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Research paper

The Dynamic Processes Mathematical Modeling in the Traction Coupling Device from Cars to the Trailers

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Abstract

The dynamic processes mathematical modeling results in the traction car coupling with a trailer are presented. It is shown that the dissipative link presence in the coupling device prevents oscillation processes, and significantly reduces the dynamic loads during transient train movement. According to research, the design of a car trailer dynamic coupling device was proposed.

Keywords*: dynamic coupling device, dynamic loads, car, towing, trailer, vehicle.*

1. Introduction

1.1. Overview

The towing a trailer by car process is accompanied by multiple sign–loads in the coupling device, which in the system "car – trailer" is an elastic link. Under certain conditions, oscillation processes arise in this system, during which the force component variable instantaneous value can far outstrip static and inertial loads. This will have an impact on the system stability, the overload of individual traction coupling elements and the fatigue progressive phenomena development in them. It also lower their work resources, or even, to an emergency.

Consequently, to improve the reliability, durability and safety in operation of the car-trains coupling traction device design should take into account the dynamic loads.

1.2. Relevance

The vehicles and car-trains stability described in many works of the local authors, among them Volodymyr Sakhno, Mykhaylo Podryhalo and others [1, 2, 3, 4, 5, 6] and the foreign once [7, 8, 9, 10, 11].

Many scientific papers are dedicated to dynamic processes investigation during the train vehicle movement. Fundamental in this direction are works [12, 13]. So in [12], the dynamic interaction between the car and trailer is considered in different motion modes, taking into account the main constructive influence and the car-trains operational factors. Parameters of the coupling devices calculation methods and selecting elastic ties are given.

Much attention is paid to dynamic interaction between car and train links in the monograph [13]. This work considers car-trains as multi-mass system which links are connected with elastic constraints.

Research on the influence car coupling devices parameters for dynamic interaction small tangible train links is carried out in [14]. In the above-mentioned works the car-train movement mathematical models contain a significant number factors of that affect the car-train links interaction nature. However, to establish the basic dynamic laws of a car interaction with a trailer you can use the two–mass dynamic model.

The traction coupling devices constructions have recently been given a significant amount of research. In works [15, 16, 17] considerable attention was paid to the traction trailers coupling devices design. With their help dynamic fluctuations were partially extinguished. Each of them has advantages, but also they have complex design and special requirements for operation.

1.3. The aim

The article purpose is to highlight the dynamic mathematical modeling processes results that arise in the traction coupling device when towing a trailer car.

2. Main body

To study the dynamic processes that arise in the traction coupling car device with a trailer, we use the mechanical theory oscillations provisions [18, 19]. The car and trailer towed by it are represented as a two–mass dynamic model (fig. 1).

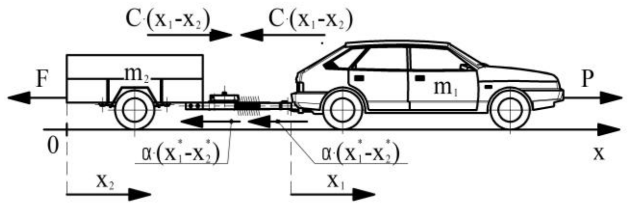

Fig. 1: Equivalent two–mass system scheme "car – trailer": m_1 – car weight, m_2 – trailer weight, x_1 – car movement, x_2 – trailer movement, P – traction force, F – trailer force, α – dissipative damper resistance, C – traction coupling device rigidity, $(x_1–x_2)$ – linear hard link movement, $(x_1`–x_2')$ – linear movement in damper

When drawing up the equivalent scheme, the following assumptions were adopted:

– a car-trains movement is considered on the horizontal path section;
– a road resistance is adopted by a constant magnitude;
– an air resistance given the low vehicle speed when towing a trailer is ignored;
– a car and trailer are considered to be absolutely solid mass bodies, m_1 and m_2 correspondingly;
– the traction coupling device rigidity C was accepted as liner magnitude in the whole possible deformations range;
– the dissipative damper resistance α is assumed to be proportional to the displacement velocity;
– there are no gaps in traction coupling device.

The 0X axis will be drawn to the car's direction.

First, to analyze the traffic system "car–trailer" with no damper, we neglect dissipative resistance forces.

We provide the car weight the m_1 name, movement – x_1, and the trailer weight – m_2 displacement – x_2.

Then, according to the Dahlberg–Lagrange principle, the "car–trailer" system can be represented as a differential equations system

$$\begin{cases} m_1 \cdot \ddot{x}_1 + C \cdot (x_1 - x_2) = P, \\ m_2 \cdot \ddot{x}_2 - C \cdot (x_1 - x_2) = -F. \end{cases} \tag{1}$$

The first system equation (1) in m_2 is multiplied, and the second – at m_1, and the first equation is sustrscted from the second

$$m_1 \cdot m_2 \cdot (\ddot{x}_1 - \ddot{x}_2) + C \cdot (m_1 + m_2) \cdot (x_1 - x_2) = P \cdot m_2 + F \cdot m_1 . \tag{2}$$

Marking the relative masses movement m_1 and m_2 through x, we get

$$m_1 \cdot m_2 \cdot \ddot{x} + C \cdot (m_1 + m_2) \cdot x = P \cdot m_2 + F \cdot m_1 . \tag{3}$$

We divide the right and left equation side (3) into m_1 and m_2. We get

$$\ddot{x} + \frac{C \cdot (m_1 + m_2)}{m_1 \cdot m_2} \cdot x = \frac{P \cdot m_2 + F \cdot m_1}{m_1 \cdot m_2} . \tag{4}$$

Equation (4) is a nonhomogeneous differential of second order equation, which describes the oscillatory processes in the "car – trailer" system.

This equation solution consists of general and partial parts, i.e.

$$x = x^* + x^{**} . \tag{5}$$

This equation solution has the form [18, 19]

$$x^* = C_1' \cdot \cos kt + C_2' \cdot \sin kt ; \tag{6}$$

$$x^{**} = \frac{P \cdot m_2 + F \cdot m_1}{m_1 \cdot m_2} \cdot \frac{1}{k^2} , \tag{7}$$

where k – is the system's own oscillation frequency, s^{-1}

$$k^2 = \frac{C \cdot (m_1 + m_2)}{m_1 \cdot m_2} . \tag{8}$$

Taking into account (8), the equation (7) takes the form

$$x^{**} = \frac{P \cdot m_2 + F \cdot m_1}{C \cdot (m_1 + m_2)} . \tag{9}$$

Substituting the equations solution (6) and (9) into equation (5) we obtain

$$x = C_1' \cdot \cos kt + C_2' \cdot \sin kt + \frac{P \cdot m_2 + F \cdot m_1}{C \cdot (m_1 + m_2)} . \tag{10}$$

Given that the dynamic load in the elastic link

$$P_d = C \cdot x , \tag{11}$$

and having accepted

$$C_1 = C_1' \cdot x , \tag{12}$$

$$C_2 = C_2' \cdot x , \tag{13}$$

we will get

$$P_d = C_1 \cdot \cos kt + C_2 \cdot \sin kt + \frac{P \cdot m_2 + F \cdot m_1}{m_1 + m_2} . \tag{14}$$

Equation (14) describes the dynamic processes that occur in the "car–trailer" system elastic link with the mutual masses m_1 and m_2 movement.

The C_1 and C_2 values in equation (14) are constant integrals that are found from the initial conditions.

Let us consider the initial conditions that correspond to the car's movement beginning

$$t = 0 ; \ \dot{P}_d = 0 ; \ P_d = 0 . \tag{15}$$

We find the first derivative of time P_d

$$\dot{P}_d = -k \cdot C_1 \cdot \sin kt + k \cdot C_2 \cdot \cos kt . \tag{16}$$

According to the accepted initial conditions (15) from equations (14) and (16) we obtain

$$C_1 = -\frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)} , \tag{17}$$

$$C_2 = 0 . \tag{18}$$

After substituting (17) and (18) into equation (14) and mathematical transformations we obtain

$$P_d = \frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)} \cdot (1 - \cos kt) . \tag{19}$$

Equation (19) describes the change in dynamic loads during transient modes in the "car – trailer" system traction coupling device without taking into account the dissipative damping device resistance. As you can see the change in dynamic loads is oscillatory in nature. Analyzing this equation we arrive at the conclusion that the maximum (peak) load in the traction coupling device occurs when $\cos kt = -1$. In this case, the dynamic load is equal

$$P_d = 2 \cdot \frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)}. \tag{20}$$

In the case of taking into account the dissipative resistance forces to the mutual movement of the car and trailer in accordance with the d'Alembert–Lagrange principle, we obtain the following differential equations system

$$\begin{cases} m_1 \cdot \ddot{x}_1 + \alpha \cdot (\dot{x}_1 - \dot{x}_2) + C \cdot (x_1 - x_2) = P, \\ m_2 \cdot \ddot{x}_2 + \alpha \cdot (\dot{x}_1 - \dot{x}_2) - C \cdot (x_1 - x_2) = -F. \end{cases} \tag{21}$$

We will carry out with the system equations the same mathematical operations as in the first case, namely, we multiply the first system equation (21) into m_2, the second – to m_1, and subtract from the first part – second and divide all the obtained members equation into m_1 and m_2

$$\begin{gathered} (\ddot{x}_1 - \ddot{x}_2) + \alpha \cdot \frac{(m_2 - m_1)}{m_1 \cdot m_2} \cdot (\dot{x}_1 - \dot{x}_2) + \\ + C \cdot \frac{(m_1 + m_2)}{m_1 \cdot m_2} \cdot (x_1 - x_2) = \frac{P \cdot m_2 + F \cdot m_1}{m_1 \cdot m_2}. \end{gathered} \tag{22}$$

Considering that

$$\ddot{x}_1 - \ddot{x}_2 = \ddot{x}, \tag{23}$$

$$\dot{x}_1 - \dot{x}_2 = \dot{x}, \tag{24}$$

$$x_1 - x_2 = x, \tag{25}$$

we write the equation (22) in the form

$$\ddot{x} + \alpha \cdot \frac{(m_2 - m_1)}{m_1 \cdot m_2} \cdot \dot{x} + \frac{C \cdot (m_1 + m_2)}{m_1 \cdot m_2} \cdot x = \frac{P \cdot m_2 + F \cdot m_1}{m_1 \cdot m_2}. \tag{26}$$

We introduce the notation

$$2 \cdot n = \alpha \cdot \frac{(m_2 - m_1)}{m_1 \cdot m_2}, \tag{27}$$

$$k^2 = \frac{C \cdot (m_1 + m_2)}{m_1 \cdot m_2}. \tag{28}$$

By taking into account (27) and (28), equation (26) takes the form

$$\ddot{x} + 2 \cdot n \cdot \dot{x} + k^2 \cdot x = \frac{P \cdot m_2 + F \cdot m_1}{m_1 \cdot m_2}. \tag{29}$$

This equation solution consists of general and partial parts

$$x = x^* + x^{**}. \tag{30}$$

The partial solution (30) will have the form of equation (9). The general solution (30) will depend on the quantities ratio n and k.
Consider the case when n equals k, as recommended in the suspension shock absorbers design [20]. In this case, the masses oscillations are absent, and their movement is aperiodic by nature.

When n = k, the general equation solution (29) has the form

$$x = e^{-nt} \cdot (C_1' \cdot t + C_2') + \frac{P \cdot m_2 + F \cdot m_1}{C \cdot (m_1 + m_2)}. \tag{31}$$

Given (11), we obtain

$$P_d = e^{-nt} \cdot C_1' \cdot C \cdot t + e^{-nt} \cdot C_2' \cdot C + \frac{P \cdot m_2 + F \cdot m_1}{C \cdot (m_1 + m_2)} \cdot C. \tag{32}$$

Let's denote

$$C_1 = C_1' \cdot C, \tag{33}$$

$$C_2 = C_2' \cdot C. \tag{34}$$

Then equation (32) will look like

$$P_d = e^{-nt} \cdot C_1 \cdot t + e^{-nt} \cdot C_2 + \frac{P \cdot m_2 + F \cdot m_1}{m_1 + m_2}. \tag{35}$$

Let us consider the dynamic system at the same initial conditions as in the study case without taking into account the dissipative damper resistance, $t = 0$; $\dot{P}_d = 0$; $P_d = 0$.
Let us find the first derivative of time P_d

$$\dot{P}_d = -n \cdot e^{-nt} \cdot C_1 \cdot t + e^{-nt} \cdot C_1 - n \cdot e^{-nt} \cdot C_2. \tag{36}$$

Taking into account the initial conditions from (35) and (36), we obtain

$$C_1 = -\frac{n \cdot (P \cdot m_2 + F \cdot m_1)}{(m_1 + m_2)}, \tag{37}$$

$$C_2 = -\frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)}. \tag{38}$$

After substituting (37) and (38) into equation (35) and mathematical transformations we obtain

$$P_d = \frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)} \cdot (1 - e^{-nt} \cdot (n \cdot t + 1)). \tag{39}$$

Equation (39) describes the change in dynamic loads during the transition modes in the "car – trailer" system traction coupling device, taking into account the dissipative damping resistance device. As we see, the change in dynamic loads is aperiodic by nature.
To calculate and analyze the dynamic efforts in the traction coupling device, we accept the VAZ–21113 car, the full weight of which is m_1 = 1580 kg [21].
According to the automobile trains operation conditions, the trailer weight should not exceed the car weight half [22], that m_2 = 790 kg. Take for the calculation the BN–20 trailer, which total weight is 700 kg [23].
Calculation will begin with the traction forces determination and resistance to the car. Power balance equation has the form

$$P = F + P_W + P_I, \tag{40}$$

where P – car thrust force, N; F – trailer resistance force, N; P_W – aerodynamic resistance force acting on the vehicle, N; P_I – inertia force, N.

After analyzing the initial conditions, we can conclude that the P_W and P_I forces depend primarily on the car–trains speed, so when they start, they have a value close to 0 and they can be neglected.
At the beginning of the motion the car traction force P, N, [24] will be

$$P = \frac{M_{d\max} \cdot i_1 \cdot i_{mt} \cdot \eta_{tr}}{r_w}, \quad (41)$$

where M_{dmax} – maximum torque value of the car engine, N · m, M_{dmax} = 130; i_1 – first position gearbox ratio transmission, i_1 = 3,636; i_{mt} – main transmission gear ratio, i_{mt} = 3,937 [21]; η_{tr} – efficiency transmission, η_{tr} = 0.9 [24]; r_k – the car wheel radius, m, we accept for the calculation the static radius r_k = 0,288 [21].

$$P = \frac{130 \cdot 3,636 \cdot 3,937 \cdot 0,9}{0,288} = 5815,4. \quad (42)$$

The trailer resistance force F, N

$$F = G_{trailer} \cdot f \cdot \cos\beta, \quad (43)$$

where β – the road angle, taking a horizontal way, β = 0º; f – resistance trailer wheels coefficient, f = 0,1 [25]; $G_{trailer}$ – the maximum possible trailer weight, N.

$$G_{trailer} = m_2 \cdot g, \quad (44)$$

where g – gravity acceleration, m / s^2, g = 9,81.

$$G_{trailer} = 700 \cdot 9,81 = 6867, \quad (45)$$

$$F = 6867 \cdot 0,1 \cdot 1 = 686,7. \quad (46)$$

The traction coupling device rigidity C, N / m is

$$C = \frac{P_{d\max}}{\Delta l}, \quad (47)$$

where P_{dmax} – the maximum dynamic force value, N; Δl – traction coupling device elongation, m.
According to (19), the maximum dynamic value effort will occur when the condition is fulfilled $\cos kt = -1$. Then (19) will look like

$$P_{d\max} = 2 \cdot \frac{P \cdot m_2 + F \cdot m_1}{(m_1 + m_2)}, \quad (48)$$

$$P_{d\max} = 2 \cdot \frac{5815,4 \cdot 700 + 686,7 \cdot 1580}{(1580 + 700)} = 4522,6. \quad (49)$$

The traction coupling device extension we find from Hooke's law [26]

$$\Delta l = \frac{P_{d\max} \cdot a}{E \cdot S}, \quad (50)$$

where a – length traction coupling device, m, a =1,5; E – is the elastic material modulus, Pa, for steel E =210·10^9; S – cross-sectional pipe area, m^2.

$$S = b^2 - d^2, \quad (51)$$

where b – the pipe outer edge length, m, b = 0,08; d – the pipe inner rib length, m, d = 0,064.

$$S = 0,08^2 - 0,064^2 = 0,0023, \quad (52)$$

$$\Delta l = \frac{4522,6 \cdot 1,5}{210 \cdot 10^9 \cdot 0,0023} = 1,4 \cdot 10^{-5}, \quad (53)$$

$$C = \frac{4522,6}{1,4 \cdot 10^{-5}} = 3,2 \cdot 10^8. \quad (54)$$

The proper oscillations frequency k, s^{-1}, according to (28)

$$k = \sqrt{\frac{3,2 \cdot 10^8 \cdot (1580 + 700)}{1580 \cdot 700}} = 812. \quad (55)$$

Oscillation period T, s,

$$T = \frac{2 \cdot \pi}{k}, \quad (56)$$

$$T = \frac{2 \cdot 3,14}{812} = 0,008. \quad (57)$$

The calculating results of dynamic forces acting in the traction coupling device adaptation for period T are given in table 1 and are shown graphically in Fig. 2.

Table 1: The forces acting on traction coupling device during transient motion processes

t, s	P_{dfree}, N, (19)	P_d, N, (39)	F, N, (43)
0	0	0	686,7
0,01	2860,217	2261,301	686,7
0,02	4205,35	2261,301	686,7
0,03	632,6036	2261,301	686,7
0,04	1179,988	2261,301	686,7
0,05	4462,78	2261,301	686,7
0,06	2176,47	2261,301	686,7
0,07	104,7576	2261,301	686,7
0,08	3488,473	2261,301	686,7

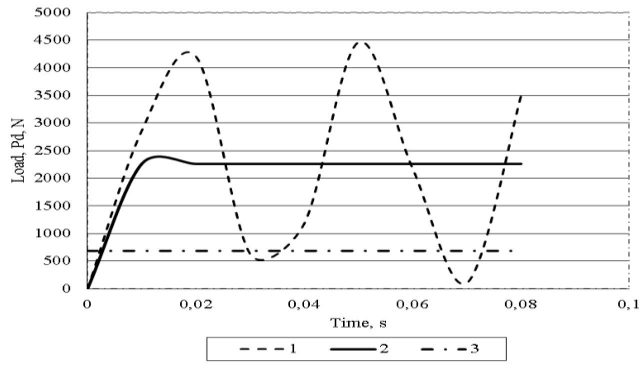

Fig. 2: Dynamic loads of a car with a trailer in the traction coupling device: 1 – without taking into account the dissipative resistance displacement forces; 2 – taking into account the dissipative resistance displacement forces; 3 – loading from rolling wheel resistance.

Analyzing Fig. 2 it can be assumed that it is advisable to install a dissipative link in the traction coupling. An example of such design is a car trailer dynamic coupling device which is present in Fig. 3.

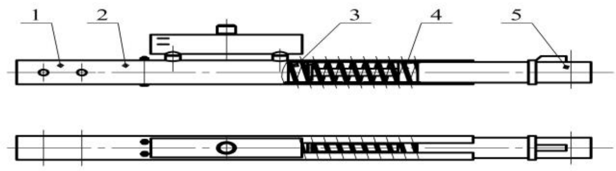

Fig. 3: The car trailer dynamic coupling device: 1 – caliper, 2 – base; 3 – shock absorber; 4 – spring; 5 – lock device

The dynamic coupling device is attached to the trailer frame by means of a caliper 1. The base 2 in the section is selected from rolled metal – a square pipe with an external 80 mm edge and 8 mm a wall thickness. With the locking device 5 it is attached to the car coupling device.

The dissipative resistance is formed by a hydraulic shock absorber 3 and a spring 4 combination.

The shock absorber consists of corps, a piston with a rod, covers and a compensation tank. The caps are fastened to the housing by a threaded connection. They have pressed rubber buffers for restricting movement and preventing piston blows. Corps of shock absorber consists of square section, which corresponds to pipe profile. In the upper part there are openings with rubber thinners, in which the compensation tank is pressed. Resistance is created by the fluid flowing through calibrated holes in the pistons, the liquid squeezed into the compensation tank, which is done by markers and control the liquid level.

The shock absorber is fixed to the base with two M10 bolts.

The spring is attached to the shock absorber by twisting. That is, the shock absorber housing on the spring side has a circular cross section. It has a circular shape groove, which corresponds to the inner spring diameter. On the groove, the spring is wound up, which is selected in such a way that the gap between the outer diameter and the wall is 1 mm – this makes it impossible to disengage them. On the other hand, the spring in the same way is attached to the lock device. A similar fastening elements construction was used in [27, 28].

Initially shock absorber piston is in the middle position – this design provides shock absorber rod movement in the two sides to 150 mm.

With the movement start, the spring taking efforts from the car coupling device, begins to stretch. In turn, the piston damper displacement creates resistance to the spring stretching.

3. Conclusions

The dynamic processes mathematical modeling in the traction coupling device from cars to the trailers gives such results:

1. An analytical dependence (19), which allows describing the change in dynamic loads traction coupling device adjusting a car with a trailer during transient's movement without dissipative resistance forces. This dependence allows establishing a general interaction characteristic between car train sections;
2. An analytical dependence (39) was obtained, which allows us to investigate the change in dynamic loads in the traction coupling device, taking into account the damper device dissipative resistance;
3. Using in the traction coupling devices dissipative link prevents oscillatory processes and significantly reduce dynamic loads during transitional car train movement.
4. It was proposed and described design features of a new dynamic coupling trailer devices as a component part of the system "car - trailer".

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