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## Statistical characteristics of strength distribution of normal sections of bended reinforced concrete elements and their analysis

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# Statistical characteristics of strength distribution of normal sections of bended reinforced concrete elements and their analysis

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**Abstract.** The results of simulation of stochastic corrosion of reinforcement and geometrical characteristics of normal rectangular cross-sections of bent concrete reinforced concrete statically determined beams (elements) with reinforcement on one level in the stretched concrete zone are given. It is proved that in solving the problems of the reliability of such structures, the real histograms are described by various statistical laws. It was also established that considering the effects of corrosion is important for determining the real statistical law of the distribution of the corrosion damage stage in various materials. Parameters of the variation of the variability of the normal section of the considered reinforced concrete structures differ depending on the initial parameter of the variability of reinforcement corrosion damage. The histogram of stress distribution in the reinforcement of the reinforced concrete element and the strength of its normal cross-section do not correspond to the histograms of normal distribution. The use of the Gauss distribution law when determining the strength of a normal section of a bent concrete element, considering the effect of corrosion, can lead to significant inaccuracies in determining the reliability parameters of such structures. Failure to consider sufficiently large asymmetry and excessive values for the resulting statistical distribution laws can lead to significant errors in solving the reliability problems of such structures.

## 1. Introduction

Recently, the tasks related to the assessment of the technical condition of existing structures, buildings and constructions, including a significant number of reinforced concrete structures, are becoming increasingly relevant. Along with determining the residual strength or bearing capacity of damaged structures, it is important to assess their reliability. It is known that one of the most common damage of reinforced concrete structures is the corrosion of the working reinforcement, as well as corrosion or exfoliation of concrete layers. These factors greatly affect the strength, carrying capacity and reliability of such structures. Therefore, conducting additional researches in the assessment of their reliability of such structures, considering the stochastic nature of these factors, is a necessary and topical task. In the problems of reliability of building structures, as a rule, a normal law of the strength distribution of a material or structure is usually adopted, which is quite convenient in determining the probability of failure-free operation of the structure, but in reality, the characteristics of the statistical law of the distribution of structural strength are due to the characteristics of the initial design parameters.



In this work, the normal sections of bended statically determined reinforced concrete beams with the reinforcement on one level in the stretched concrete zone under the action of uniformly distributed load are investigated (Figure 1). And also, the statistical characteristics of the strength distribution in the form of bending moment and characteristics of stress distribution in the reinforcement located in the stretched concrete zone of a normal section are determined.

## 2. Analysis of recent research and publications

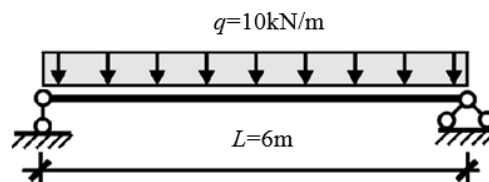
The work of researchers [2,3, 8-10] is devoted to the development of basic approaches to the assessment of the reliability of building structures. Significant development of the reliability theory of building structures acquired in the work of researchers [4, 5]. In [6] an analysis of the influence of corrosion on the statistical characteristics of steel rolling profiles was made. One of the basic works where methods for assessing the reliability of reinforced concrete structures are work [4]. In this case, the lack of results of the statistical simulation of the stochasticity of different initial design parameters of reinforced concrete structures should be noted. To a large extent, this is due to the complexity of using analytical expressions to determine the characteristics of histograms of resultant distributions.

## 3. The purpose of this work

We studied and analyzed the stochastic influence of various design parameters on the characteristics of the distribution of strength in the normal sections of bent concrete elements. In particular, the analysis of the influence of corrosion of stretched reinforcement and stochastic geometric characteristics of concrete cross-section on the distribution of strength of these structures. At the same time, this influence is modelled by different initial laws of statistical distribution.

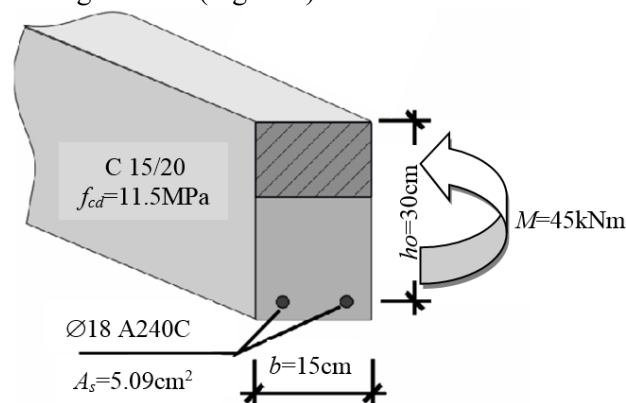
## 4. The main material

For the above-mentioned researches, a statically determined reinforced concrete beam is considered for the action of a uniformly distributed load, the calculation scheme of which is shown in Figure 1.



**Figure 1.** Calculation scheme of reinforced concrete beam

The concrete strength on the compression is taken to be equal C15/20 [1]. The height of the rectangular normal section of the reinforced concrete beam is selected at the span moment  $M=45$  kNm. The height of the compressed zone is 10.8cm. The working reinforcement is two rods  $\text{Ø}18$  A240C. Section reinforcement percentage is 1.13 (Figure 2).



**Figure 2.** Scheme of reinforced concrete beam cross-section.

The condition of reliability with respect to the strength or bearing capacity of a bent concrete element will be recorded as:

$$M \leq f_{yd} \cdot A_s \cdot \left( h_0 - \frac{f_{yd} \cdot A_s}{2 \cdot f_{cd} \cdot b} \right) \tag{1}$$

When modeling the influence of corrosion on the main working reinforcement, when its cross-sectional area decreases, it is convenient to express the condition of reliability of the reinforced concrete section due to the stresses in the reinforcement  $\sigma_s$ , which is compared with the corresponding statistical characteristics of the strength of this reinforcement  $f_{yd}$ . In this case, the reliability condition is written as  $\sigma_s \leq f_{yd}$ . The stresses in the reinforcement are described by the square equation:

$$\sigma_s^2 \cdot \frac{A_s^2}{2 \cdot f_{cd} \cdot b} - \sigma_s \cdot A_s \cdot h_0 + M = 0. \tag{2}$$

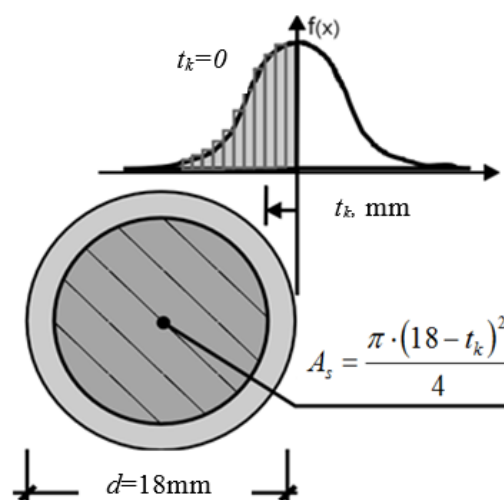
Solving the equation (2) determined the statistical characteristics of the distribution of stresses in the reinforcement, considering the stochasticity of various initial factors:

$$\sigma_s = f(f_{cd}, b, A_s, h_0, M) \tag{3}$$

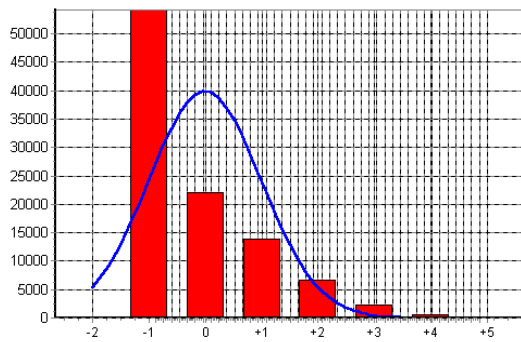
The influence on the characteristics of stress distribution in the reinforcement  $f_{yd}$  of each of the different initial factors was determined by mathematical modeling considering their stochasticity. The scheme of determining the area of the residual cross-section of the reinforcement in the modeling of different reinforcement corrosion depth  $t_k$  according to the Gauss law is shown in Figure 3.

Features of the implementation of mathematical modeling of these studies using a PC are given in [7]. The number of realizations of random variables in these studies was taken at 100.000. In Figures 4–7, the results of mathematical modeling on the PC of the variability of the above-mentioned factors, which determine the strength and reliability of such structures, are given. These drawings include histograms of tension distribution in the reinforcement and the strength of the normal section.

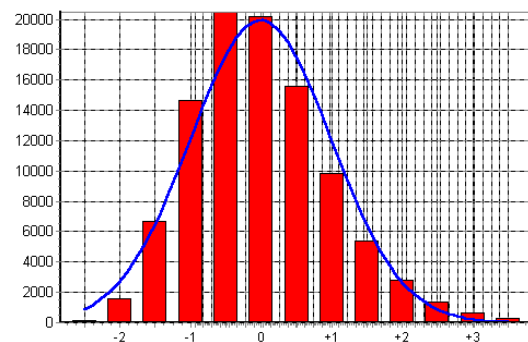
The arguments of all histograms are normalized to zero mathematical expectations. For these histograms, the theoretical Gaussian curve is selected.



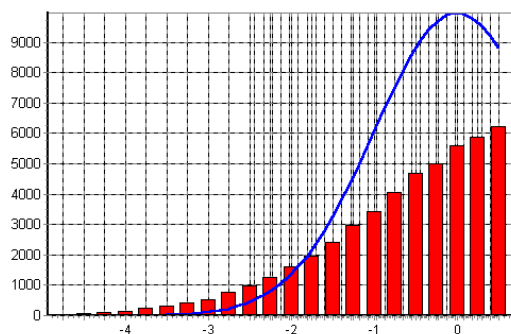
**Figure 3.** Scheme for determining the area of the residual section.



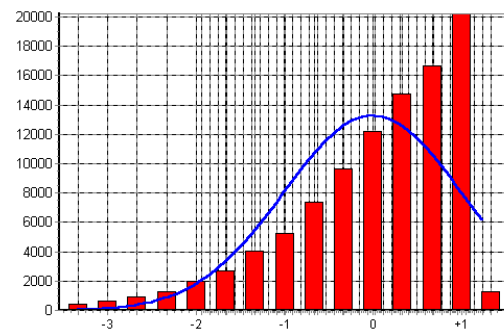
**Figure 4.** Stress distribution histogram in reinforcement during modeling of the influence of its corrosion by the Gauss law  $\sigma_s = f(f_{cd}, b, A_s, h_0, M)$



**Figure 5.** Stress distribution histogram in reinforcement during simulation stochasticity of concrete beam height  $h (h_o = f(h))$ ,  $\sigma_s = f(f_{cd}, b, A_s, h_0, M)$ .



**Figure 6.** Histogram of the strength distribution of the normal section during modeling reinforcement corrosion influence on the Gauss law,  $M = f(f_{cd}, b, A_s, f_{yd}, h_0)$ .



**Figure 7.** Histogram of the strength distribution of the normal section during modeling variability of the reinforcement strength  $f_{yd}$ ,  $M = f(f_{cd}, b, A_s, f_{yd}, h_0)$ .

### 5. Conclusions.

The analysis of the received histograms for the distribution of the parameters of the cross sections reliability of such structures shows that they can be described by different statistical laws. This should be considered when assessing the reliability of the considered structures. The consideration of the effects of corrosion is important to determine the true statistical law of the distribution of the depth of penetration of corrosion in structures materials. This can greatly affect the resultant distribution of the investigated parameter. Statistical variation standard of strength variation in the normal section of the considered elements linearly depends on the output standard in reinforced corrosion damage. The histogram of stress distribution in the reinforcement of the reinforced concrete element and the strength of its normal cross-section often do not correspond to the histograms of the normal Gaussian distribution. Failure to consider sufficiently large values of asymmetry and excess for the resulting statistical distribution laws can lead to significant errors in solving the reliability problems of such structures. The use of the Gauss distribution law when determining the strength characteristics of a normal section of a bending reinforced concrete element, considering the effect of corrosion, can lead to significant inaccuracies in determining the reliability parameters of such structures.

### 6. References

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