - 1\right)(k-1)^2}{(k-2)^4 \eta_{(1)}^2} - \frac{\eta_{(1)}}{3(k-2)}, \quad k \neq 2 \\ \omega_1 &= \frac{\eta_{(1)}}{6}\left(4 - \eta_{(1)}\right), \; k = 2 \end{aligned}\right\}; \tag{6}$$

$$\left.\begin{aligned} \varphi_1 &= \frac{(k-1)^2(1 - c^2 + 2c\ln c)}{(k-2)^5 \eta_{(1)}^3} - \frac{c - 2(k-1)^2 - 1}{6(k-2)^2}, \; k \neq 2 \\ \varphi_1 &= \frac{\eta_{(1)}}{30}\left(10 - 3\eta_{(1)}\right), \; k = 2 \end{aligned}\right\}, \tag{7}$$

where $c = 1 + k\eta(1) - 2\eta(1)$.

It is proposed to determine the equivalent N_{si} by the expression:

$$N_{si} = \sigma_{si} A_{si}, \tag{8}$$

where A_{si} – area of the i-th reinforcement rod.

The stress σ_{si} is determined depending on the strain ε_s of reinforcement.

$$\text{at } 0 \le \varepsilon_s < \varepsilon_{s0}, \; \sigma_s = E_s \varepsilon_s; \tag{9}$$

$$\text{at } \varepsilon_{s0} \le \varepsilon_s \le \varepsilon_{uk}, \; \sigma_s = f_{yd} \tag{10}$$

where $\varepsilon_{s0} = f_{yd} / E_s$ – reinforcement strain at the design yield stress f_{yd};

ε_{uk} – characteristic value of the ultimate strain of reinforcement;

$$\varepsilon_{si} = \frac{\varepsilon_{c(1)} y_{si}}{X}. \tag{11}$$

The coordinate y_{si} of N_{si} application point in the rebars in the section of the column (Figure 3) is determined by

$$y_{si} = X - y_{0,si}\cos\theta - x_{0,si}\sin\theta. \tag{12}$$

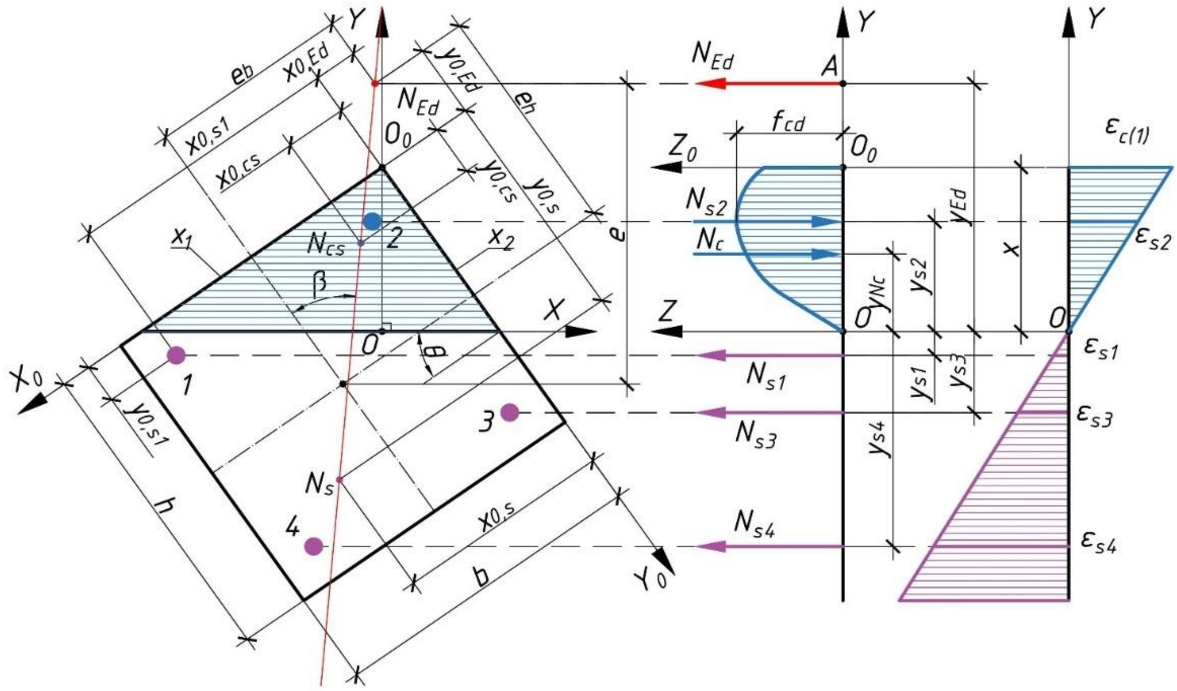

Figure 3. Design scheme of stresses, strains and forces in the cross-section of a biaxially bent reinforced concrete column with the triangular compressed area

The coordinate of the N_{Ed} application point is calculated by the dependence

$$y_{Ed} = X + e - 0{,}5b\sin\theta - 0{,}5h\cos\theta\,, \tag{13}$$

in which e – eccentricity of N_{Ed} application about the geometric center of the column cross-section, $e = e_b\sin\theta + e_h\cos\theta$;

e_b, e_h – eccentricities of N_{Ed} application about the geometric center of the column cross-section in the direction b and h, respectively;

b, h – breadth and height of the column cross-section, respectively.

After substituting (3) – (13) into equations (1) – (2), two equations with four unknown parameters, namely: η, X, θ, N_{Ed} are obtained. Two additional conditions are applied for an analytical solution to the problem.

The first condition is the application of the criterion of complete concrete strength of the compressed area of a reinforced concrete member [21] to determine the fiber strain of concrete $\varepsilon_{c(1)}$ (η) in the composition of a biaxially bent column in the ultimate limit state. In its physical essence, it is a criterion for the destruction of concrete in the compressed area at the extreme value of the force N_c, i.e. at the maximum filling of compressive stress diagram. The criterion is determined by studying the function $N_c = f(\varepsilon_{c(1)})$, at the extremum, which can also be equivalent to studying the function of the coefficient ω_1 of the fullness of the compressive stress diagram in concrete of the compressed area in the form of the condition:

$$\frac{\partial\omega_1}{\partial\eta} = 0\,, \tag{14}$$

where $\eta = \varepsilon_{c(1)} / \varepsilon_{c1}$,

ε_{c1} – compressive strain in the concrete at the peak stress.

The application of the criterion is acceptable provided that the corresponding strain ε_s of the tensile reinforcement of the reinforced concrete column is in a certain range of its deformation $0 < \varepsilon_s \le \varepsilon_{s0}$, which corresponds to the elastic work of the reinforcement.

The solution to equation (14) is a dependence that allows determining the ultimate strain of concrete depending on the coefficient k of concrete physical and mechanical properties

$$\eta_o = \frac{e^{\lambda} - 1}{k - 2},\ k \ne 2\,, \tag{15}$$

where λ is the root of the characteristic equation

$$e^{\lambda}\left(2k(k-2)(\lambda-1)+2\lambda-3\right)-e^{3\lambda}+2e^{2\lambda}+2k(k-2)+2=0\,.$$

For the value $k = 2$

$$\frac{\partial\omega_1}{\partial\eta} = \frac{2-\eta}{3}. \tag{16}$$

Representing (16) in the form of an equation according to (14), it is obtained $\eta_o = 2.0$.

The results of analyzing expression (14) at its extremum are presented in the diagram of the ultimate values of concrete fiber strain levels of triangular compressed area of reinforced concrete elements (Figure 4). The resulting diagram may be used in calculations.

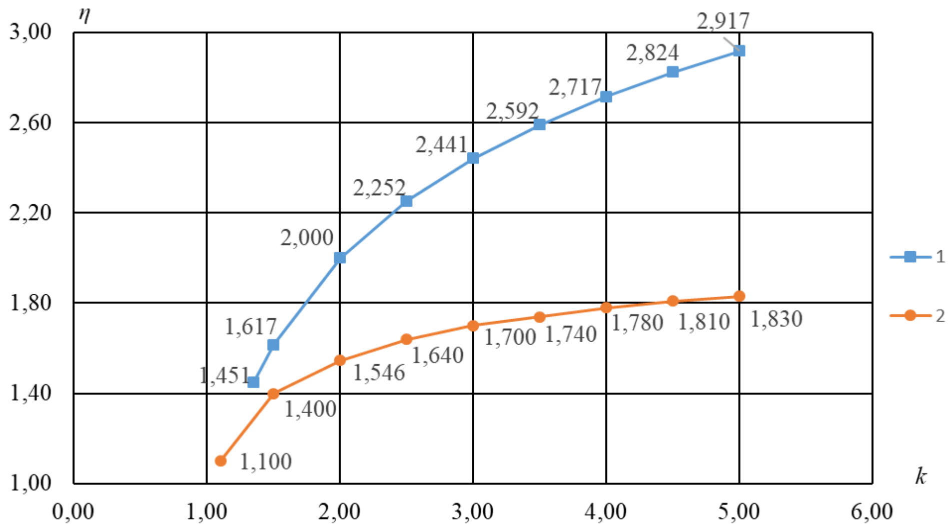

Figure 4. Diagrams of the ultimate values of the concrete fiber strain levels of the triangular compressed zone of the cross-section of a biaxially bent column: 1 – η_o according to (14) at $\sigma_s < f_{yd}$ and 2 – η_u according to the extreme criterion [23] at $\sigma_s = f_{yd}$, depending on the parameter k

Table 2. The values of the parameters η_o, ω_1 and φ_1 for biaxially bent columns with a triangular form of concrete compressed area at $\sigma_s < f_{yd}$, depending on the concrete grade C

Concrete grade	C12/15	C16/20	C20/25	C25/30	C30/35	C32/40	C35/45	C40/50	C45/55	C50/60
η_o	2,500	2,426	2,350	2,296	2,250	2,203	2,163	2,135	2,088	2,038
ω_1	0,765	0,752	0,738	0,728	0,719	0,710	0,702	0,696	0,686	0,675
φ_1	0,286	0,284	0,281	0,279	0,277	0,275	0,274	0,273	0,271	0,268
φ_1/ω_1	0,374	0,377	0,381	0,383	0,385	0,388	0,390	0,39	0,394	0,398

The parameters ω_1 and φ_1, which depend on $\eta = \eta_o$ at the ultimate limit state, are calculated according to (6) and (7), taking into account the data of graph 1 in Figure 4, and are summarized in Table 2 depending on the concrete grade C for practical application.

Based on the adopted theorem, the angle β of the intersection line of the force plane with the column cross-section plane (Figure 3) can be expressed through the coordinates of the intersection points of the action lines of the forces performing in this plane with the column cross-section plane in the $X_0O_0Y_0$ coordinate system as follows:

$$\mathrm{tg}\beta = \frac{x_{0,cs} - x_{0,s}}{y_{0,cs} - y_{0,s}}, \quad (17)$$

$$\mathrm{tg}\beta = \frac{x_{0,Ed} - x_{0,s}}{y_{0,Ed} - y_{0,s}}, \quad (18)$$

where $x_{0,s}$, $y_{0,s}$ – coordinates of the intersection point of N_s action line with the cross-sectional plane of the column, which are determined by the expressions

$$x_{0,s} = \frac{\sum_{j=1}^{m} \sigma_{sj} A_{sj} x_{0,sj}}{\sum_{j=1}^{m} \sigma_{sj} A_{sj}}, \quad (19)$$

$$y_{0,s} = \frac{\sum_{j=1}^{m} \sigma_{sj} A_{sj} y_{0,sj}}{\sum_{j=1}^{m} \sigma_{sj} A_{sj}}, \quad (20)$$

where σ_{sj} – stress in the j^{th} tensile rebar;
A_{sj} – area of the j^{th} tensile rebar;
$x_{0,sj}$, $y_{0,sj}$ – coordinates of the location of the j^{th} tensile rebar in system $X_0O_0Y_0$;
m – amount of tensile rebars;
$x_{0,cs}$, $y_{0,cs}$ – coordinates of the intersection point of the action line of N_{cs} with the cross-sectional plane of the column, which are calculated according to the dependencies

$$x_{0,cs} = \frac{N_c x_{0,c} + N'_s x'_{0,s}}{N_c + N'_s}, \quad (21)$$

$$y_{0,cs} = \frac{N_c y_{0,c} + N'_s\, y'_{0,s}}{N_c + N'_s}, \tag{22}$$

where N_c, N'_s – resultant forces in compressed concrete and compressed rebars, respectively;
$x_{0,c}$, $y_{0,c}$ – coordinates of N_c application point, which are determined depending on its shape;
$x'_{0,s}$, $y'_{0,s}$ – coordinates of N'_s application point, which are determined by the expressions

$$x'_{0,s} = \frac{\sum_{k=1}^{n-m} \sigma'_{sk} A'_{sk}\, x'_{0,sk}}{\sum_{k=1}^{n-m} \sigma'_{sk} A'_{sk}}, \tag{23}$$

$$y'_{0,s} = \frac{\sum_{k=1}^{n-m} \sigma'_{sk} A'_{sk}\, y'_{0,sk}}{\sum_{k=1}^{n-m} \sigma'_{sk} A'_{sk}}, \tag{24}$$

in which σ'_{sk} – stress in the k^{th} compressed rebar;
A'_{sk} – area of the k^{th} compressed rebar;
$x'_{0,sk}$, $y'_{0,sk}$ – coordinates of the location of the k^{th} compressed rebar in the $X_0O_0Y_0$ system;
$n – m$ – amount of compressed rebars in the cross-section;
$x_{0,Ed}$, $y_{0,Ed}$ – coordinates of the intersection point of N_{Ed} action line with the cross-sectional plane of the column.

Equating dependencies (17) and (18), it is obtained

$$\frac{x_{0,cs} - x_{0,s}}{y_{0,cs} - y_{0,s}} = \frac{x_{0,Ed} - x_{0,s}}{y_{0,Ed} - y_{0,s}}. \tag{25}$$

The coordinates ($x_{0,c}$, $y_{0,c}$) of the point of intersection of the action line of N_c with the cross-sectional plane of the column may be determined by the formulas:

$$x_{0,c} = \frac{S_{c,Y_0}}{N_c}, \tag{26}$$

$$y_{0,c} = \frac{S_{c,X_0}}{N_c}, \tag{27}$$

in which $S_{c,Y0}$, $S_{c,X0}$ are the static moments of the spatial figure of the compressed concrete stress diagram about the axes Y_0 and X_0, respectively (Figure 3).

Using the stress diagram function over the compressed concrete area, formulas were obtained for determining the coordinates of N_c application point in the triangular compressed area of concrete in the system $X_0Y_0Z_0$ in the form:

$$x_{0,c} = \frac{X}{\sin\theta}\frac{\omega_1 - \varphi_1}{2\omega_1}, \tag{28}$$

$$y_{0,c} = \frac{X}{\cos\theta}\frac{\omega_1 - \varphi_1}{2\omega_1}. \tag{29}$$

Since the coordinates (x_{0c}, y_{0c}) are determined by the angle θ according to (28) and (29), after their substitution in the dependences (17) and (18), and then into the equation (25), an implicit relationship describing the function $\theta = f(\beta)$ is derived. This function enables the evaluation of the neutral axis angle θ of inclination, assuming a triangular configuration of the compressed concrete area.

3. Results

The joint solution of equations (4) and (5), taking into account the data in Table 2 as well as the dependencies defined by relations (25), (28), and (29), makes it possible to determine the complete set of unidentified parameters characterizing the stress–strain state of a biaxially bended reinforced concrete column in a cross-section with a triangular compressed zone. Consequently, the bearing capacity of such elements can be evaluated under the condition that the tensile reinforcement remains within the elastic range. The applicability of the proposed calculation approach is illustrated through a numerical example.

3.1. Example

Given: a corner (1) column of a 16-story frame building with flat plate floors (Figure 2b) with a square cross-section of dimensions b = 400 mm, h = 400 mm (Figure 5), made of concrete C20/25 [22] (f_{cd} = 14.5 MPa, E_{cd} = 23 GPa) reinforced with 4Ø25A400C (A_{si} = 490.9 mm^2, f_{yd} = 435 MPa, E_s = 210 GPa, ε_{s0} = 0.00207). The rebars are located at a distance a = 50 mm from cross-section faces. The eccentricities of N_{Ed} application are $e_b = M_{y,Ed} / N_{Ed}$ = = 53.31 / 197.58 = 270 mm and $e_h = M_{z,Ed} / N_{Ed}$ = 54.02 / 197.58 = 273 mm (Table 1).

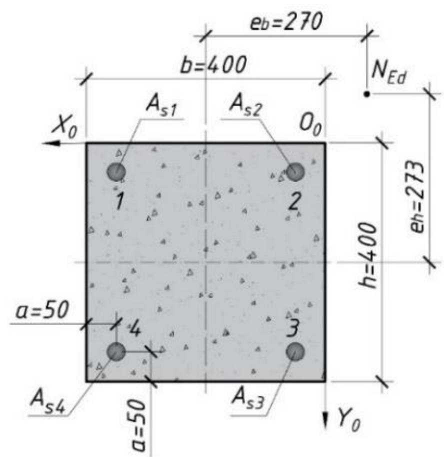

Figure 5. Column cross-section

Find: the longitudinal force N_{Rd} that the column can withstand.

The possible shape of the compressed area of concrete is a triangle or a trapezoid. The calculation for the triangular shape of the compressed concrete area is performed. The neutral axis depth and the angle θ of neutral axis inclination are determined by the joint solution of equations (5) and (25), expressing their components in terms of X and θ. In this case, the Excel software package "Solution Search" add-in was used.

As a result of the calculation, it is obtained: X = 237.84 mm^2; θ = 44.84 º.

The shape of the compressed concrete area is a triangle, because: $X < b\sin\theta$ = 400sin44.84 º= 282.05 mm, $X < h\cos\theta$ = = 400cos44.84 º= 283.63 mm.

The stresses and strains in the rebars are checked.

Strain of rod 1 according to (11)

$$\varepsilon_{s1} = \frac{\varepsilon_{c(1)} y_{s1}}{X} = \frac{0,00404 \cdot (-44,41)}{237,84} = -0,00075,$$

where by (12) using Figure 5

$$y_{s1} = X - x_{0,s1}\sin\theta - y_{0,s1}\cos\theta = 237,84 - 350\sin 44,84° - -50\cos 44,84° = -44,41 \text{ mm}.$$

Since $|\varepsilon_{s1}| = |-0,00075| < \varepsilon_{s0} = 0,00207$, then the stress in rod 1 does not reach the yield point and is determined by (9)

$$\sigma_{s1} = E_{s1}\varepsilon_{s1} = 210000 \cdot (-0,00075) = -158,50 \text{ MPa}.$$

Strain of rod 2 according to (11)

$$\varepsilon_{s2} = \frac{\varepsilon_{c(1)} y_{s2}}{X} = \frac{0,00404 \cdot 167,13}{237,84} = 0,00284$$

where by (12) using Figure 5

$$y_{s2} = X - x_{0,s2}\sin\theta - y_{0,s2}\cos\theta = 237,84 - 50\sin 44,84° - -50\cos 44,84° = 167,13 \text{ mm}.$$

Since $|\varepsilon_{s2}| = |0,00284| > \varepsilon_{s0} = 0,00207$ then the stress in rod 2 reaches the yield point and is determined by (10) $\sigma_{s2} = f_{yd} = 435$ MPa.

Strain of rod 3 according to (11)

$$\varepsilon_{s3} = \frac{\varepsilon_{c(1)} y_{s3}}{X} = \frac{0,00404 \cdot (-45,60)}{237,84} = -0,00078,$$

where by (12) using Figure 5

$$y_{s3} = X - x_{0,s3}\sin\theta - y_{0,s3}\cos\theta = 237,84 - 50\sin 44,84° - -350\cos 44,84° = -45,60 \text{ mm}.$$

Since $|\varepsilon_{s3}| = |-0,00078| < \varepsilon_{s0} = 0,00207$, then the stress in rod 3 does not reach the yield point and is determined by (9)

$$\sigma_{s3} = E_{s3}\varepsilon_{s3} = 210000 \cdot (-0,00078) = -162,73 \text{ MPa}.$$

Strain of rod 4 according to (11)

$$\varepsilon_{s4} = \frac{\varepsilon_{c(1)} y_{s4}}{X} = \frac{0,00404 \cdot (-257,13)}{237,84} = -0,00437$$

where by (12) using Figure 5

$$y_{s4} = X - x_{0,s4}\sin\theta - y_{0,s4}\cos\theta = 237,84 - 350\sin 44,84° - -350\cos 44,84° = -257,13 \text{ mm}.$$

Since $|\varepsilon_{s4}| = |-0,00437| > \varepsilon_{s0} = 0,00207$ then the stress in rod 4 reaches the yield point and is determined by (10)

$$\sigma_{s4} = -f_{yd} = -435 \text{ MPa}.$$

It is obtained that the stresses in two of the three tensioned rebars (1 and 3) do not reach the yield point, therefore the use of the coefficients ω_1 and φ_1 (Table 2), determined using the criterion of complete use of the concrete strength of the compressed area (14), is confirmed for the problem under consideration.

The coordinate of N_{Ed} application of the external longitudinal force according to (13):

$$y_{Ed} = X + e - 0,5b\sin\theta - 0,5h\cos\theta = 237,84 + 383,96 - -0,5 \cdot 400\sin 44,84° - 0,5 \cdot 400\cos 44,84° = 338,96 \text{ mm},$$

where eccentricity $e = e_b\sin\theta + e_h\cos\theta$ = 270sin44.84 º+ 273cos44.84 º= 383.96 mm.

The coordinate of the point of N_c application is calculated by (4):

$$y_{Nc} = \frac{\varphi_1}{\omega_1} X = \frac{0,277}{0,738} \cdot 237,84 = 89,27 \text{ mm}.$$

The longitudinal force N_{Rd}, which the column can perceive under given conditions, is determined by taking $N_{Rd} = N_{Ed}$, from equation (2), in which by (3) and (8)

$$N_c = \frac{f_{cd} X^2 \omega_1}{\sin 2\theta} = \frac{14,5 \cdot 237,84^2 \cdot 0,738}{\sin 2 \cdot 44,84°} = 605333 \text{ N} = 605,33 \text{ kN},$$

$$N_{s1} = \sigma_{s1} A_{s1} = -158,50 \cdot 490,90 = -77807 \text{ N} = -77,81 \text{ kN},$$

$$N_{s2} = \sigma_{s2} A_{s2} = 435 \cdot 490,90 = 213542 \text{ N} = 213,54 \text{ kN},$$

$$N_{s3} = \sigma_{s3} A_{s3} = -162,73 \cdot 490,90 = -79882 \text{ N} = -79,88 \text{ kN},$$

$$N_{s4} = \sigma_{s4} A_{s4} = -435 \cdot 490,90 = -213541 \text{ N} = -213,54 \text{ kN}.$$

By equation (2)

$$N_{Rd} = N_c + \sum_{i=1}^{n} N_{si} = 605,33 - 77,81 + 213,54 - 79,88 - -213,54 = 447,64 \text{ kN}.$$

The longitudinal force that may be perceived by the column, calculated by the finite element method using the RFEM6.04.0008 software package (Trial Version for teachers was used), is N_{Rd} = 424.19 kN at X = 243.9 mm, θ = 44.67 º.

4. Conclusions

1. The nonlinear model of the stress–strain behaviour of reinforced concrete columns subjected to biaxial bending has been refined through the introduction of a criterion ensuring full utilization of concrete strength. This criterion, unlike the extreme criterion, enables a comprehensive assessment of the load-bearing capacity of such columns in flat-plate frame structural systems, while considering the physical and mechanical characteristics of concrete as a pseudoplastic material and the condition of elastic behaviour of the tensile reinforcement.
2. The concrete fiber strains in biaxially bent columns with a triangular compressed concrete zone at the ultimate limit state have been analytically determined using the criterion of complete utilization of concrete strength in reinforced concrete elements.
3. The nonlinear deformation model of the stress–strain state of reinforced concrete elements under biaxial bending has been enhanced and generalized to a universal form, allowing simulation that accounts for the plastic properties of concrete.
4. Analytical relationships have been derived for the calculation of equivalent internal forces in the compressed concrete zone and the coordinates of their points of application, depending on the geometry of the compressed area and the cross-sectional shape. These relationships are intended for engineering evaluation of the load-bearing capacity of biaxially bent reinforced concrete columns in flat-plate frame structural systems.
5. The applicability of the proposed method for calculating column load-bearing capacity has been validated, providing a basis for its use in the design of flat-plate frame columns with an optimal degree of longitudinal reinforcement.

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