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Abstract. The article is devoted to the calculation of elevated steel silos for seismic actions. The purpose of the study is developing methods for estimation of reliability function of the elevated steel silos under seismic action, on the basis of which the authors obtained an analytical expressions suitable for engineering analysis. The spectral technique for determining seismic actions is used, as well as methods of probability theory and mathematical statistics. The steps are formulated for determining the reliability function of structures of elevated silos, taking into account the probabilistic nature of geotectonic excitations and stochastic properties of the material of structures. The dynamic response of elevated silos is analyzed by varying the direction of the seismic action. The methods proposed in the article and the formulated results can be applied both for direct probabilistic analysis of elevated steel silos and for codification seismic actions within the framework of the spectral technique.

1. Introduction

Among the natural sources of danger, the main place, no doubt, belongs to earthquakes. A significant part of the Earth's surface is seismically active. The greatest seismic activity is concentrated in the Pacific belt. Another zone of high seismic activity is the Alpine belt, which crosses Europe and continues in a latitudinal direction across the entire Eurasian continent. This zone covers, in particular, the southwestern part of Ukraine, Moldova, part of Romania and Central Asia. Maps of seismic zoning of the territory of Ukraine as of 2023 are given in the norms [1]. The maps show the approximate boundaries of the territories within which the estimated intensity of earthquakes varies from 5 to 10 points. There are also estimates of the approximate frequency of strong earthquakes with an average period of occurrence of 500, 1000, and 5000 years.

The earthquake tremors **E** on the site is a superposition of seismic waves arriving at the site from epicenter region. These tremors E form a stream of random events characterized by macroseismic parameters: intensity, maximum acceleration, tremor duration, parameters of spectral composition. All these parameters are random variables. The purpose of analysis for seismic resistance is to ensure in the event of earthquakes: most of human lives are protected, most of damage is limited and most of structures important for civil protection or possessor remain operational. The random nature of the seismic events and the limited resources available to counter their effects are such as to make the attainment of these goals only partially possible and only measurable in probabilistic terms. The extent of the protection that can be provided to different categories of structures, which is only measurable in probabilistic terms, is a matter of optimal allocation of resources and is therefore expected to vary from country to country, depending on the relative importance of the seismic risk with respect to risks of other origin and on the global economic resources. The structures shall be designed and constructed to withstand the design seismic action without local or global collapse, thus retaining its structural integrity and a residual load bearing capacity after the seismic events. The design seismic action is expressed in terms of the reference seismic action associated with a reference probability of exceedance P_{seis} or a reference return period T_{seis}.

The accident rate of steel silos worldwide is much higher than that of other industrial structures. The collapse of a silo can lead to environmental pollution and jeopardize not only the operation of the enterprise as a whole, but also the life and health of people. The seismic loads pose a particular danger, as they create high-intensity dynamic loads on the silo. These loads and the dynamic response of the silo they cause are probabilistic in nature and must be described using probability theory and mathematical statistics. These are very specific and complex branches of mathematics, little known to the average engineer. In addition, seismic analysis of steel silos is complicated by the need to take into account the properties of bulk materials and their contact with the silo body. These circumstances force us to develop relatively simple methods (if possible analytical) for calculating steel silos for seismic effects for wide engineering use based on the finite element method. The analytical representation of seismic loads and the use of the finite element method make it possible to realistically describe the stress-strain state of steel silo structures and solve problems of contact of bulk material with the silo wall.

The research on the behaviour of steel silos under seismic action is relevant [2-10]. This is evidenced by the latest publications, which provide analytical models and models of finite elements for performing seismic analysis [11-17]. However, a detailed analysis of these publications allows us to formulate the following general shortcomings in the proposed methods (both analytical and numerical) for calculating steel silos for seismic loads. Firstly, this is a description of the seismic effect using the technique of random variables, rather than random processes. Secondly, the statistical characteristics of foundation soils are not taken into account. Thirdly, assessing the reliability of silo structures only in the displacement space. Finally, fourthly, the use of parametric reliability theory, instead of considering a systemic reliability theory, which makes it possible to build logical probabilistic connections between individual components of the silo-soil system.

2. Probabilistic model for seismic analysis

The probabilistic analysis of steel silos for seismic resistance, a distinction must be made between silos directly supported on the ground or on the foundation and elevated silos, supported on a skirt extending to the ground, or on a series of columns, braced or not (see Fig. 1).

The main effect of the seismic action on ground silos are the stresses induced in the shell wall due to the response of the contents of the silos. The main concern in the seismic design of elevated silos is the supporting structures and its ductility and energy dissipation capacity. Under seismic conditions, the pressure exerted by the particu-late material on the walls, the hopper and the bottom, may increase over the value relative to the condition when there is no seismic action. For design purposes this increased pressure is deemed to be found only from the inertia forces acting on the stored material due to the seismic action. The model to be used for the determination of the seismic action effects shall reproduce accurately the stiffness, the mass and the geometrical properties of the containment structures, shall account for response of the contained particulate material and for the effects of any interaction with the foundation soil. In addition, silos shall be analyzed by considering elastic behavior of the silos shell and of its supporting structure, if any, unless proper justification is given for performing a nonlinear analysis.

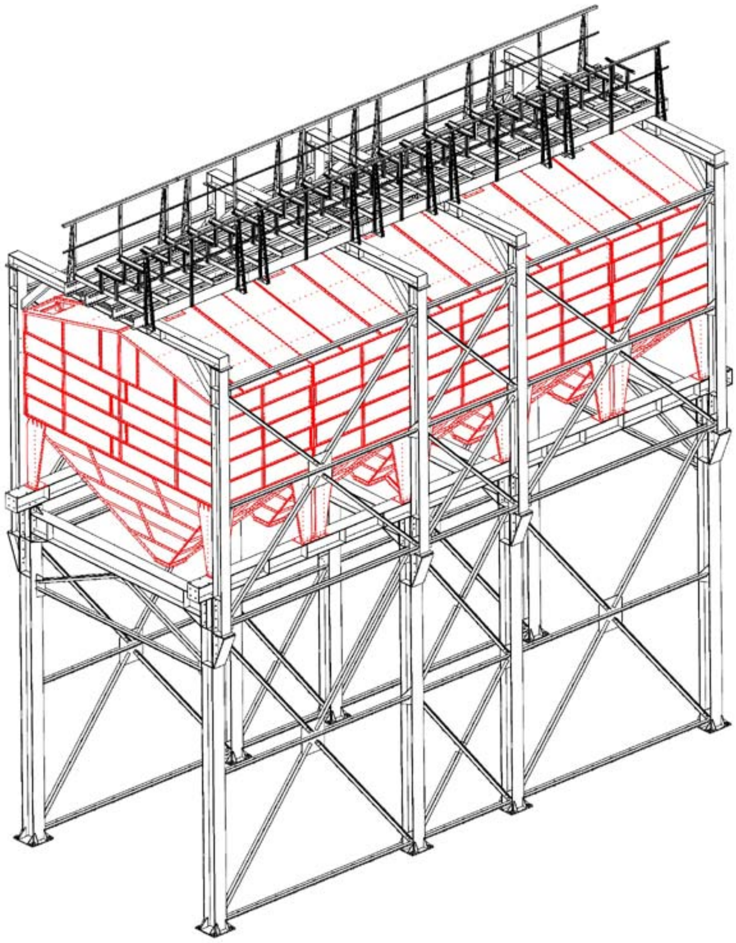

Figure 1. General 3D-view of classic elevated steel silo.

The earthquake tremors **E** on the site form a stream of random events. The random events will be divided into classes $\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_j, \ldots, \mathbf{\Phi}_m$ that differ, for example, by the level of intensity of seismic effects. Suppose that the events of each class j form independent stationary flows E with intensity λ_j over a period of time $[0, T_{ef}]$. The probability of an event occurring $E \in \mathbf{\Phi}_j$ in a given time period k times is denoted by $Q_k(\mathbf{\Phi}_j \,|\, T_{ef})$. Let $P_U(\mathbf{\Phi}_j)$ is the probability that an accident (silo structure failure) will not occur when the event occurs $E \in \mathbf{\Phi}_j$. Then, neglecting the impact of damage caused by previous events and taking into account the independence of event streams from different classes, we calculate the reliability function as the probability that an emergency situation will never occur during a time interval $[0, T_{ef}]$ [18].

$$P_U(T) = \prod_{j=1}^{m} \left[Q_0(\mathbf{\Phi}_j; T_{ef}) + \sum_{k=1}^{\infty} P_U^k(\mathbf{\Phi}_j) Q_k(\mathbf{\Phi}_j; T_{ef}) \right] \quad (1)$$

where $Q_0(\mathbf{\Phi}_j; T_{ef})$ is probability that the event $E \in \mathbf{\Phi}_j$ will never occur in the time interval $[0, T_{ef}]$. If the stream of random events $E \in \mathbf{\Phi}_j$ is Poisson, then the probability $Q_k(\mathbf{\Phi}_j \mid T_{ef})$ of occurrence of k events in the time interval $[0, T_{ef}]$ can be expressed by the formula

$$Q_k(\mathbf{\Phi}_j; T_{ef}) = \exp(-\lambda_j T_{ef})(\lambda_j T_{ef})^k / k! \quad (2)$$

Substituting (2) into (1), after a number of algebraic transformations, we will have an expression

$$P_U(T) = \exp\left[-\sum_{j=1}^{m} \lambda_j T_{ef} Q_U(\mathbf{\Phi}_j) \right] \quad (3)$$

where $Q_U(\mathbf{\Phi}_j) = 1 - P_U(\mathbf{\Phi}_j)$ is the probability that an accident (structure failure) will occur when the event occurs $E \in \mathbf{\Phi}_j$; λ_j is intensity of a stationary stochastic process on an interval $[0, T_{ef}]$.

If $\lambda_j T Q_U(\mathbf{\Phi}_j) \ll 1$ for all j and all T_{ef}, then a simple formula for the reliability function follows from Eq. (3)

$$P_U(T) \approx 1 - \sum_{j=1}^{m} \lambda_j T_{ef} Q_U(\mathbf{\Phi}_j) \quad (4)$$

Since the bearing capacity of silo structures in design probabilistic earthquake resistance calculations is specified with an accuracy of a random value $\tilde{R}$, the complete reliability function of the structure is determined based on the method of conditional reliability functions [18] in the form

$$P_U(T) = \int_L^{\infty} P_U(R \mid T_{ef}) f_R(R) dR = \int_L^{\infty} f_R(R) dR - \int_L^{\infty} f_R(R) \sum_{j=1}^{m} \lambda_j T_{ef} Q_U(\mathbf{\Phi}_j \mid R) dR \quad (5)$$

where $f_R(\bullet)$ is the density distribution of the bearing capacity of the silo structure; $Q_U(\mathbf{\Phi}_j \mid R)$ – the conditional probability of failure of the silo structure, that is, the probability of failure calculated under the assumption that the bearing capacity of the structure is deterministic and determined by the value R; L is the left limit of integration (as rule $L = 0$, but in some cases, for example, for the model of absolute maximum loads [18, 19], $L > 0$).

In the following considerations, we will assume that $\tilde{q}_j(t)$ are onedimensional stochastic processes that have the same physical meaning and play the role of a single parameter, with the accuracy of which the seismic action is identified. An example is the earthquake acceleration $\tilde{a}_j(t)$ at some site or forces, moments and stresses in the supporting structure of the silo. At the same time, the class of seismic action characterizes the focal area, while acceleration (and, accordingly, internal forces) are taken into account in the calculation regardless of the source of the earthquake. In most practically important cases, seismic effects $\tilde{q}_j(t)$ (and their corresponding stress $\tilde{S}_j(t)$) can be represented in the form of stationary ergodic stochastic processes with an average duration τ_E.

The conditional probability of failure of structure in Eq. (5) can be estimated in two ways.

The first approach proposed in paper [18], under the name "model of absolute maxima", consists in equating the average number of positive crossings $N_+(\Phi_j \mid R; \tau_E)$ of a stochastic process of stress of a certain level with the probability $Q_U(\Phi_j \mid R; \tau_E)$ of crossing this level, i.e.

$$Q_U(\Phi_j \mid R; \tau_E) \approx N_+(\Phi_j \mid R; \tau_E), \text{ for condition } N_+(\Phi_j \mid R; \tau_E) \ll 1 \tag{6}$$

where τ_E is an average duration of stochastic process.

At the same time, there is always an elementary inequality.

$$Q_U(\Phi_j \mid R; \tau_E) \le N_+(\Phi_j \mid R; \tau_E) \tag{7}$$

Therefore, Eq. (6) will always give a strict upper estimate for the probability $Q_U(\Phi_j \mid R; \tau_E)$. How accurate this estimate depends on the frequency structure of an individual earthquake and how far the level R is from the level of mathematical expectation of the process $\tilde{S}_j(t)$.

The second approach, which is called "exponential model", is based on Poisson distribution and for case of a stationary stochastic process determines the probability $Q_U(\Phi_j \mid R; \tau_E)$ in the form

$$Q_U(\Phi_j \mid R; \tau_E) \approx 1 - \exp\left[-N_+(\Phi_j \mid R; \tau_E)\right] \tag{8}$$

The accuracy with which the probability assessment $Q_U(\Phi_j \mid R; \tau_E)$ is performed depends on the type of stochastic process $\tilde{S}_j(t)$ emission stream per level R. If a positive crossing by a level R process can be interpreted as an event in a Poisson flow, then expression (8) makes it possible to find an exact solution for the conditional probability of failure $Q_U(\Phi_j \mid R; \tau_E)$. Otherwise, the estimate will be performed with some error, the value of which will depend on how different the emission flow is from the Poisson one.

To obtain complete reliability functions, we substitute Eqs. (6) and (8) into Eq. (5). For elevated steel silos it considers each case separately. In the statistical theory of seismic stability, there are only two suitable probabilistic models: the model of absolute maxima and the exponential model. These models are discussed below.

3. The model of absolute maxima for seismic resistance of elevated steel silos

The mathematical sense of absolute maximum model is the following expression

$$P_U(T) = \int_{S_{0,j}}^{\infty} f_R(R)dR - \int_{S_{0,j}}^{\infty} \sum_{j=1}^{m} \lambda_j T_{ef} N_+(\Phi_j \mid R; \tau_E) f_R(R)dR \tag{9}$$

where $S_{0,j}$ is the characteristic maximum of a stochastic process $\tilde{S}_j(t)$, that is, the level for which the equation $N_+(\Phi_j \mid S_{0,j}; \tau_E) = 1$ is satisfied.

The first part of this formula determines the probability that the bearing capacity of the structure $\tilde{R}$ is above the level of the characteristic maximum of the stochastic force process $S_{0,j}$. The other part can be expanded by writing an expression for the average number of positive intersections of a random process $\tilde{S}_j(t)$ level [18, 19]

$$N_+(\Phi_j \mid R; \tau_E) = \hat{S}_j \frac{\omega_{e,j} \tau_E}{\beta_{\omega,j} \sqrt{2\pi}} f_S(R) = \frac{f_S(R)}{f_S(S_{0,j})} \tag{10}$$

where $\omega_{e,j}$, $\beta_{\omega,j}$, $\hat{S}_j$ and $f_S(\bullet)$ is, respectively, the effective frequency, the bandwidth coefficient, the standard and the law of distribution of the ordinate of a stochastic process $\tilde{S}_j(t)$ from $\mathbf{\Phi}_j$ a class of seismic actions (for simplicity, it is assumed that the distribution density of a stochastic process $\tilde{S}_j(t)$ has the same analytical form for all classes of seismic actions).

Substituting Eq. (10) into Eq. (9), we will have

$$P_U(T) = \int_{S_{0,j}}^{\infty} f_R(R)dR - \int_{S_{0,j}}^{\infty} \sum_{j=1}^{m} \left[\lambda_j T_{ef} f_S(R) / f_S(S_{0,j}) \right] f_R(R)dR \tag{11}$$

The final formula for estimating the reliability function of silo structures will be obtained by passing to the normalized form, taking into account that the value $\tilde{R}$ (yield strength of steel) is described by the normal distribution law

$$P_U(T) = \frac{1}{\sqrt{2\pi}} \int_{Z_{0,j}}^{\infty} \exp\left(-0.5Z^2\right)dZ - \frac{1}{\sqrt{2\pi}} \int_{Z_{0,j}}^{\infty} \sum_{j=1}^{m} \left[\lambda_j T_{ef} f_{nS} \frac{E_j(Z)}{f_{nS}(\gamma_{0,j})} \right] \exp\left(-0.5Z^2\right)dZ \tag{12}$$

where $f_{nS}(\bullet)$ and $\gamma_{0,j}$ are respectively the normalized distribution density and the normalized characteristic maximum of the stochastic process $\tilde{S}_j(t)$ of seismic action; V_R, $V_{S,j}$ are coefficients of variation of the bearing capacity and stress process $f_{nS}(\bullet)$; $p_j = \hat{R} / \hat{S}_j$ is the ratio of standards; $Z_{0,j} = \gamma_{0,j} p_j^{-1} + p_j^{-1} V_{S,j}^{-1} - V_R^{-1}$ is the lower limit of integration; $E_j = Zp_j + p_j V_R^{-1} - V_{S,j}^{-1}$ is the dimensionless argument of the function $f_{nS}(\bullet)$.

4. The exponential model for seismic resistance of elevated steel silos

The mathematical sense of exponential model is the following expression

$$P_U(T) = \int_{S0}^{\infty} \exp\left[-\sum_{j=1}^{m} \lambda_j T_{ef} N_+(\mathbf{\Phi}_j \mid R; \tau_E) \right] f_R(R)dR \tag{13}$$

Substituting Eq. (5) for $N_+(\mathbf{\Phi}_j \mid S_{0,j}; \tau_E)$, we get the expression

$$P_U(T) = \frac{1}{\sqrt{2\pi}} \int_{Z_{0,j}}^{\infty} \exp\left[-\sum_{j=1}^{m} \left[\lambda_j T_{ef} f_{nS} \left[E_j(Z) \right] / f_{nS}(\gamma_{0,j}) \right] \right] \exp\left(-0.5Z^2\right)dZ \tag{14}$$

Eqs. (12) and (14) were proposed, apparently, by the authors for the first time and are a further development of the general procedure for reliability estimate of steel structure elements developed in works [20-22]. They make it possible to take into account the frequency-time structure of geotectonic excitations, the actual laws of the distribution of force effects in silo structures (efforts, stresses) and solve the problem of the reliability of steel structures within the framework of this work.

It is important to note that the theory presented above should primarily be considered as a research tool, because its direct use for engineering calculations is still associated with serious mathematical and computational difficulties. However, it can be used to create standard calculations that are simple in form. One possible way is to create special curve graphs that resemble the wellknown "acceleration spectra". The calculation of steel silos for seismic impact will then be replaced by a calculation for the action of horizontal forces corresponding to a certain constant acceleration. This acceleration is determined depending on the period of natural oscillations of the silo. However, in contrast to the wellknown "acceleration spectra" the developed special curve graphs form families of curves, each of which corresponds to a certain probability of destruction during a specified service life or, conversely, to a certain service life with a specified probability of destruction. These families of curves can be constructed using the methods of statistical theory of seismic resistance.

5. The example of a probabilistic analysis of an elevated silo for seismic resistance

As an example, consider a partial case $m = 1$. Consider an earthquake model, according to which the acceleration of the base $\tilde{a}(t)$ on which the elevated silo is located is the segment of the realization of a stationary normal stochastic process with a mathematical expectation equal to zero and a standard $\hat{a}$. We will present reaction of the silo and its supporting structure to seismic load in the form of a differential equation (a linear system with one degree of freedom is considered)

$$\ddot{u}(t) + 2\beta\omega_0\dot{u}(t) + \omega_0^2 u(t) = -\tilde{a}(t) \quad (15)$$

where ω_0 is the natural oscillation frequency of the structure; β – damping coefficient.

We will assume that the natural period of oscillations of the elevated silo, the main shaking period, as well as the characteristic correlation time of the process $\tilde{a}(t)$ are small enough compared to the duration τ_E of the intense phase of the earthquake. Also, let the damping be small enough so that $\beta^2 \ll 1$. Then in expression (15) we can take $\ddot{u}(t) + \tilde{a}(t) \approx -\omega_0^2 u(t)$. The forces arising in the supporting structure will be represented in proportion to the acceleration of the structure using the proportionality factor k – $S(t) = k \cdot \omega_0^2 u(t)$. Probabilistic fluctuations of an elevated silo are shown in Fig. 2 and 3.

The calculation model of the silo was created in the SCAD Office 21.1 finite element analysis software, the basis of which is the displacement method. The model of the silo was formed in the form of a set of rods and plates with the appropriate conditions for joining the elements to the nodes: placing ties in the nodes to limit the degrees of freedom, arranging end hinges to release mutual linear and angular displacements, introducing rigid inserts to move the elastic parts of the rods away from the nodes adjacency. The silo shell sheets and hoppers are modeled with flat end elements of type 44 "quadrilateral shell element" and linear structures with rod element type 10 – "universal spatial rod element". The solid bodies were used to model particulate material: density 900 kg/m^3 and effective elastic modulus 10 MPa.

Two finite element models of steel silos were considered, each of which consisted of 580 elements and 400 nodes. The models differed in that the silos of the first model were connected to the rod frame, while the silos of the second model were not connected. The dynamic load on the silos was created by vibrations of the base soil with acceleration of a given intensity and in a given direction. The acceleration itself was considered as a random function of time with a quasistationary frequency structure. The forces in the frame elements and the displacement of the frame nodes were calculated using direct integration of the equation of motion.

Within the framework of these remarks, Eq. (10) will take the form

$$N_+(\mathbf{\Phi}_j \mid \gamma; \tau_E) = \exp\left[0.5\left(\gamma_0^2 - \gamma^2\right)\right], \qquad \gamma_0 = \sqrt{2\ln\left[\omega_0\tau_E / (\pi\beta_\omega)\right]} \quad (16)$$

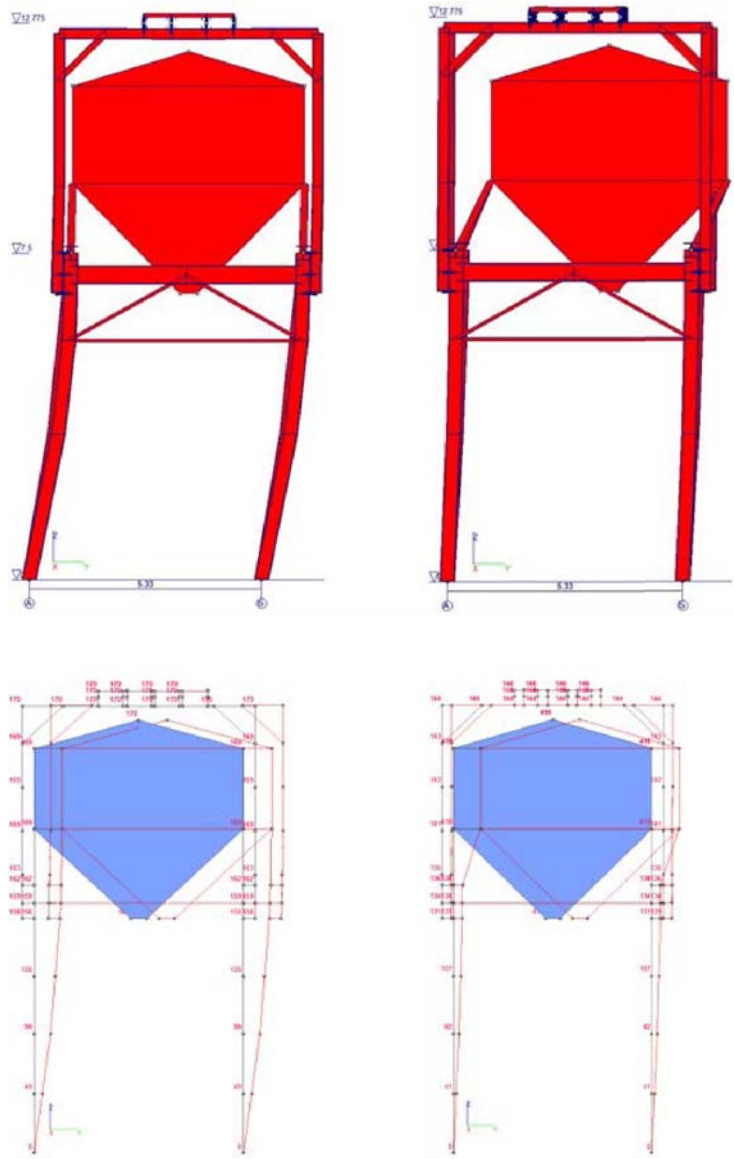

Figure 2. Stochastic fluctuations of an elevated silo in the transverse direction: for the case when the silo is rigidly connected to the supporting structure (left) and when the silo is not connected to the supporting structure (right).

Taking into account that the design acceleration standard is $\hat{a} = \omega_0^2 \hat{u}$ ($\hat{u}$ is the process $\tilde{u}(t)$ standard), for Eqs. (12) and (14), respectively, we get

$$P_U(T) = \frac{1}{\sqrt{2\pi}} \int_{Z_{0,j}}^{\infty} \exp\left(-0.5Z^2\right) dZ - \frac{\lambda T_{ef}}{\sqrt{2\pi}} \int_{Z_0}^{\infty} \exp\left\{0.5\left[\gamma_0^2 - E^2(Z)\right]\right\} \exp\left(-0.5Z^2\right) dZ \qquad (17)$$

$$P_U(T) = \frac{1}{\sqrt{2\pi}} \int_{1/V_R}^{\infty} \exp\left[\lambda T_{ef} \exp\left\{0.5\left[\gamma_0^2 - E^2(Z)\right]\right\}\right] \exp\left(-0.5Z^2\right) dZ \qquad (18)$$

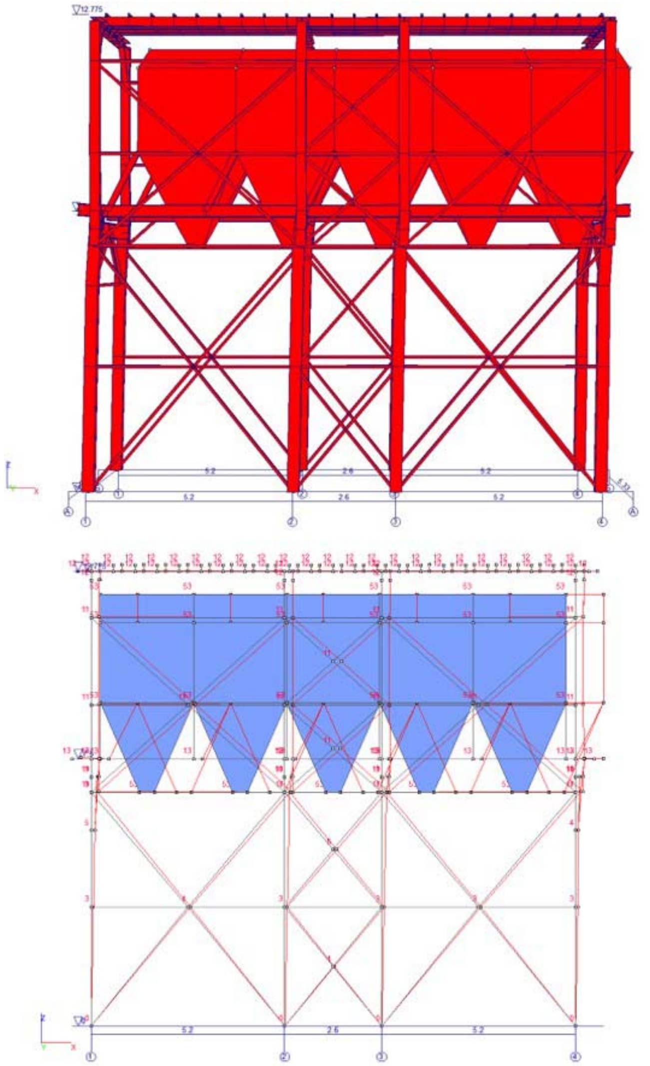

Figure 3. Stochastic fluctuations of an elevated silo in the longitudinal direction.

Eqs. (17) and (18) in combination with Eq. (16) are of particular interest, since their right-hand sides include a complex expression that combines different parameters: the natural frequency of the silo ω_0, the duration of the intense phase of the earthquake τ_E, the earthquake repeatability parameter λ, the probabilistic characteristics of loads on the silo, silo service life T_{ef}, stochastic characteristics of the structure material. On the one hand, these parameters characterize the stochastic nature of geotectonic excitation, on the other hand, the deterministic and probabilistic properties of the elevated silo itself, which experiences seismic impacts.

6. Conclusion

The approach given in the article to estimation of reliability function of elevated silos, which are subjected to seismic actions, is not associated with serious mathematical and computational difficulties, and thus can be used directly in practice. Note that the seismic effect should be described by a model of a stationary random process with zero mathematical expectation and standard deviation according to the norms [1]. In this case, the reaction spectrum of the silo should belong to the class of spectra with a fractional-rational function.

The described technique can be used not only for direct probabilistic analysis of silos for earthquake resistance, but also for creating normative calculations that are simple in form. The limitation of the presented material is the application of the described approach only for elevated silos that work linearly. However, if the non-linearity of the structures is quite small, then the statistical linearization method, which is widely used in the general theory of the reliability of building structures, can be successfully used in this case as well. Further steps to improve the methods of normalization of seismic loads are to introduce several types of calculated shocks that differ in repeatability, maximum accelerations and spectral composition.

The proposed method is universal and makes it possible to analyze the reliability of steel silos for cases where the seismic impact is schematized in the form of both stationary and non-stationary multidimensional random process. In this case, reliability calculations can be performed both for a completely filled silo and for a partially filled silo with an uneven surface of the stored product backfill. The use of the proposed probabilistic method instead of the normative deterministic method described, for example, in EN 1998-4, makes it possible to reduce the metal consumption of silos by 10-20% (depending on the design solutions of the silos and the seismicity of the construction area).

It should also be added that the standard [1] in force in Ukraine for the design of buildings and structures for seismic impacts does not contain any recommendations for the design of steel silos. Thus, engineers do not understand how to perform seismic calculations for such structures, especially taking into account real earthquake accelograms. Therefore, the information presented in the article can be considered as the basis for creating a regulatory methodology for calculating steel silos for seismic impacts.

References

[1] State building codes 2014 DBN V.1.1-12 *Construction in Seismic Regions of Ukraine* (Kyiv: MinRegion of Ukraine)
[2] Demir A and Livaoglu R 2023 *Soil Dyn. Earthq. Eng.* **165** 107664
[3] Yang J, Zhang F, Li P and Jing H 2023 *Structures* **47** 595-606
[4] Mehretehran A and Maleki S 2021 *Eng. Fail. Anal.* **120** 105100
[5] Jing H, Wang X, Yang J and Chen H 2022 *J. Constr. Steel Res.* **198** 107515
[6] Wang M, Sun Z, Sun J, Cui L and Wu Y 2023 *Structures* **48** 318-330
[7] Semko V, Mahas N, Fenko O, Sirobaba V and Hasenko A 2019 *IOP Conf. Ser: Mater. Sci. Eng.* **708** (1) 012071
[8] Barabash M, Pisarevskyi B and Bashinsky Ya 2020 *Teh. glas.* **14**(1) 55-59
[9] Yang J, Feng G, Jing H and Zhang F 2023 *J. Constr. Steel Res.* **211** 108207
[10] Lupattelli A, Kita A, Salciarini D, Venanzi I and Ubertini F 2023 *Structures* **55** 310-323
[11] Kanyilmaz A and Castiglioni C 2017 *Eng. Struct.* 1**43** 447-497
[12] Rotter J and Hull T 1989 *Eng. Struct.* **11** (3) 139-147
[13] Mehretehran A and Maleki S 2018 *Thin-Walled Struct.* **131** 654-667
[14] Bartsch H *et al* 2020 *J. Constr. Steel Res.* **173** 106264
[15] Zeng C, Li X and Wang Y 2020 *Biosyst. Eng.* **190** 87-96
[16] Wang L, Nagarajaiah S, Shi W and Zhou Y 2022 *Eng. Struct.* **271** 114963
[17] Nateghi F and Yakhchalian M 2011 *Procedia Eng.* **14** 3050-3058
[18] Makhinko A 2012 *Probabilistic Design of Communication Towers* (Poltava: PoltNTU)
[19] Bolotin V 1984 *Forecasting the Resource of Machines and Structures* (Moscow: Mashinostroenie)
[20] Makhinko A and Makhinko N 2021 *Steel Silos for Grain Storage* (Kyiv: Stal)
[21] Pichugin S, Makhinko A and Makhinko N 2020 *Lect. Notes Civ. Eng.* **73** 183-192