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ANALYSIS OF EFFICIENCY OF THE BIOINSPIRED METHOD FOR DECODING ALGEBRAIC CONVOLUTIONAL CODES

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Показано, що для підвищення достовірності передавання інформації у безпроводових телекомунікаційних системах широко застосовуються згорткові коди спільно з різноманітними методами декодування. Наведено загальні принципи синтезу та параметри алгебраїчних несистематичних згорткових кодів з довільною швидкістю кодування та максимально досяжною кодовою відстанню.

Представлено основні етапи біоінспірованого методу декодування алгебраїчних згорткових кодів з використанням механізму випадкового зміщення. Показано, що сутність представленого методу декодування полягає у застосуванні процедури диференційної еволюції з евристично визначеними параметрами. Також у даному методі використовується інформація про надійність прийнятих символів для знаходження найбільш надійного базису узагальненої породжувальної матриці. Додатково застосовується механізм випадкового зміщення для модифікації прийнятої послідовності з метою здійснення біоінспірованого пошуку на основі різних найбільш надійних базисів узагальненої породжувальної матриці.

За результатами досліджень визначено, що біоінспірований метод декодування алгебраїчних згорткових кодів забезпечує більшу ефективність у порівнянні з алгебраїчним методом декодування у каналі зв'язку з адитивним білим гауссовим шумом. В залежності від параметрів алгебраїчного згорткового коду та необхідного коефіцієнту помилок енергетичний виграш від кодування становить від 1,6 дБ до 3 дБ. Показано, що представлений біоінспірований метод декодування може бути використаний для згорткових кодів з великою довжиною кодового обмеження.

При цьому представлений метод декодування алгебраїчних згорткових кодів програє за ефективністю методу декодування Вітербі та турбокодам при достатній кількості ітерацій декодування

Ключові слова: безпроводові телекомунікаційні системи, згорткові коди, алгебраїчна структура, декодування, біоінспірований пошук

1. Introduction

The introduction of new telecommunications services and the need to provide access to information resources over a large territory predetermines the relevance of application of wireless telecommunication technologies.

A variety of noise resistant encoding methods are used to ensure the desired reliability of information transmission in telecommunication systems, based on these technologies. Specifically, convolutional codes are widely used in wireless telecommunication systems [1]. Synthesis of random codes of this class is based on a computer search that is characterized by high computational complexity and does not guarantee obtaining codes with the required properties [2]. Convolutional codes with the algebraic structure, the so-called algebraic convolutional codes, were proposed to remove these constraints. These codes are based on generalization of the provisions of the block code theory, have high structural code characteristics and improved properties.

In paper [3], the polynomial representation of convolutional codes, which made it possible to identify their cyclic structure and to obtain the codes with a large free distance, was proposed. In paper [4], the algebraic approach to decomposition of any convolution code into several sub-codes was presented and the parameters of the obtained cyclic convolutional codes were given. In article [5], the concept of cyclic convolutional codes was developed and a relationship between the polynomial and vector of representation of these codes based on the generalized circular matrices was found. In paper [6], the principles of construction of the double cyclic convolutional codes were presented and it was shown that separate codes of the given class are close to optimal. In article [7], an innovative approach to the implementation of the cyclic structure of convolutional codes and the corresponding binary codes based on polynomials of a special kind was proposed. In [8, 9], algebraic principles of the construction of convolutional codes with the given encoding rate and the necessary code distance that have the properties similar to the classical block codes were presented. Paper [10] proposed an algebraic approach to determining generator polynomials and structural parameters of convolutional codes through the selected non-binary cyclic block code.

The classic decoding of random convolutional codes by the Viterbi method is based on the search in the code trellis with the use of probabilistic procedures. Due to the high computational complexity, this decoding method can be applied only to the codes with a small code constraint length. In addition, the possible algebraic structure of a code is not taken into account during implementation of this method [11].

Thus, the research aimed at further improvement and development of the methods for decoding convolutional codes by using the information on their algebraic structure, as well as determining their characteristics and properties, should be considered relevant.

2. Literature review and problem statement

At present, most methods of decoding algebraic convolutional codes are based on the approach that takes into consideration the cyclic structure of these codes by analogy with block codes.

Paper [12] presents the algebraic method for decoding a particular class of cyclic convolutional codes, which guarantees correction of a fixed number of errors. In this case, the limit of successful decoding is determined by the minimum code length of a code. According to this method, the accepted sequence is divided into separate blocks. After accumulating several blocks, decoding of the first of the obtained blocks of the accepted sequence begins. Partial decoding of the determined cyclic block code is carried out at each stage of the method.

In article [13], the principles of construction and the method of decoding the convolutional codes with special cyclic structure and the maximum achievable code distance were shown. The decoding method based on the procedure of finding the positions and error values in accordance with the principle of decoding by a minimum distance was presented. In this case, the computational complexity of this method practically does not depend on the size of the finite field and grows polynomially with an increase in code memory. Due to these characteristics, the presented decoding method can be applied to convolutional codes with high structural characteristics.

Paper [14] considers the features of formation of a code sequence of convolutional codes, parameters of which are assigned by a generator polynomial of the selected cyclic code. In this paper it was shown that decoding these codes can be carried out by serial decoding of codewords of the given cyclic code. According to this method, the syndrome for each block of the accepted sequence is determined. After that, the errors in the accepted block are found by means of the calculation of the system of equations in the finite field and the codeword is recovered.

Through processing of hard decisions, the above algebraic methods for decoding the convolutional codes are characterized by relatively low correction ability and low efficiency.

At the same time, known advantage of likelihood decoding of convolutional code is the possibility of taking into consideration the information about reliability of accepted symbols, that is, the implementation of soft decision processing. This direction of development of the methods for decoding the random convolutional codes, specifically, the codes with the ring structure.

Specifically, a two-phase approach to decoding cyclic closed convolutional codes based on the Viterbi method was proposed in [15]. This approach uses the assumption that all possible states in the code trellis can act as an initial state with an equal likelihood. At the first stage, the search for the state in the grid trellis, which has the highest credibility using a modified Viterbi method with soft output, is performed. After finding this state, the circular properties of these codes are used and Viterbi decoding is applied that begins and ends on the selected state, determined at the previous stage. The efficiency of this decoding method by the criterion of decoding error is close to optimal. This method has a fixed and relatively low computational complexity in the given range of the signal/noise ratio.

In paper [16], there is a comparative analysis of the efficiency of different approaches to the Viterbi list decoding of convolutional codes with the ring structure that is used to transmit short information sequences. It was shown that in this case the highest efficiency is ensured by the circular parallel Viterbi decoding by lists that are characterized by the smallest computational complexity.

The method of decoding by a maximum likelihood of cyclically closed convolutional codes with reduced complexity was proposed in [17. This method is based on bi-directional search by the first priority. The presented approach ensures steady computing complexity of decoding with an increase in the information sequence length and the code constraint length of the convolution code.

A promising development of the methods for soft decoding of convolutional code implies the application of new approaches based on the mathematical apparatus of natural computing.

In paper [18], the method for decoding the convolutional codes with an arbitrary encoding rate based on recurrent neural networks was proposed and encoding optimization strategies were given. The research conducted in the work showed that this decoding method provides reliability of information transmission at the level of the Viterbi method, but with a lower computational complexity and the possibility of parallel information processing.

Paper [19] presented the adaptive method of soft decoding of convolutional codes based on artificial neural networks. In this paper, training of a neural network was carried out according to the principle of teacher-guided learning, and computer modeling was used for the optimization of systematic parameters of the method. In this paper it was shown that the efficiency of the proposed decoding method is close to that of the Viterbi method. Characteristic properties of this method include the adaptability to the conditions of information transmission, reduction of training and decoding time, non-iterative principle of information processing and the possibility of parallel decoding.

A significant disadvantage of the above methods of soft decoding of convolutional codes based on the search in the code trellis and the use of neural networks is the rapid increase in computational complexity at an increase in code constraint length, which significantly narrows the area of their application in practice. In addition, to enhance efficiency and to decrease computational complexity of decoding, it is advisable to take into account the algebraic structure of the selected convolutional code.

On the other hand, an innovative approach in the theory of block codes is representation of a decoding problem in the form of an optimization problem for the solution of which it is advisable to apply bioinspired procedures, specifically, genetic algorithms.

The methods for decoding algebraic block codes based on the joint application of the genetic algorithm, information sets and the Chase method are proposed in paper [20]. This research revealed that the efficiency of the presented hard decoding method is similar to that of the Berlecamp-Messi method. In addition, the conducted research implies that the proposed method of soft decoding of block codes ensures the energy gain from encoding at the level of the existing combined decoding method.

To reduce the computational complexity of soft decoding of block codes based on the genetic algorithm, it is proposed in [21] to use the verification matrix, that is, to perform decoding in the binary space. The characteristics of this approach at different parameters of the genetic algorithm and the assigned parameters of the selected classical block codes were studied. It was shown that the proposed approach provides additional gain from encoding in comparison with the existing decoding methods for various models of communication channels.

Article [22] presented the methods for decoding linear block codes in the binary space based on the compact genetic algorithm with an increased size of the tournament The work proposed different approaches to increasing the size of the tournament and determined the influence of the parameters of the selected genetic algorithm on the quality of decoding of algebraic block codes. It was shown that the proposed decoding methods provide high enough gain from encoding and are characterized by a reduced computational complexity.

Generalization of these results in the case of convolutional codes was proposed in [23]. This paper presents the main stages of the method for soft decoding of algebraic convolutional codes based on bioinspired procedures. The key stage of this method is an application of the generalized bioinspired search to determine the predicted code sequence. The additional components of this method include finding the most reliable basis of the generalized generator matrix of an algebraic convolution code and the application of the random shift mechanism. This decoding method can be regarded as further development of the methods for decoding convolutional codes based on natural computations and as an alternative to hard decoding of algebraic convolutional codes.

However, in paper [23], only the general conceptual idea of this approach was presented and no information on the characteristics of the presented decoding method for assigned conditions of information transmission was given. Specifically, an important task is to determine the efficiency of this method for decoding algebraic convolutional codes during the application of the specific bioinspired procedures and the influence of additive noise, which causes the need to conduct further research in this direction.

3. The aim and objectives of the study

The aim of this study was to assess the efficiency of the bioinspired method of decoding of algebraic convolutional codes in the communication channel with additive white Gaussian noise (AWGN).

To accomplish the aim, the following tasks have been set:

– to analyze the principles of synthesis and to determine parameters of algebraic non-systematic convolutional codes with arbitrary encoding rate;

– to consider the main stages and determine the features of the bioinspired method for decoding algebraic convolutional codes using the random shift mechanism;

– to assess the efficiency of the bioinspired method for decoding algebraic convolutional codes at the influence of AWGN.

4. Methods for analysis of efficiency of bioinspired decoding of algebraic convolutional codes

4. 1. Principles of synthesis and parameters of algebraic convolutional codes

In paper [10], it was shown that a generalized generator polynomial of a convolution code is actually a generator polynomial of some non-binary block code, which completely determines the characteristics of this convolutional code.

Consider the principles of synthesis of algebraic non-systematic convolutional codes with arbitrary encoding rate $R=k_0/n_0$ and the maximum achievable free code distance d_∞ based on generator polynomial (N, K, D) of the Reed-Solomon code.

Let us assume that an infinite sequence of information symbols divided into frames of the length of k_0 arrive at the input of a convolutional encoder:

$$i=(i_{0,1},\ldots,i_{0,k_0},i_{1,1},\ldots,i_{1,k_0},i_{2,0},\ldots,i_{2,k_0},\ldots),$$

where $i_{i,j}$ is the information symbols united into frames of k_0 elements, $i_{i,j}\in GF(q)$, $i=0, 1, 2, \ldots, j=1, 2, \ldots, k_0$.

Then the infinite information sequence of a non-systematic convolutional code with encoding rate $R=k_0/n_0$ can be written down in a polynomial form of notation:

$$i(x)=(i_{0,1},\ldots,i_{0,k_0})+(i_{1,1},\ldots,i_{1,k_0})x+(i_{2,0},\ldots,i_{2,k_0})x^2+\ldots. \quad (1)$$

The sets of information symbols of a non-systematic convolutional code with encoding rate $R=k_0/n_0$ can be considered as the elements of field $GF(q^m)$, which is an extension of the output field $GF(q)$, and in this case the length of an information frame is supplemented with zeros to value $m=n_0$.

Then the information polynomial (1) will be represented in the following way:

$$i(x) = I_0 + I_1 x + I_2 x^2 + \dots, \qquad (2)$$

where I_i is the information symbols, $I_i \in GF(q^m)$, $i=0, 1, 2, \dots$.

On the other hand, polynomial (2) corresponds to an information vector of non-finite length, obtained as a result of reading the coefficients at a formal variable x:

$$i = (I_0, I_1, I_2, \dots). \qquad (3)$$

Therefore, we obtained non-binary information symbols of the given convolution code form set $H \subseteq GF(q^m)$, the size of which is $|H| = q^{k_0} \le q^m$, at $k_0 \le m$.

According to [10], a non-systematic convolutional code with encoding rate $R = k_0/n_0$ can be algebraically assigned by a generator polynomial, which is actually a generator polynomial of the Reed-Solomon code:

$$G(x) = (x - \alpha^b)(x - \alpha^{b+1})\dots(x - \alpha^{b+D-2}), \qquad (4)$$

where α^b, α^{b+1}, ... , α^{b+D-2} are the roots of polynomial $G(x)$, which belongs to field $GF(q^m)$; b is the non-negative integer; D is the minimum code distance of Reed-Solomon code.

After the computations, the generalized generator polynomial of the convolution code (4) can be represented as:

$$G(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \dots + \alpha_u x^u, \qquad (5)$$

where u is the memory of the convolutional code that corresponds to the number of check symbols in a codeword in the Reed-Solomon code, $u=D-1$; α_0, α_1, ... , α_u are the roots of polynomial $G(x)$, which belongs to field $GF(q^m)$.

Then the process of convolutional encoding of an information sequence in the polynomial form corresponds to multiplication of information polynomial (2) by the generalized generator multinomial (5):

$$c(x) = i(x)G(x) = C_0 + C_1 x + C_2 x^2 + \dots, \qquad (6)$$

where C_i is the code symbols of a convolutional code, $C_i \in GF(q^m)$, $i=0, 1, 2, \dots$.

To obtain the q-th code sequence, it is necessary to display code symbols, obtained in accordance with (6), in the sets of the elements of field $GF(q)$ that correspond the frames of a code sequence of length n_0:

$$c(x) = (c_{0,1}, \dots, c_{0,n_0}) + (c_{1,1}, \dots, c_{1,n_0})x + (c_{2,0}, \dots, c_{2,n_0})x^2 + \dots, \quad (7)$$

where $c_{i,k}$ is the code symbols united in frames by n_0 elements, $c_{i,k} \in GF(q)$, $i=0, 1, 2, \dots$, $k=1, 2, \dots, n_0$.

Therefore, the described above process of convolutional encoding can be represented in a vector form using the infinite generator matrix that is represented in the generalized form. For convenience of the vector representation of the encoding process by the algebraic non-systematic convolutional code with encoding rate $R = k_0/n_0$, we will transform the generalized generator polynomial (5) as follows:

$$G(x) = G_0 + G_1 x + G_2 x^2 + \dots + G_u x^u, \qquad (8)$$

where G_0, G_1, ... , G_u are the symbols that belong to field $GF(q^m)$, which assign the form of the mutual connection of convolution coder registers. Then a generalized generator matrix of the algebraically assigned non-systematic convolutional code with encoding rate $R = k_0/n_0$ will take the following form:

$$G = \begin{pmatrix} G(x) \\ xG(x) \\ x^2 G(x) \\ \dots \end{pmatrix}. \qquad (9)$$

Therefore, the process of convolution encoding in the vector form of notation corresponds to the multiplication of information vector (3) by generator matrix (9):

$$c = iG = (C_0, C_1, C_2, \dots)$$

with further display of the symbols of field $GF(q^m)$ into vectors over field $GF(q)$ to obtain the code vector, which corresponds to polynomial (7):

$$c = (c_{0,1}, \dots, c_{0,n_0}, c_{1,1}, \dots, c_{1,n_0}, c_{2,0}, \dots, c_{2,n_0}, \dots).$$

In [10], it was shown that non-systematic (n_0, k_0, V) convolutional code with encoding rate $R = k_0/n_0$ over field $GF(q)$, algebraically assigned by generalized generator polynomial (8) or by generator matrix (9), has the following parameters: information frame length $k_0 = \log_q(H)$ $(H \subseteq GF(q^m))$; code frame sequence length $n_0 = m$; code memory u; code constraint length $V = uk_0$; length of information block $k = (u+1)k_0$; code block length $n = (u+1)n_0 = kn_0/k_0$; free code distance $d_\infty \ge D$.

4. 2. Bioinspired method for decoding algebraic convolutional codes

Restrict the length of the information sequence that arrives at the input of the convolutional coder to value K, then taking into account (6), we will get the code polynomial:

$$C(x) = C_0 + C_1 x + C_2 x^2 + \dots + C_{N-1} x^{N-1}. \qquad (10)$$

In this case, the polynomial (10) can be formally represented in the form of code polynomial (N, K, D) of the Reed-Solomon code, and the corresponding binary display of this code sequence takes the form:

$$c(x) = (c_{0,1}, \dots, c_{0,n_0}) + \\ + (c_{1,1}, \dots, c_{1,n_0})x + \dots + (c_{N-1,0}, \dots, c_{N-1,n_0})x^{N-1}. \qquad (11)$$

Let us assume that information transmission using this algebraic convolutional code is implemented with the use of binary phase modulation, then the code sequence (11) can be represented in the polynomial form by the corresponding bipolar code sequence:

$$v(x) = (v_{0,1}, \dots, v_{0,n_0}) + \\ + (v_{1,1}, \dots, v_{1,n_0})x + \dots + (v_{N-1,0}, \dots, v_{N-1,n_0})x^{N-1}, \qquad (12)$$

where $v_{i,k}$ is the bipolar code symbols, united in frames by n_0 elements, $v_{i,k} \in \{1, -1\}$, $i=0, 1, 2, \dots$, $k=1, 2, \dots, n_0$.

Then at the output of the communication channel with AWGN, the accepted sequence in the polynomial form equals to:

$$r(x)=(r_{0,1},...,r_{0,n_0})+$$
$$+(r_{1,1},...,r_{1,n_0})x+...+(r_{N-1,0},...,r_{N-1,n_0})x^{N-1}, \quad (13)$$

where $r_{i,k}$ are the accepted symbols, united into frames by n_0 elements, $r_{i,k}\in R$, $i=0, 1, 2, ..., k=1, 2, ..., n_0$.

It should be noted that the matrix representation unambiguously corresponds to the polynomial representation of algebraic convolutional codes based on (11)–(13). In this case, the information, code and the accepted sequences can be represented as vectors, and the generalized generator polynomial of an algebraic convolutional code (5) corresponds to the generalized generator matrix (9).

Taking into consideration the fact that algebraic convolutional codes can be represented as binary display of Reed-Solomon codes, that is, actually in the form of long binary ($N'=Nn_0, K'=Kn_0$) block codes, the bioinspired method for soft decoding of these codes using the random shift mechanism was proposed in paper [23].

The main stages of this method for decoding algebraic convolutional code based on the procedure of differential evolution [24] are given below.

Stage 1. Initialization.

Determining the initial iteration $l=0$, the maximal number of decoding iterations L, parameters of the procedure of differential evolution (population size is NP, maximal number of generations is $L_{\max}$, impact factor a, probability of «crossover» b), magnitudes of random shift θ and formation of accepted sequence $q=(q_0, q_1, ..., q_{N'-1})$, where $q_i=|r_i|$.

Stage 2. Ordering the accepted sequence based on the information about reliability of symbols.

Location of positions of accepted sequence q according to decreasing reliability of elements $q_i=|r_i|\geq q_j=|r_j|$, which determines permutation π_1 at $l=0$ and permutation π_1' at $l>0$.

Stage 3. Finding the most reliable basis of the generator matrix of the algebraic convolutional code.

Ordering the columns of generator matrix (9) based on π_1 at $l=0$ or π_1' at $l>0$, determining the most reliable basis of the given matrix and transformation of the obtained matrix into the systematic form G_s at $l=0$ or G_s' at $l>0$ based on permutation π_2 at $l=0$ or π_2' at $l>0$.

Stage 4. Bioinspired search for the predicted code sequence of an algebraic convolutional code with the use of the procedure of differential evolution.

Step 4. 1. Initialization of the population of vectors of decisions (information sequences of an algebraic convolutional code) and establishment of the number of initial generations $g=1$.

The first vector of decisions $\tilde{i}_{1,g}$ is formed by hard decision for the first K' symbols of ordered sequence q, which corresponds to the most reliable information sequence. The rest of vector of solutions $\tilde{i}_{2,g}$, $\tilde{i}_{3,g}$, ..., $\tilde{i}_{NP,g}$ are generated randomly.

Step 4. 2. Selection of objective vector of solutions $\tilde{i}_{v,g}$, $v\in[1, NP]$.

Step 4. 3. Formation of the mutation vector of decisions with the use of the operator of differentiating mutation.

$$\tilde{i}_{v,g+1}=\tilde{i}_{r_1,g}+a\cdot(\tilde{i}_{r_2,g}-\tilde{i}_{r_3,g}), \quad (13)$$

where $\tilde{i}_{r_1,g}$, $\tilde{i}_{r_2,g}$, $\tilde{i}_{r_3,g}$ are the random vectors of decisions form current generation g, $\tilde{i}_{r_1,g}\neq\tilde{i}_{r_2,g}\neq\tilde{i}_{r_3,g}\neq\tilde{i}_{v,g}$, $v\in[1, NP]$; a is the impact factor, $a\in[0, 2]$.

Step 4. 4. Formation of the trial vector of solutions using the crossover operator:

$$\tilde{i}'_{v,g+1}=(\tilde{i}'_{w,v,g+1},\tilde{i}'_{w,v,g+1},...,\tilde{i}'_{w,v,g+1}), \quad (14)$$

where $\tilde{i}'_{w,v,g+1}$ is the elements of the trial vector of solutions, $w=1, 2, ..., K'$, $v\in[1, NP]$, which are determined in the following way:

$$\tilde{i}'_{w,v,g+1}=\begin{cases}\tilde{i}_{w,v,g+1}, & \text{if } U_w\leq b \text{ or } w=w_r,\\ \tilde{i}_{w,v,g}, & \text{else,}\end{cases}$$

where U_w is the random number, which is generated by the uniform distribution law in the range of [0, 1]; b is the «crossover» probability, $b\in[0, 1]$; w_r is the random index determined by the element of the trial vector of solutions.

Step 4. 5. Encoding the obtained trial vector of solutions (information sequence) with the use of the modified generator matrix of algebraic convolutional code G_s (or G_s').

Step 4. 6. Performance of the selection operation.

Assessment of the quality of a trial vector (and the corresponding code sequence of an algebraic convolutional code) by calculation of the non-correlation function, which plays the role of an objective function in the procedure of differential evolution:

$$E(r,v)=\sum_{i:\, r_i\cdot v_{i,k}<0}|r_i|, \quad (15)$$

where $|r_i|$ is the «reliability» of accepted symbols that is determined by the absolute value (amplitude) of symbols.

If for the trial vector of solutions (14) the objective function (15) has a lower value than for the objective vector of solutions (13), then a given vector replaces the objective vector of solutions in the next generation $g=g+1$. Otherwise, the current objective vector of solutions transfers into the next generation $g=g+1$.

Step 4. 7. If the maximal number of generations $g\leq L_{\max}$, proceed to step 4.1; otherwise, the best vector of solutions (and the corresponding code sequence of the algebraic convolutional code) is determined and we proceed to stage 5.

Stage 5. Application of random shift to the elements of the accepted sequence.

If the number of iterations $l\leq L$, we proceed to stage 2 after their formation of the elements of trial vector r':

$$r_i'=r_i+\theta,$$

where θ is the random shift, which is a binary random magnitude that accepts with equal probability value $\pm a$, where a is a real number.

In [25], it was shown that the use of this mechanism makes it possible to perform randomly the exchange in positions between the groups of symbols of the accepted sequence with varying reliability. Actually, the presented mechanism enables the formation of a new most reliable basis with the best characteristics by excluding some error positions.

Stage 6. Formation of assessment of the transmitted code sequence with the help of inverse display and completion of the decoding process.

After reaching the maximum number of decoding iterations, there occurs the formation of the estimate of the transmitted code sequence by the reverse display of the best code sequence obtained at stage 4:

$$y'=(y_0', y_1', ..., y_{N'-1}')=\pi_2'[\pi_1'[r]].$$

Thus, a key feature of the presented method for decoding algebraic convolutional codes implies determining the predicted code sequence using the bioinspired search. In addition, an important component of this method is finding the most reliable basis for different trial vectors, which are obtained using the random shift mechanism.

In this case, the most reliable basis directly for the accepted sequence is in zero iteration and the new most reliable bases for modified accepted sequences obtained by applying the mechanism of random shifts are formed in the following iterations. This makes it possible to generate new code sequences of an algebraic convolutional code using updated sets of the most reliable bases for the assigned accepted sequence, which leads to an increase in efficiency of the bioinspired search for the predicted code sequence.

Computational complexity and efficiency of the presented method for decoding algebraic convolutional codes mainly depends on the selected bioinspired procedure and the magnitude of random shift, the selection of which is determined by the characteristics of the communication channel and other factors.

4. 3. Procedure for estimating efficiency of the bioinspired method for decoding algebraic convolutional codes

To assess the efficiency of the bioinspired method for decoding algebraic convolutional codes, we developed a computer model of a wireless telecommunication system that takes into consideration specific features of information transmission in the communication channel with AWGN with the use of the given types of codes and the presented decoding method. In this case, this model provides the ability to change the parameters of algebraic convolutional codes, the magnitude of interference in the communication channel and makes comparison of the presented method with the existing decoding methods [11, 14]. The basic steps of computer modeling of the process of information transmission in a wireless telecommunications system are shown below.

Step 1. Establishment of the parameters for algebraic convolutional code, assigning a generator polynomial (generalizing generator matrix).

Step 2. Establishment of parameters of bioinspired method for decoding with random shift.

Step 3. Establishment of the range of the signal/noise ratio and the information sequence length.

Step 4. Formation of the information sequence.

Step 5. Formation of the code sequence based on the generalized generator polynomial (generalized generator matrix).

Step 6. Transformation of the code sequence into the signal of binary phase modulation.

Step 7. Formation of AWGN and its adding to the modulated signal.

Step 8. Decoding the accepted sequence with the use of the bioinspired method for decoding with random shift.

Step 9. Determining the error coefficient for the selected decoding methods.

Step 10. If a maximum value of the signal/noise ratio has been achieved and an information message has been completely transmitted, proceed to step 11, otherwise, proceed to step 4.

Step 11. Displaying the dependence of error coefficient on the signal/noise ratio for the selected decoding methods.

Thus, software implementation of this algorithm makes it possible to estimate energy efficiency when using the bioinspired method for decoding the algebraic convolutional codes and to compare it with the existing decoding methods.

When performing experimental studies with the use of the developed computer model, we used the following settings:

1) code type – algebraic convolutional code with assigned parameters;

2) range of signal/noise ratio – from 0 to 9 dB;

3) number of the transmitter information messages (code sequences of a convolutional code) – 1000;

4) parameters of the bioinspired method for decoding based on the procedure of differential evolution [24] – maximal number of decoding iterations $L=50$; magnitude of random shift $\theta=0.1$; parameters of the procedure of differential evolution – size of population $NP=20$, maximal number of generations $L_{\max}=100$, impact factor $a=0.7$, «crossover» probability $b=0.9$.

5. Results of analysis of efficiency of the bioinspired method for decoding the algebraic convolutional codes

Comparison of modeling results in the channel with AWGN of the represented bioinspired decoding method with random shift, the algebraic decoding method and the Viterbi method for algebraic (n_0, k_0, V) convolutional codes with the selected parameters is shown in Fig. 1, 2.

Analysis of Fig. 1 reveals that the represented bioinspired decoding method with random shift compared with the algebraic decoding method for algebraic (3, 1, 3) convolution code provides the energy gain from encoding of 1.6 dB at error coefficient 10^{-2}; 2.4 dB at error coefficient 10^{-3}; about 3 dB at error coefficient 10^{-4}. At this, the proposed decoding method is less efficient than the Viterbi method in the range of low signal/noise ratio (about 1 dB).

It follows from the diagrams shown in Fig. 2 that the proposed decoding method for algebraic (4, 1, 8) convolutional code compared to the algebraic decoding method provides the energy gain of encoding of around 2.4–2.7 dB at error coefficient less than 10^{-2}. In addition, it also follows from Fig. 2 that at an increase in code constraint length, the efficiency of the presented decoding method decreases in comparison with the Viterbi method (the loss is about 1.5 dB).

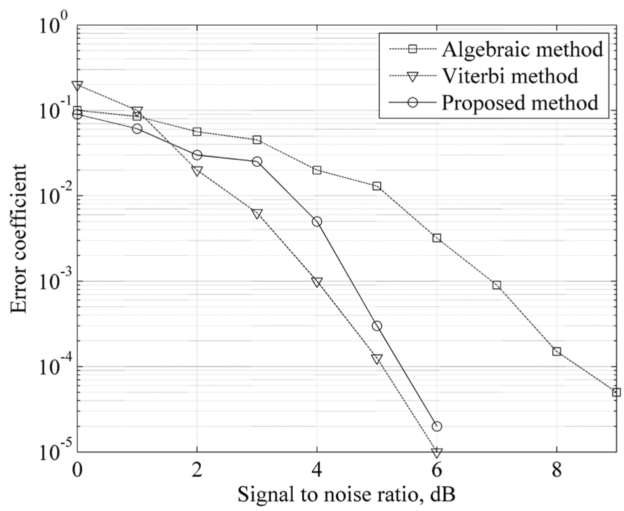

Fig. 1. Dependence of coefficient of errors from the signal/noise ratio for algebraic (3, 3, 1) convolution code

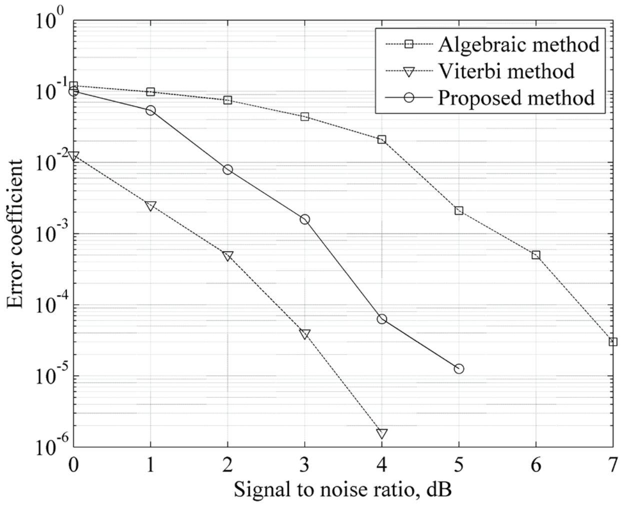

Fig. 2. Dependence of error coefficient on the signal/noise ratio for algebraic (4, 1, 8) convolution code

Comparison of modelling results in the channel with AWGN of the proposed decoding method and the algebraic decoding method for algebraic (5, 1, 16) convolutional code is shown in Fig. 3. Because of a large code constraint length, decoding of this convolution code is impossible to implement by the Viterbi method. That is why the efficiency of the bioinspired decoding method with random shift was compared with the characteristics of the turbo code for different number of iterations of decoding (Fig. 3). This turbo code has encoding rate $R=1/2$, and consists of recursive systematic convolutional codes with a code constraint length of $V=4$.

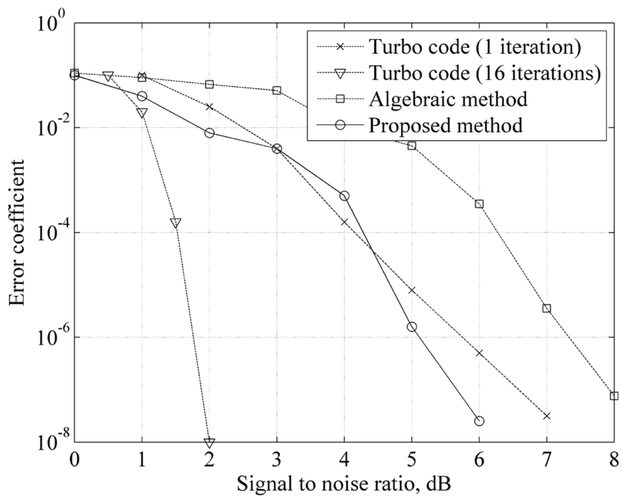

Fig. 3. Dependence of error coefficient on the signal/noise ratio for algebraic (5, 1, 16) convolutional code and turbo code

It follows from the diagrams presented in Fig. 3 that the bioinspired decoding method in comparison with the algebraic decoding method for algebraic (5, 1, 16) convolutional code provides energy gain from encoding of 2.1 dB at error coefficient 10^{-2}; 2.2 dB at error coefficient 10^{-5}; 2.3 dB at error coefficient 10^{-7}. It also follows from Fig. 3 that in the range of a high signal/noise ratio, the characteristics of this convolutional code during the application of the proposed decoding method exceed the characteristics of the turbo code after 1 iteration of decoding by 1 dB. At a decrease in the number of iterations up to 16, efficiency of a turbo code considerably exceeds the characteristics of the algebraic convolutional code, although it has a significant code constraint length.

Table 1 gives numerical results on the efficiency of decoding methods that were obtained in the course of experimental research for algebraic convolutional codes with the selected parameters in the assigned range of error coefficient.

Table 1

Numerical results concerning efficiency of the methods for decoding algebraic convolutional codes

Parameters of a code	Error coefficient	Signal/noise ratio at algebraic decoding, dB	Signal/noise ratio at bioinspired decoding, dB	Energy gain from encoding, dB
(3, 1, 3)	10^{-2}	5.1	3.5	1.6
	10^{-3}	6.9	4.5	2.4
	10^{-4}	8.4	5.4	3
(4, 1, 8)	10^{-2}	4.3	1.9	2.4
	10^{-3}	5.5	3.1	2.4
	10^{-4}	6.6	3.9	2.7
(5, 1, 16)	10^{-2}	4	1.9	2.1
	10^{-3}	5.6	3.7	1.9
	10^{-4}	6.3	4.3	2

The analysis of Table 1 reveals that at the assigned values of error coefficient for the selected algebraic convolutional codes, energy gain from encoding when using the bioinspired decoding method in comparison with the algebraic decoding method is from 1.6 dB to 3 dB. In addition, it should be noted that the use of the bioinspired method with random shift for decoding the algebraic convolutional codes with high encoding rate ($R=1/3$, $R=1/4$) provides higher efficiency.

6. Discussion of results of analyzing efficiency of the bioinspired method for decoding algebraic convolutional codes

The methods for soft decoding of convolutional codes based on the search in the code trellis and the application of neural networks [15–19] can be used only for convolutional codes with a small code constraint length ($V\leq 9$) due to the rapid growth of computational complexity. According to the research results, the presented method ensures decoding of the convolutional code with a quite large code constraint length ($V\leq 16$).

In comparison with the existing methods for decoding convolutional codes with the algebraic structure, using the mathematical apparatus of the theory of finite fields and linear algebra [12–14], the bioinspired decoding method is based on provisions of the theory of stochastic optimization and additional heuristic procedures. As a result, the simplification of the formal representation of this decoding method is achieved and visibility of its main stages is ensured. In addition, a significant advantage of the presented decoding method is the possibility of taking into consideration the information of reliability of the symbols received from the communication channel, that is, the implementation of soft decoding of algebraic convolutional codes. Due to this, the presented decoding method in comparison with the algebraic decoding method [14] in a given range of the signal/noise ratio ensures an average energy gain from encoding of about 2 dB.

The shortcomings of the presented method for decoding the algebraic convolutional codes include uncertainty at selecting the operating parameters for the assigned characteristics of the communication channel, such as the type of the bioinspired procedure and its parameters. The proposed decoding method has high enough computational complexity of technical implementation. In addition, the presented method for decoding algebraic convolutional codes is less efficient than the Viterbi decoding method and turbo codes at sufficient number of iterations of decoding.

The performed research can be considered as the component for improving the characteristics of wireless telecommunication systems through the use of convolutional codes that should ensure specific reliability of information transmission while providing some services.

In further studies, it is planned to formalize the selection rules at the key stage of the presented method of decoding of a particular bioinspired procedure and its parameters depending on the conditions of information transmission. In addition, it is advisable to carry out a comparative analysis of efficiency of the application of different bioinspired procedures when implementing the proposed decoding method.

The results obtained in the course of research are independent and can be used to upgrade the existing wireless telecommunication systems or when developing advanced telecommunication technologies.

7. Conclusions

1. Convolutional codes are widely used, along with various decoding methods, to increase the reliability of information transmission in wireless telecommunication systems. Convolutional codes with the special algebraic structure, the parameters of which are completely assigned by the generalized generator polynomial or the corresponding generalizing generator matrix, can be represented as long binary block codes.

2. The key feature of the presented bioinspired method for decoding the algebraic convolutional codes implies the use of a certain bioinspired procedure with the heuristically determined parameters as a search mechanism. In addition, this method uses information about the reliability of accepted symbols at each decoding iteration in order to find the most reliable basis of the generalized generator matrix. Using this approach makes it possible to generate more accurate trial information and code sequences at the stage of bioinspired search.

The random shift mechanism, which is designed to modify the accepted sequence for the purpose of bioinspired search based on a variety of the most reliable bases of the generalized generator matrix, is additionally used to enhance efficiency of decoding.

3. The bioinspired method for decoding algebraic convolutional codes based on the procedure of differential evolution ensures higher efficiency in comparison with the algebraic decoding method in the communication channel with AWGN. Depending on the parameters of the algebraic convolutional code and the required error coefficient, energy gain from encoding is from 1.6 dB to 3 dB. In addition, in contrast to existing soft decoding methods, the presented decoding method can be used for convolutional codes with a large code constraint length.

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