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RESEARCH OF DATA DIAGNOSTIC METHODS IN A RESIDUE NUMBER SYSTEM

Abstract. One of the effective ways to increase the productivity and reliability of data processing is the use of non-traditional types of machine arithmetic. Currently, there is considerable interest in alternative number systems, one of which is a non-positional number system based on the Chinese remainder theorem. The existing methods of control and diagnosis of data presented in the residue number system are low operational operations due to the complexity of implementation. The use of the results of solving this problem, namely the research and analysis of the main shortcomings of existing methods for diagnosing data presented in the non-positional number system, will contribute to the further development of the theory and practice of error-correcting coding in the residue number system. This, in turn, expands the area of effective use of the residue number system as a number system for computer systems and a component for processing integer data.

The purpose of the research is to analyze existing methods for diagnosing data based on the use of a non-positional number system in residual classes.

Subject of research: methods and algorithms for diagnosing data presented in the form of residues from the division of a given residue number system.

The main hypothesis of the research: the low operational efficiency of the process of diagnosis of constantly growing arrays of data reduces the practical use of the residue number system.

The article considers two main methods for diagnosing data that are presented in a residue number system. The main drawback of these methods is the significant time required to diagnose data in the residue number system. This is because considerable time is needed to sequentially carry out the procedures included in the process of diagnosing data in the residue number system. The presence of the named disadvantage requires further improvement of methods for diagnosing data errors in terms of increasing the efficiency of the diagnostic process. The development and practical application of new methods for operational data diagnostics will increase the overall efficiency and feasibility of using non-position code structures in the residue number system in computing systems.

Key words: residue number system, data control, speed of data diagnostics, position-independent code structure, alternative aggregate of the number

JEL classification: C0, C02, C6, C61, C65, C69, Z19.

Statement of the problem. The need for timely data processing by computer means in real time of large amounts of information in a short period of time leads to possible aging or even loss of important information. Increasing the efficiency of computer systems largely depends on the principles of construction and operation of data processing systems. Currently, there is an active search for various ways to increase the speed and reliability of data processing, based on the use of new principles and ideas. In particular, information processing methods based on the use of codes presented in a non-positional number system in residual classes.

The results of the analysis of modern trends in the development of computer systems and components functioning in the residue number system (RNS) confirm the existence of the following conflict situation. On the one hand, between the existing possibility of significantly increasing the speed of performing integer arithmetic operations in the RNS. On the other hand, the low efficiency of the existing systems and means of control, diagnostics and correction of the results calculated in the RNS due to the significant time of the implementation of the listed procedures.

It is obvious that in this case, the existing means of monitoring and diagnostics computer systems and components in the RNS inhibit the wide potential capabilities of fast data processing inherent in non-positional machine arithmetic.

This circumstance necessitates the formulation and solution of a new, important and relevant scientific and technical problem – the development methods for operational diagnostics of data components of a computer system operating in the RNS, without reducing the processing speed of integer operations.

Analysis of recent studies and publications. A number of works, both domestic and foreign, are devoted to the construction of corrective non-positional (modular) codes [1-4]. So well-known scientists in the field of the non-positional number system described in detail the properties of the RNS, namely the equality of residues, which has its characteristics and advantages.

In [1] proposed two-dimensional correction codes – based on residue number system – to improve the data transmission robustness in wireless sensor networks. These codes are characterized by high correction characteristics as well as the simplified coding procedure.

In [2] proposes to use the results on the distribution of error intervals to develop a new method for detecting and correcting errors. The method makes it possible to simultaneously translate data from modular representation into positional representation and correct distorted information, which significantly reduces the time spent on detecting and correcting errors.

In [3] the possibility of using non-positional codes capable of detecting and correcting errors that arise in the dynamics of the data processing process is shown.

In [4], the structure of the correcting code in the RNS is described, which is the basis for constructing codes capable of detecting and correcting errors in the process of implementing modular operations.

In [5] proposed method makes it possible to reduce the time of diagnosing errors in RNS-based data, which increases the efficiency of diagnosis. But this method is based on the use of the information redundancy of the code, which is not always acceptable.

At the same time, the aforementioned publications do not resolve the contradiction between the high speed of implementation of integer arithmetic operations in the RNS and the low efficiency of data control and diagnostics operations. To eliminate this problem, it is necessary to investigate existing methods of increasing the efficiency of diagnostics of integer data presented in the system of residual classes, without reducing the productivity of information processing.

The purpose of the article is a detailed research of data diagnostic methods operating in the RNS to find existing ways to increase the reliability and efficiency of this process.

Methodology. Data diagnostics in the RNS is understood as the process of determining distorted residues in a non-positional code structure (NCS) presented in the following form $A_{SRC} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| ... \| a_{n+k})$, where n and k are the number of informational and control bases m_i $(i = \overline{1, n+k})$ in the ordered $(m_i < m_{i+1})$ RNS respectively. To carry out the procedure for diagnosing errors in the NCS, it is necessary to introduce a certain information redundancy. The degree of information redundancy R, which determines the corrective ability of the code, is estimated by the value $d_{\min}{}^{(RNS)}$ of the minimum code distance (MCD). In RNS, the value of MCD is determined by the ratio $d_{\min}^{(RNS)} = k + 1$. With one control base, the MCD is equal to $d_{\min}^{(RNS)} = 2$. In accordance with the coding theory, with the minimum code distance $d_{\min}^{(RNS)} = 2$ the distortion of only one of the residues (a single error) of the code word in the RNS can be reliably established [6].

As noted in [7], due to the influence of the properties of the RNS on the data processing, in some cases, it is possible to correct one-time (in one NCS residue) data errors with the introduction of minimal $(k = 1)$ information code redundancy. One of the cases of correction due to such a property of the RNS as the independence of NCS residues is the case of error correction only of the final calculation result. A typical example for this case is the possibility of implementing a data error correction procedure in the presence of one control base without stopping the process of intermediate calculations (in the dynamics of a computational process (DCP)). To implement this procedure, it is necessary to diagnose intermediate results of calculations based on the use of the concept of an alternative set of numbers (AS) in the RNS [8, 9].

The basis of the research carried out in the work was the theory of numbers, the theory of computational processes and systems, as well as the theory of coding in a non-positional number system. When solving research problems, the theory of interference-resistant coding in the non-positional counting system in the RNS was used, as well as the sections on the theory of divisibility and the theory of comparisons of the number theory.

The main part. By the definition of AS $W(\tilde{A}) = \left\{ m_{l_1}, m_{l_2}, ..., m_{l_\rho} \right\}$ of an incorrect number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| ... \| a_{n+k})$ in the RNS, we mean the totality $\{m_{l_\kappa}\}$ $(\kappa=\overline{1,\rho})$ of ρ bases, on which the correct (not distorted) NCS $A_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|...\|a_{n+k})$ can differ only in one of the residues from the totality of possible incorrect numbers. Note that the occurrence of only a single error is assumed (in only one remainder a_i, where $i=(\overline{1,n+1})$). In this case, the distorted NCS in the RNS will appear in the form $\tilde{A}_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|a_{n+1})$.

If, in the process of solving a problem in DCP, as intermediate results of calculations, we obtain a set of values of the numbers A_j $(j=\overline{1,n+1})$ in RNS, then by a conditional alternative set (CAS) $\overline{W}(A)=W(A_1)\wedge W(A_2)\wedge...\wedge W(A_j)$ of an incorrect number $\tilde{A}_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|a_{n+1})$ we mean a set of bases on which calculation errors are possible, taking into account the nature of the AS of the previous incorrect results calculations in the course of solving the problem. Or in other words, CAS is understood as the totality of the bases of the RNS common for each AS $W(\tilde{A})=\{m_{l_1},m_{l_2},...,m_{l_\rho}\}$. Serial calculation of CAS makes it possible to reduce the number of bases for which errors are possible. In [6], it is said that, with a sufficient length of the computation chain, there is such a value of ρ, under which the condition $\overline{W}(A_j)=\{m_i\}$, where m_i is the specific base of the RNS for which an error occurs in the residue a_i of the incorrect number $\tilde{A}_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|a_{n+1})$ Further, if necessary, a correction of the residue a_i of the number $\tilde{A}_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|a_{n+1})$ is carried out modulo m_i by methods known in the RNS. Based on the foregoing, the essence of data diagnostics in the RNS lies in finding the AS $W(\tilde{A})=\{m_{l_1},m_{l_2},...,m_{l_\rho}\}$ of the number $A_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|...\|a_{n+k})$.

Thus, while conducting the diagnosis of intermediate calculation results, there is a need for frequent diagnosis of calculation results, i.e. in the frequent definition of the AS of the numbers. As always, the main requirement for the data control process is the high speed of the implementation of the diagnostic procedure. In the case of diagnosis in RNS, this is a high diagnostic efficiency, i.e. high speed determination of the AS $W(\tilde{A})=\{m_{l_1},m_{l_2},...,m_{l_\rho}\}$ of the numbers in RNS.

Consider the two main practical methods for diagnosing data presented in the RNS.

First method. For the first method, an alternative set $W(\tilde{A})=\{m_{l_1},m_{l_2},...,m_{l_\rho}\}$ of the incorrect number $\tilde{A}_{RNS}=(a_1\|a_2\|...\|a_{i-1}\|a_i\|a_{i+1}\|...\|a_n\|a_{n+1})$ can be determined by sequentially checking each of the bases m_i $(i=\overline{1,n})$ of the RNS as follows. A set of numbers that have the same value of residues on all bases of the

RNS, as the number $\tilde{A}$, except for one specific base, and differing only in the values of possible residues on this basis, is determined. Among this set of numbers there may not be a single correct number, or there may be only one correct number. In the latter case, the resulting number is included in the AS of the incorrect checked number $\tilde{A}$. For the first method, similar checks are supposed to be carried out sequentially for each of the information bases of the RNS (the control base is always included in the set of bases of the AS). The result of such sequential checks determines the AS $W(\tilde{A}) = \left\{ m_{l_1}, m_{l_2}, ..., m_{l_\rho} \right\}$.

Second method. This method is also based on the determination of the AS $W(\tilde{A}) = \left\{ m_{l_1}, m_{l_2}, ..., m_{l_\rho} \right\}$. In this case, the procedure for diagnosing NCS is carried out on the simultaneous parallel calculation of all possible projections $\tilde{A}_{i\ RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_{i+1} \| ... \| a_n \| a_{n+1})$ of the incorrect number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$, and their subsequent comparison with the value $M = \prod_{i=1}^{n} m_i$ of the information numerical interval $0 \div M - 1$ for a given RNS. It was proved [1] that the necessary and sufficient condition for the entry of the bases of the RNS into the AS $W(\tilde{A}) = \left\{ m_{l_1}, m_{l_2}, ..., m_{l_\rho} \right\}$ of the number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$ is the correctness $(A_{i\ RNS} < M)$ of its projection $\tilde{A}_{i\ RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_{i+1} \| ... \| a_n \| a_{n+1})$.

Parallelization of the procedure of calculating all possible projections $\tilde{A}_{i\ RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_{i+1} \| ... \| a_n \| a_{n+1})$ of the incorrect number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$ increases the efficiency of determining the AS.

The application of the second method makes it possible to somewhat speed up the process of determining the AS $W(\tilde{A}) = \left\{ m_{l_1}, m_{l_2}, ..., m_{l_\rho} \right\}$ of the number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$, due to the possibility of simultaneously determining the values of projections $\tilde{A}_j$ of the incorrect number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$. However, note that for the second method, the procedure for determining the AS of a number contains the following basic operations: transferring the number $\tilde{A}_{RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_i \| a_{i+1} \| ... \| a_n \| a_{n+1})$ from the RNS to the PNS; transferring the projections $\tilde{A}_{i\ RNS} = (a_1 \| a_2 \| ... \| a_{i-1} \| a_{i+1} \| ... \| a_n \| a_{n+1})$ of the incorrect number $\tilde{A}_{RNSn+1}$ from the RNS to the PNS and the operation of comparing the numbers with the value M. In the RNS, these operations belong to non-positional operations requiring significant time and hardware costs for their implementation.

The disadvantage of the methods considered is the long time it takes to implement the data diagnostic process in the RNS, i.e. low efficiency of data diagnostics. This is because considerable time is needed to sequentially carry out the procedures included in the process of diagnosing data in the RNS.

Conclusion. The article studies practical methods for diagnosing data errors presented in the RNS. The disadvantage of the methods considered is the long time it takes to implement the data diagnostic process in the RNS. This is because considerable time is needed to sequentially carry out the procedures included in the process of diagnosing data in the RNS. The presence of the named disadvantage requires further improvement of methods for diagnosing data errors in terms of increasing the efficiency of the diagnostic process. The development and practical application of new methods for operational data diagnostics will increase the overall efficiency and feasibility of using non-position code structures in the RNS in computing systems.

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