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Concrete and Reinforced Concrete Strength under Action of Shear, Crushing and Punching Shear

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Abstract. Sufficiently general technique for calculating strength based on mathematical concrete plasticity theory apparatus, variational method with virtual velocities principle application, discontinuous solutions, and ultimate load magnitude upper estimate is proposed. Ultimate state simultaneous existence condition on failure entire surface is criterion for plasticity theory applicability. Strength problems of flat and spatial concrete and reinforced concrete elements are solved under crushing, shear and punching shear for various loading applications, and punching shear slabs by central and eccentric load. The adopted kinematic destruction schemes reflect specifics of problems under consideration and allow us to take into account factors determining strength. Results of experimental researches that confirming cases of failure elements taken are adduced. Higher convergence of theoretical strength with an experimental one was obtained in comparison with calculation according to the norms.

1. Introduction

Concrete and reinforced concrete constructions and their elements under shear action (beams and slabs in shear forces acting zone, short elements, slabs keyed joints with girders, girders with columns and columns with foundations, wall panels joints, cast-in-site and precast constructions contact seams, shell slabs with each other and board elements joints), local compression (columns joints, pre-stressed constructions end faces, walls and supports bridges in places where slabs, beams, lintels are supported) and punching shear (foundations and floor slabs, double-curvature shells) are widely distributed in building practice.

Currently significant experimental material about concrete and reinforced concrete elements strength under shear [1 – 3], crushing [4, 5] and punching shear [6 – 9], various factors influence on strength had been accumulated, numerous empirical dependences for ultimate load determination are proposed. These dependencies dominance obtained for certain experimental conditions and their extension to other cases leads to insufficient such calculations accuracy and elements optimization unreliability performed on their basis. Therefore, developing sufficiently general and reasonably accurate theory for calculating concrete and reinforced concrete elements strength under uniaxial inhomogeneous stress-strain states is actual. This theory should use well-known standard programs and be accessible to wide users range, unlike now popular, but requiring complex creation, time-consuming finite element method programs [10 – 11].

At compressive hydrostatic pressure, promising, from our viewpoint, is general technique development for calculating strength using the concrete plasticity theory. Methods based on this basis

and give reliable results are already known [2, 3, 12 – 14]. Concrete ideal plasticity theory variational method for calculating elements strength under action of shear, crushing and punching shear in Poltava National Technical Yuri Kondratyuk Univercity was developed [15, 16].

2. Main preconditions and sequence of design

The following preconditions are used to develop the method: for concrete on destruction stage ideal plasticity assumption is applied, in this case condition of simultaneous limit state existence in more brittle (tensioned) and more plastic (compressed) concrete destruction zones [17] is defining for its application possibility; Balandin - Geniev concrete strength condition [12] is assumed what considered as a plastic potential; strains velocities with stresses relationship is found from plastic straining associated principle; the rigid-plastic solid model and problems solution in discontinuous velocities functions are used.

Consider solving sequence concrete prism strength the simplest task under central compression. The kinematical possible element destruction scheme is assumed, that is outline of destruction surface (velocities breaks) S_l, dividing element into being considered absolutely rigid parts, making mutual movement with some velocities in destruction stage. Herewith parameters determining destruction surface and individual element parts movement velocities are introduced. According to experimental data, prisms are destroyed by inclined to vertical plane at angle 30 ... 40° by shear. The plane AB (figure 1, a) is determined by inclination angle β and divides prism into two rigid disks I and II. Herewith prism upper part makes relatively lower motion with a velocity $V (V_1, V_2)$. On the surface AB uniformly distribute limit normal σ_u and tangential τ_u stress, determining from the Balandin - Geniev strength condition.
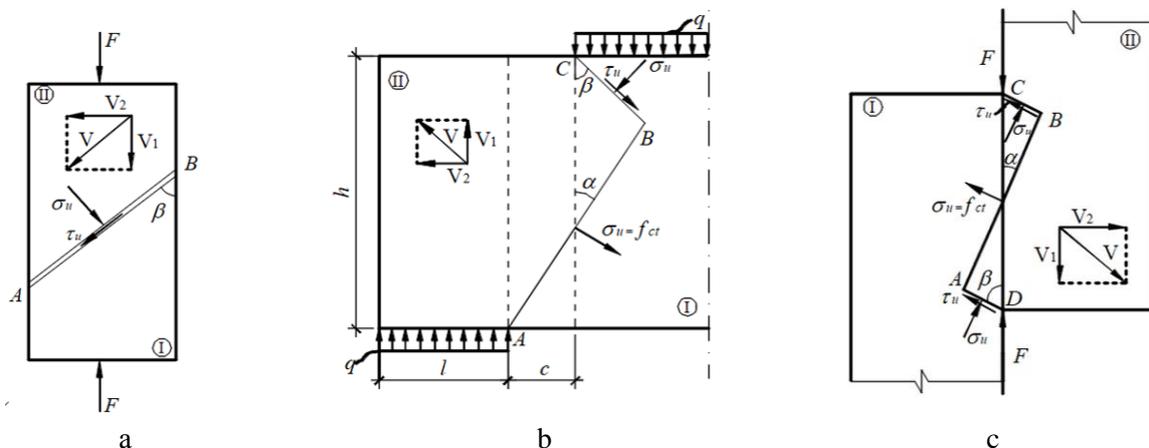

Figure 1. Kinematic possible destruction scheme of concrete prism (a), beam-wall (b)
(if $a = 0$ its E. Mersh's sample) and A. Gvozdev's sample (c)

There are a normal ΔV_n and tangential ΔV_t velocity breaks (jumps) components to surface S_l, which area is $a^2 / \sin \beta$, here is a – prism base size.

Velocity jumps components are expressed in terms of geometric parameters (in our case, angle β) and velocities V_1 and V_2

$$\Delta V_t = V_2 \sin \beta + V_1 \cos \beta, \quad \Delta V_n = V_2 \cos \beta - V_1 \sin \beta .\tag{1}$$

Virtual velocities principle functional is recorded [18], which on real stress-strain state reaches a minimum

$$I = m \left[2B \left(1 + \frac{1}{4} \left(\frac{V_2 \cos \beta + V_1 \sin \beta}{V_2 \cos \beta - V_1 \sin \beta} \right)^2 \right)^{0,5} - 1 \right] \left(V_2 \cos \beta - V_1 \sin \beta \right) \frac{a^2}{tg \beta} - F_u V_1 .\tag{2}$$

Equating the functional (2) to zero, there is an expression for ultimate load F_u through angle β and velocities ratio $k_j = V_j/V_F$, where $V_F -$ velocity of load application point (in our case $k = V_2/V_1$)

$$F_u = m\left[2B\sqrt{(k - tg\beta)^2 + 0.25(ktg\beta + 1)^2} - (k - tg\beta) \right] a^2/tg\beta B,\qquad(3)$$

here $m = f_c - f_{ct}$, $B^2 = \left(1 + \chi/(1-\chi)^2\right)/3$, $\chi = f_{ct}/f_c$.

Unknowns to this task are the maximum load F_u, plane inclination angle the AB to vertical β and velocities ratio k. From the ultimate load minimum condition (3), unknown quantities β and k are determined, then calculates F_u.

Solution reliability obtained is confirmed by equality $F_u/a^2 = f_{c,prizm}$.

Solving strength problems, kinematic possible schemes reflect element destruction specificity under consideration. Destruction surface consists of a different parts number, differing stress-strain state.

Flat elements in angular crushing case, keyed joints with a size ratio $l_k/h_k < 0.25$, truncated concrete wedges simulating a compressed concrete zone over a dangerous inclined crack are destructed on one plane.

For example, fracture surface consists of two planes for a keys shear with ratio $l_k/h_k \geq 0.25$. In addition to shear with compression section, there is a platform acting tensile stresses. It is regarded as the main $-\sigma_u = f_{ct}$. In this case, unknowns number increases (the BC platform inclination angle α to vertical is added), and conditional ultimate load function minimum is searched. Conditions are equilibrium equations $- \sum M = 0$ relative to characteristic points, partially taking into account possible rotation on kinematic scheme. The E. Mersh's samples and short concrete beams destruction surface consists of two described segments (figure 1, b)

For the G. Gvozdev's sample, proposed for determining concrete stress under «pure shear», fracture surface three sections are given in calculation scheme: two sections corresponding to shear with compression and one corresponding to tension, acting main stresses (figure 1, c). In this case, as an additional condition in search ultimate load minimum, equilibrium equation is also used: all forces projections sum on horizontal axis.

Similar fracture surface and elements movement kinematics are inherent for single-keyed joint with seam.

In case of one-sided plane central crushing, loaded along entire thickness, we also have three segments of fracture surface: two areas shear with compression, bounding seal wedge forming under loading area, and splitting plane connecting wedge top to support surface [19]. In this case, sample is divided into three rigid disks. Additional conditions for ultimate load function study on an extremum are not required.

Compression and reinforcement in strength problems are accounted for as an external load. There is reinforcement work separate accounting possibility, located within tear and shear zones. In reinforcement located in shear zone, nagel effect is taken into account.

The developed method is also used in slab calculations under punching shear. The adopted kinematic possible slab destruction scheme is shown in figure 2. Destruction takes place on complex surface with broken line, two segments consists – upper section, located in compressed concrete zone, with inclination to vertical β and height x and lower one, inclined to horizontal at angle α, simulating crack formed during loading. Thus, sample is divided into two disks: I, moving at velocity V_1 relative to sample other II.

The expression for determining maximum load has form

$$F_u = 2m(B^2 tg\beta + 0,25/tg\beta)x(h_k + b_k + 2xtg\beta).\qquad(4)$$

The geometric characteristics of fracture zone $tg\beta$ and x are unknowns to this problem.

Reinforcement work is accounted for by introduction of an additional condition: all forces zero sum projections on horizontal axis.

When solving central punching shear problem, we use following assumptions: angle β is assumed to be same from all 4 sides, crack horizontal projection is $c=1.5d$ (which corresponds to $\alpha = 30$ °). Dependence is proposed for determining stresses ratio in concrete compressed zone to stresses in reinforcement as function of reinforcement coefficient ρ and parameter $\chi = f_{ct} / f_c$.
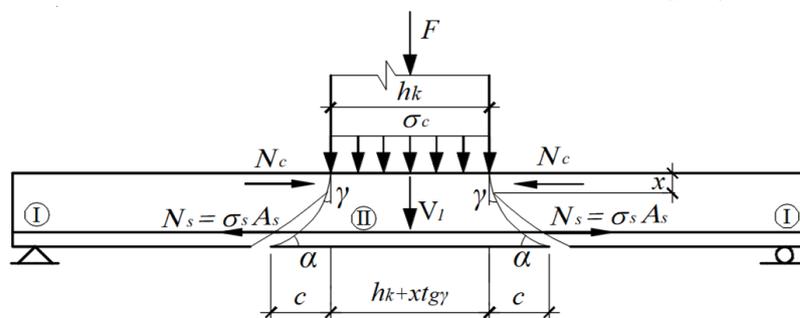

Figure 2. Kinematic reinforced concrete slab scheme with central punching shear

The calculation scheme for eccentric punching shear is similar, as shown in figure 2 but loading area is thus limited to column compressed zone. To determine it, reinforced concrete classical theory problem is solved. The actual compressive stresses triangular diagram within this height is replaced by an equivalent rectangular stress, and then central punching shear problem is solved.

Comparing the proposed calculation method with the finite element method, it should be noted that in proposed method, physical equations inaccuracy is partially compensated by specifying plausible kinematic considering element destruction mechanism.

3. Experimental researches

Experiments program included four directions. Samples were tested in order to obtain information on failure nature, ultimate load magnitude and influence strength factors.

First direction is devoted to experimental study of the samples proposed by A. Gvozdev and E. Mersch, as the best known for determining concrete stress under «pure shear» [20]. The failure nature of these samples, as well as of wall beams, corresponds to that adopted in theoretical solutions (figure 1, b, c). Destruction has externally brittle character. At the same time, intensive deformation localization is observed in a thin layer on failure surface. This is evidenced by a significant increase in deformation value recorded by resistive strain gages, with decrease in their base from 50 to 10 mm and intersection by fracture surface.

Second direction included testing of prototypes simulating compressed concrete zone over dangerous inclined crack and single-keyed joints work. Concrete wedges are loaded on verge of truncating using a special device that decomposes the vertical force onto tangent T and normal N components. The tangential stresses were directed from both the right angle and to it. Depending on direction T and ratio value T/N, two cases of fracture under shear one plane are observed: near right angle and near obtuse angle. External shear phenomenon manifestation in wedges is similar to that observed in prisms. In single-keyed joints, key depth to height ratio l_k / h_k was varied; loading area inclination angle ψ; compression level σ/f_c; coefficient reinforcement ρ_{sw} (reinforcement nature placement in key height: one or two levels); key width t_j; concrete type and class. Results obtained made it possible to make proposals for key depth to height ratio, key profile shape, seam width, compression level, reinforcement limit percentage, prospects for using fiber-reinforced concrete for concreting joints [21].

Third direction is devoted to testing samples-cubes with rib size of 150 and 200 mm. Load from slabs of the PG-125 press was transferred through metal stamp 50×50 mm. Various schemes of applying load were considered. Samples failure nature confirms correctness of chosen kinematic schemes in calculating their strength.

Fourth experiments direction on testing reinforced concrete slabs under punching shear provided studying nature their destruction and determining ultimate load, depending on load eccentricity amount application and longitudinal reinforcement amount. Test samples slab part consisted, which was reinforced with welded mesh, and column fragment with console. Slabs were based on frame. Frame spans elements provided free slabs punching shear. All samples were destroyed by a pyramid whose lower base is bounded by closed contour with rounded corners close to rectangle. Closed crack was formed under central punching shear along column base perimeter on upper slab edge. Load application with eccentricity leads to reduction in pyramid edge area, on three sides contour crack position does not change, and only crack part that is parallel to short slab side and more distant from load application point is displaced. On the slab upper edge, first crack appears at junction more compressed column short side, then the cracks appear along long edge that do not close on opposite column short side. This confirms theoretical premise that, in eccentric punching shear case, pyramid upper base is not entire cross column section, but some of it. Theoretical strength convergence analysis with the experimental one is performed.

Statistical parameters of the estimation: for samples A. Gvozdev $\bar{X} = F^{calc}/F^{test} = 1.04$ with variation coefficient $v = 17.4\%$ and E. Mersh $\bar{X} = 0.86$, $v = 16.7\%$; for beams-walls $\bar{X} = 0.88$, $v = 18.4\%$ and truncated concrete wedges $\bar{X} = 0.9$, $v = 12.2\%$; for single-keyed joints at concrete shear keys $\bar{X} = 0.94$, $v = 13.9\%$, compressed concrete keys $\bar{X} = 0.92$ and $v = 13.9\%$; reinforced concrete keys $\bar{X} = 0.99$ and $v = 4.6\%$ at joint failure for concrete and reinforced joints $\bar{X} = 0.9$, $v = 27.6\%$; plane and three dimensional concrete and reinforced concrete elements under crushing $\bar{X} = 0.86...1.21$ with v from 9.5 to 18%; under slabs central punching shear $\bar{X} = 0.98$, $v = 12\%$.

Conclusion

On the variational method basis in concrete theory ideal plasticity with breaks velocity functions application, strength problems under shear and crushing of concrete and reinforced concrete elements under different load application schemes were solved. The received relationships are exacter in comparison with normative by taking into account to the stress-strain state in each concrete case. This account is carried out by means of appropriate kinematic failure scheme displaying each concrete specificity task and introduction in design a number of factors: key depth to height ratio h/l_{loc}, seam width, key profile shape, keys cross section, compressed level (keyed joints under shear); parameter h/l_{loc}, frictional forces between sample surface and load surface, and location of load site (under crashing). Various reinforcement work character is taken into account within zones tension and shear.

Reinforced concrete slabs strength design under punching shear is offered which takes into account longitudinal reinforcement and force application eccentricity influence. In all problems, both strength concrete characteristics are taken into account: compression and tension.

Experimental researches results confirmed kinematic schemes adopted in theoretical solutions, and also influence of factors determining strength. Concrete and reinforced concrete elements theoretical strength under shear, crushing and punching shear, found on offered method well enough converges with tested.

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