

# Optimizing Forms and Size of Windows for Energy Conservation

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## Abstract

The article deals with the problem of optimizing the form and size of the windows. This is important because of significant losses of heat through windows. The article demonstrates how to improve methods accepted as standard in Ukraine for calculating the natural illumination and insolation of houses.

**Keywords:** insolation, natural illumination, solar map, saving energy

## 1. Introduction

The conducted researches allow us to determine the optimal energy form of the building [1], the location of energy generating windows on the angles of buildings [2], the rational orientation of window openings and location on the edges of buildings [3] in order to save on heating costs. The issue of determining the shape and size of windows has not been resolved. Let's consider this question in more detail.

The minimal size of a window is calculated basing on the recommended relations of the area window to the floor area. The minimal windows area is calculated in most countries by using the daylighting factor (DF)

$$e = E_{int} / E_{ext} \cdot 100 \text{ (in percents),} \quad (1)$$

in which  $E_{int}$  is the natural light exposure of the design point on the working plane in the room, and  $E_{ext}$  is the outdoor horizontal light exposure under open sky.

The standard values of DF are defined for the critical external light exposure (equaling 5000 luxes in Europe) from the condition of light exposure sufficiency of the design point for performance of a prospective kind of visual work.

In the case of cloudy sky, the DF is calculated by using various analytical and grapho-analytical methods [4]. In Ukraine and some other countries, the DF is calculated in two stages.

At the first stage, the geometrical DF (GDF), denoted by  $\square$ , is calculated by (1) under the following restrictions:

- (i) Brightness of the sky is identical for all directions.
- (ii) The window is an aperture in a wall or covering, without glazing and frames.
- (iii) All light exposure is caused by skylight and reflections are not considered.

At the second stage, GDF is multiplied by some factors that take of restrictions (i)-(iii).

The GDF is calculated by using the radial projection of a window

to the sphere of unit radius (whose center coincides with the design point) with the subsequent re-projection of the obtained compartment of sphere to the working plane by beams that are perpendicular to it. In this case we have

$$\varepsilon = \sigma / \pi \cdot 100. \quad (2)$$

In Ukraine, the grapho-analytical calculation of GDF is realized by the method of beams. In this method the equi-bright hemisphere of sky is broken by two plane pencils into 10000 fragments whose projections to the horizontal plane are equal. The first pencil consists of 100 planes that pass through some diameter of the base of hemisphere, and the second one consists of 100 planes that are perpendicular to this diameter. If the number  $N$  of fragments of sky hemisphere that are visible from the design point is known, then equation (2) can be rewritten in the form

$$\square = 0,01N. \quad (3)$$

The number  $N$  can be found by Graphs I and II. The beams of Graph I are the lines of crossing the planes of the first pencil with the plane that passes through the centre of hemisphere perpendicularly to the diameter. The beams of Graph II are the lines of crossing of any plane in the first pencil with the planes of the second pencil.

The standard design procedure of DF [5] does not use all resources of the method of beams. For example, it does not use the method of calculating DF for light openings that are partially eclipsed by surrounding buildings. It does not use the method of calculating DF for inclined windows. It does not consider the light coming from the earth surface to the design points on inclined surfaces. In Section 2 we show how to correct this.

The second important function of windows is the insolation of housings. In Ukraine the duration of insolation is normalized. It is considered equaling to the duration of insolation in some design point, usually the window center. This method allows finding the time of insolation only for windows whose contour of external

aperture is similar to the contour of glazing, and only if both contours are flat, convex, and central-symmetric. It is possible for other forms of windows that the design point is not insulated but solar beams get into a room. To find insolation in this case, we propose to use the method of boundary surface insolation, which is considered in Section 3.

## 2. Natural illumination

The following formulas can be used for calculating DF:

- For lateral illumination:

$$e = \left( \sum_{i=1}^{m_i} \varepsilon_{s_i} q_i + \sum_{j=1}^{m_j} \varepsilon_{b_j} R_j + \sum_{k=1}^{m_k} \varepsilon_{e_k} A_k \right) r_1 \frac{\tau_{\Sigma}}{K_f}; \quad (4)$$

- For upper illumination:

$$e = \left[ \varepsilon + \frac{\sum_{p=1}^N \varepsilon_p}{N} (r_2 K_l - 1) \right] \frac{\tau_{\Sigma}}{K_f}; \quad (5)$$

$$\varepsilon = \sum_{i=1}^{m_i} \varepsilon_{s_i} q_i + \sum_{j=1}^{m_j} \varepsilon_{b_j} R_j,$$

- Here  $\varepsilon_{s_i}, \varepsilon_{b_j}, \varepsilon_{e_k}$  are GDF in design point; they make allowance

to direct light from  $i$ -th site of the sky, to light reflected from the  $j$ -th opposite building, and to light reflected from the  $k$ -th fragment of the ground surface;  $q_i$  makes allowance to the brightness of  $i$ -th fragment of the sky;  $R_j$  makes allowance to relative brightness of  $j$ -th opposite house;  $A_k$  is the albedo of  $k$ -th fragment of the ground;  $m_i, m_j, m_k$  are the numbers of fragments of the sky, of opposite houses, and of fragments of the ground that are visible from the window in design point;  $r_1, r_2$  makes allowance to diffuse light reflected from surfaces of premises at lateral and top illumination, respectively;  $\tau_{\Sigma}$  is the common factor of optical transmission of windows;  $K_f$  makes allowance to cleanliness of the window;  $K_l$  makes

- allowance to type of lantern of ceiling light;  $N$  is the number of design points on characteristic cut of a house (at least 5). The values of factors  $R_i, q, r_1, r_2, \tau_{\Sigma}, K_f$  and  $K_l$  are found by [5].

- The design value of DF in each design point is defined as the total value of DF at this point from all windows.

- Figure 1 illustrates the finding of  $\varepsilon_s$  for a window of any form not shaded surrounding buildings in design point lying on a plane of general position.

First of all, we find the geometrical centre  $D$  of the part of window located above the plane of horizon.

Then we find the line  $a$  of crossing of the window plane with the working plane. Through the design point  $A$ , we draw the plane  $\square = ABC$  that is perpendicular to  $a$ , in which  $C$  is the orthogonal projection of  $D$  to  $\square$ .

The part of window located above the plane of horizon is replaced by a rectangle 1234 of the same area that is constructed as follows. We inscribe it into a rectangle 1'2'3'4' with sides 1'4' and 2'3' that are parallel to  $a$ . Define the area  $F_{1'2'3'4'} = l \cdot m$  of rectangle 1'2'3'4' and area  $F_s$  of the considered part of the window. Then we calculate  $k = \sqrt{F_s / F_{1'2'3'4'}}$ . Accepting the point  $D$  as the center of symmetry, we build a rectangle 1234 with sides 12 = 34 =  $l = k \cdot l'$ ; 23 = 14 =  $m = k \cdot m'$ ; the sides of 1234 must be parallel to the corresponding sides of 1'2'3'4'.

Further, we superpose Graph I with the plane  $ABC$  in such a way

that the base of the graph coincides with the line  $AB$ , and its pole  $O$  coincides with  $A$ . Calculate the number of beams  $n_1$  passing through the window 1234 to the point  $A$  in Graph I.

We superpose the Graph II with the plane  $ACD$  in such a way that the pole  $O$  of the graph superposes with  $A$ , and its base with the line of crossing of the working plane with the plane  $ACD$ . Then we calculate the number of beams  $n_2$  that pass through the window 1234 to  $A$ . The GDF from the celestial sphere  $\square_s$  is defined by (3) in which  $N = n_1 \square n_2$ .

Similarly, we find the value  $\square_e$  from the part of the window under the plane of the horizon. If the window is shaded by the neighboring houses, then the part of window  $w_{b_j}$  that is shaded, its

part  $w_{s_i}$  through which the firmament is observed, and its part  $w_{e_k}$

through which the ground is observed, are considered as separate windows.

This method for calculating  $\square_s, \square_b$  and  $\square_e$  is realized in MathCAD.

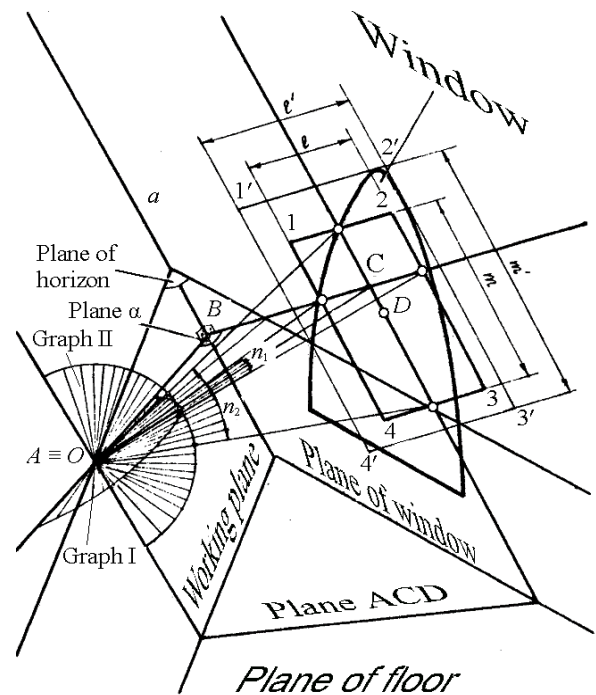


Fig. 1. Finding  $\varepsilon_s$  by Graphs I and II

## 3. Insolation

It is convenient to calculate the time of insolation by using solar cards. The solar card is the stereographic projection from Nadira's point of the celestial hemisphere with the solar trajectories put on it, to the base plane of the hemisphere. Usually the pole of solar card is superposed with the design point, and the window, shading elements of the facade, and opponent buildings are stereographically projected to the solar card [6]. This method is very illustrative and informative.

For calculating the duration of insolation in rooms with windows of complicated form, we propose to use the method of boundary surface. By the boundary surface of insolation we mean the surface of glass cover of windows. The room is insulated if the boundary surface of insolation is insulated. The method is based on a space transformation that transforms the boundary surface of insolation to the point in the center of solar card. Consider this transformation presented on Figure 2.

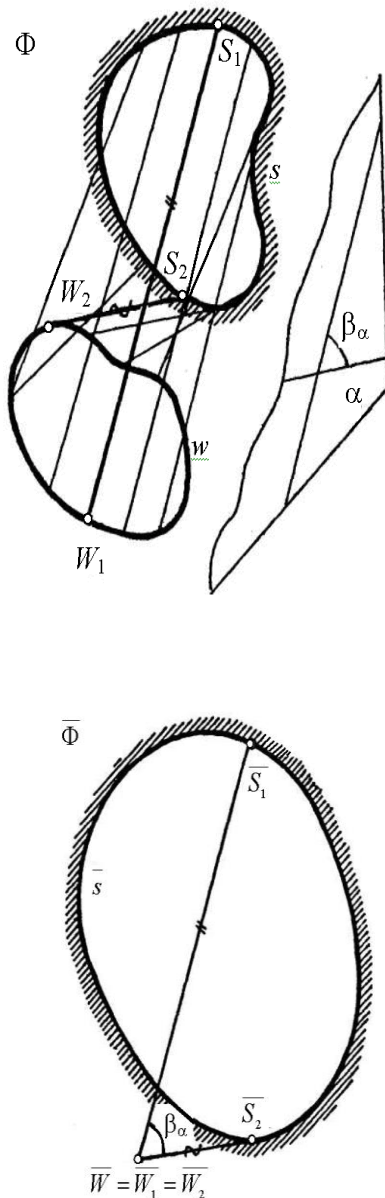


Fig. 2. Space transformation

Let the plane of glass cover is bounded by a curve  $w$ , and solar beams get into the room through the area bounded by a curve  $s$ . The horizontal area of insolation is filling by traces of all vertical planes that cross both focal curves. If we take from this two-parameter set any pencil of planes with a parallelism plane  $\square$ , then we can define for this bunch a pair of crossed straight lines  $W_1S_1$  and  $W_2S_2$  that have the maximal corner of crossing  $\square$ .

Let a curve  $w$  of space  $\Phi$  correspond to a point  $\bar{W}$  of the space  $\bar{\Phi}$ . Draw from the point  $\bar{W} = \bar{W}_1 = \bar{W}_2$  line segments  $\bar{W}_1\bar{S}_1$  and  $\bar{W}_2\bar{S}_2$  that have the same sizes and directions as  $W_1S_1$  and  $W_2S_2$ .

Consider the points  $\bar{S}_2$  and  $\bar{S}_2$  of space  $\bar{\Phi}$  as corresponding to  $S_1$  and  $S_2$  of  $\Phi$ . Thus, the curve  $s$  is transformed to the curve  $\bar{s}$ . The time of isolation of the point  $\bar{W}$  through light opening  $\bar{s}$  is equal to the time of insolation of glass cower  $w$  through the light opening  $s$ , which is equal to the time of insolation of a room that is disposed behind the light opening.

#### 4. Heat engineering

The mathematical simulation of processes in heat engineering has been well developed: There are programs (based on the finite element method) that construct temperature and humidity fields within constructions. There are geometrical models of the fields of freezing of constructions and of the physical fields of radiant energy [7].

It may be interesting to simulate the physical processes of a heat transfer within closed air interlayers. In thermodynamics, these processes are described by criteria equations linking the numbers of Nusselt  $Nu$ , Grashof  $Gr$  and Prandtl  $Pr$  [8]. However, these equations are known only for closed air interlayers that are plane and located horizontally or vertically. But modern architects often use double-glazed windows, having another position, or even non-planar.

#### 5. Heat engineering determination of the optimal thickness of the interglass space in double-glazed windows

One of major factors that determine the value of the heat transfer resistance of double-glazed windows is the thickness of the interglass space. It is important to know the thickness for which the magnitude of the convective component of heat transfer is minimal. Though the transmission by convection amounts at most 1/5 of the heat passing through the interglass space, the use of a double-glazed window with optimal thickness of the interglass space can raise its energy-conservation attribute noticeably.

Equations linking main thermal and physical characteristics of a closed air interlayer are known if it is positioned horizontally or vertically. In thermodynamics, they are named *criteria equations* and connect the Nusselt ( $Nu$ ), Grashof ( $Gr$ ) and Prandtl ( $Pr$ ) numbers. One may apply these equations to double-glazed windows and obtain the dependence

$$\alpha_k = f(\tau_w, \tau_c, \delta, \varphi) \quad (6)$$

of the coefficient  $\alpha_k$  of convective heat transfer upon the temperature  $t_w$  of the warmer glass, the temperature  $t_c$  of the colder glass, the thickness  $\delta$  of the interglass space, and the angle of inclination  $\varphi$  with respect to horizontal ( $\varphi = 0$  or  $\varphi = \pi/2$ ).

With help of the computer algebra system *MathCAD*, we constructed diagrams that present this dependence for several often used values of thickness  $\delta$  of double-glazed windows positioned horizontally or vertically; we assume that the colder glass is posed over the warmer glass if the interlayer is horizontal. This dependence is presented in Figure 3 for  $\delta = 12$  mm.

If  $\varphi = 0$ , then the function  $f(t_w, t_c, \delta, 0)$  obtained from criteria equations is continuous at all points except for the points of the regular surfaces  $Gr=10^4$  and  $Gr=4 \cdot 10^5$  (in Figure 3, the dashed line denotes points of the surface  $Gr=10^4$ ). Of course, this is a gap of the mathematical model; there is a sharp but continuous conversion from the heat transfer via thermal conductivity if  $Gr < 10^4$  to the heat transfer via cellular convection if  $10^4 < Gr < 4 \cdot 10^5$ , and then to the heat transfer via turbulent convection if  $Gr > 4 \cdot 10^5$ .

For  $\varphi = \pi/2$  the conversion from thermal conductivity to convection takes place on the surface  $GrPr \approx 10^3$ .

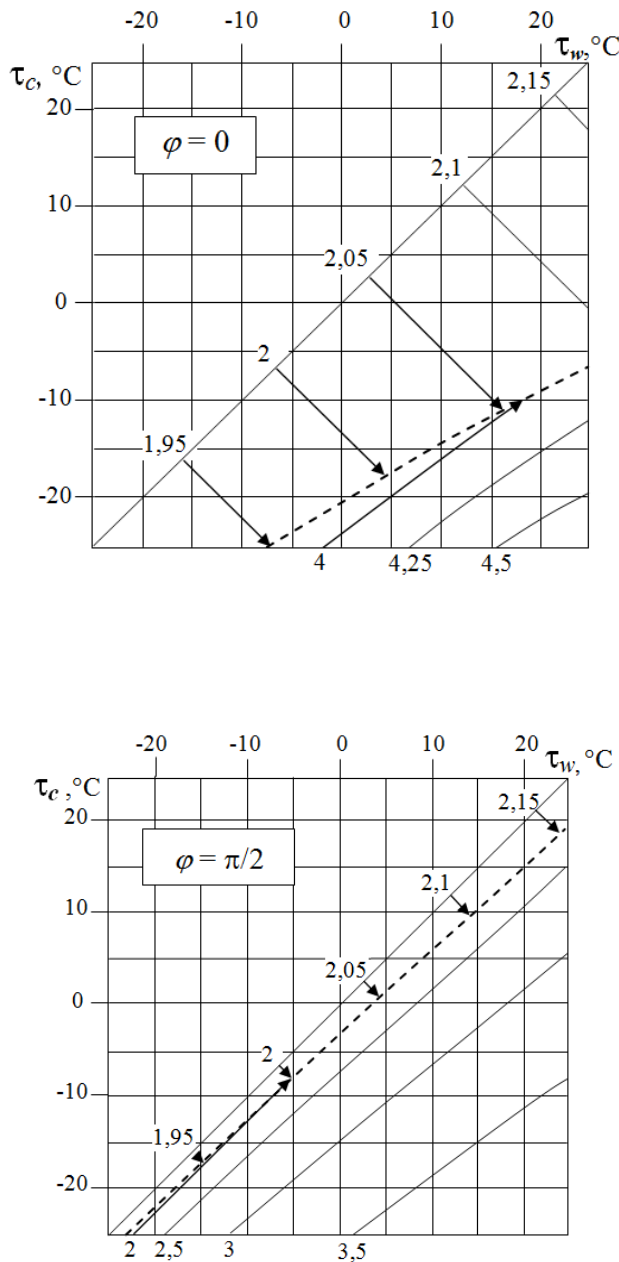
We propose to use a sinusoidal function for the smoothing of  $f(t_w, t_c, \delta, 0)$  along these surfaces.

The criteria equations are unknown if a closed air interlayer has an arbitrary angle of inclination  $\varphi$  with respect to horizontal, although angled windows are often made in mansard roofs. We propose to calculate the coefficient of convective heat transfer for an angled double-glazed window as follows:

$$\alpha_k = \alpha_{k,h}(1 + \cos 2\varphi)/2 + \alpha_{k,v}(1 - \cos 2\varphi)/2, \quad (7)$$

where  $\alpha_{k,h}$  and  $\alpha_{k,v}$  are the values of the coefficient of convective heat transfer of the same window positioned horizontally or vertically.

We actualized this technique in the program Double-Glazed for calculating the heat transfer resistance of double-glazed windows.



**Fig. 3.** Diagrams for calculating the coefficient  $\alpha_k$  of convective heat transfer ( $\delta = 12$  mm)

This program works in the computer algebra system *MathCAD*. With *Double-Glazed*, one may calculate, for his climatic region, the optimal thickness of a double-glazed window wherein the heat leakage through the window is minimal.

## 6. Unsolved geometrical problems of lighting engineering

**Natural lighting.** A geometrical simulation of natural lighting was developed by many authors [9]. The authors assumed an overcast

sky and the allocation of luminosity of the roof of heaven according to the Munn-Spenser law; this is rationally since it essentially simplifies calculations. The most actual unsolved geometrical problems remain the following:

1. To align the coefficient of natural lighting depending on orientation of a window and geographical coordinates of the building.
2. To develop a technique for registering light losing in solar-protection devices.
3. To create a radically new technology of registering light reflected by countering buildings.
4. To improve the technique of accounting the influence of a reflected light from internal surfaces within the building.

**Insolation.** This part of building physics is the most investigated by geometers [10-11]. It may be explained by growing interest to the solar-power engineering. However, the authors study only the direct component of solar radiation. As a rule, they neither appreciate its decay in the atmosphere, nor its decay is taken into account by empirical coefficient. Presently, there is information enough to simulate a direct solar radiation with provision for its decay in the atmosphere, and to simulate a dissipated or total insolation: We know an averaged distribution on the globe of the transmission capacity of the atmosphere, the annual course of cloudiness for different geographical areas, and so on.

It is necessary to reconsider the approach to rating housing insolation since the duration of insolation deficiency characterizes the quality of insolation. The values of solar radiation must be rating with respect to different spectral bands.

**Solar protection.** There is a need to give recommendations concerning choosing a type of stationary solar-protecting devices depending on orientation of windows for different climatic districts of Ukraine. The recommendations must take account of the boundaries of daily overheating [12].

## 7. Conclusions

The methods that we consider allow calculating natural illumination and house insolation, and to find the area and form of windows. They may help to raise the energy effectiveness of buildings. What is very relevant in Ukraine.

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