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Research paper

Features of the Silo Capacities' Calculation at the Unsymmetrical Wind Load

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Abstract.

The paper deals with summarizing research concerning analytical technique of calculation of vertical cylindrical capacities for grain storage under the influence of unsymmetrical wind influences. In particular, the analysis of vertical stiffeners' work was conducted and illustrated the body deformation for high and low capacities, depending on the number of rigid elements. Considering the construction's form, which is an axially-symmetric shell of rotation, the main equations for determining efforts, displacements and the series of calculation coefficients were formed. Considering the character of the wind load's influence, there were also made propositions considering defining normal and tangential strains in cross-section, longitudinal directions and also radial, circular and longitudinal displacements, accordingly to the general theory. was made The graphical nomograph for the convenient calculation, which could be used as an alternative when searching for intermediate calculation functions was made. In addition, capacities with different edges' fortification are considered, for example, strengthened by absolutely rigid circle, free or fixed by elastic circle. General assessment of the total deflected mode of the capacity's construction was made, depending on the number of decomposition coefficients, which were put in calculation. Conclusions are accompanied with graphic isometric understandings, which were obtained on the basis of practical calculation.

Keywords: *Cylindrical silos; Harmonica of the load; Internal forces; Thin-walled shell; Wind loads.*

1. Introduction

Vertical capacities for grain storage refer to the thin-walled constructions, which have increased sensitiveness to the wind load. In such cases complexity of learning of construction's behavior is stipulated by asymmetric allocation of loads, which are inherent to the given influences. The most precise method for determining full spectrum of internal efforts and displacements of the capacity's construction is the finite element method, which is recommended to be used in practical calculations. The majority of the scientific researches about this question are also based on using 3D finite element models, which is absolutely relevant and effective solution [1, 2, 3, 4]. But formation of the alternative analytical technique is an important procedure, which in future could be used for preliminary assessment of the project solution or solving a series of adjacent problems. Besides, obtained results are the landmark for the designer, which in future allow avoiding technical errors and probable inaccuracies when making computer model of the FEM.

2. Work of the Vertical Rigid Elements

Wind load at the silos received by stiffeners and the body, which in general case are an axially symmetric shell of rotation for cylindrical capacities. Changing of the internal efforts N_p, M_p in vertical stiffeners and their radial deflection w_z, first of all, depends on the number of elements of rigidity n_p correspondence of the overall dimensions $\Delta_w = H_w/D_w$ (maximum height to the diameter of cross-section) and the method of fortification of points of the finite-element method. The character of this dependence will be determined by extent of the zone of edge effect on the ends of the shell. For sections enough remote from the points of stiffeners' fortification the lines of capacity's deflection under the influence of asymmetric load will be look like as on the fig. 1. This image of deformation is more inherent to high flexible capacities.

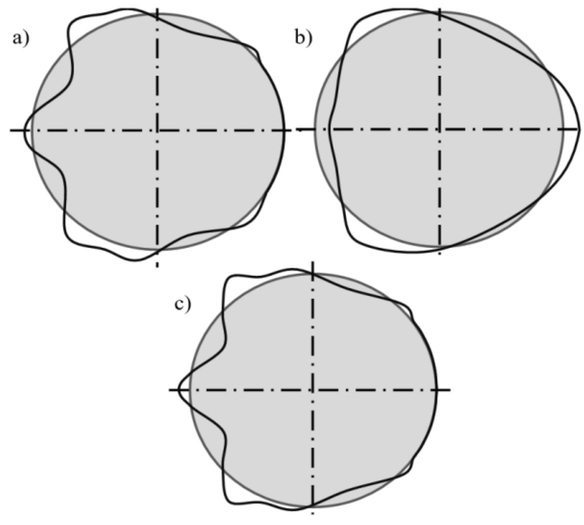

Fig. 1: The character of high capacities' deformation under the influence of the wind load: (a) without stiffeners; (b) $n_p = 5$; (c) $n_p = 50$

Near the bearing zone the line of deflection has another form (fig. 2), which is the characteristic feature for buildings of small aspect ratio, where the edge effect is spread at almost all height.

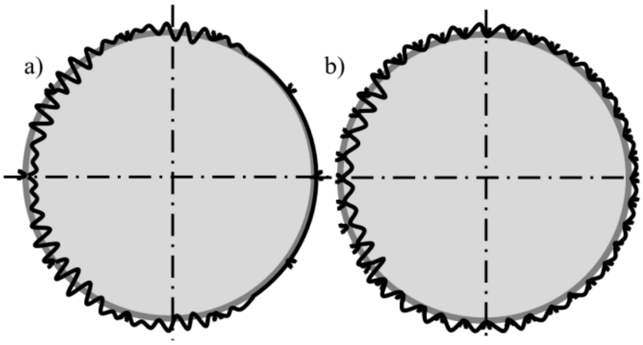

Fig. 2: The character of low capacities' deformation from the flat sheets under the influence of the wind load: (a) $n_p = 5$; (b) $n_p = 40$

For analytical determination of the effort N_p in every stiffener at the z level from its top we could use the formula

$$N_p(z) = \pi g D_w z / n_p , \tag{1}$$

where g is the intensity of the compressing load.

The given expression is not considering compatible work of external framework with covering and provides a certain supply in assessment of the numerical values.

3. Deflected Mode of the Body Sheets

Using of the general precondition that all efforts of compression are perceived only by vertical stiffeners, we can equate to zero meridional and circular strains in the sheets of capacity's body and assume that they work only for axially symmetric and asymmetric radial loads. Considering that the form of the capacity's storage is a shell without abrupt changing of geometric, the problem of determining strains and deformations of such shell under the influence of axially symmetric load $p(x)$ reduces to solving the differential equation [5-9]

$$\frac{d^4 w(x)}{dx^4} + 4k_w^4(x) = \frac{p(x)}{D_r} , \tag{2}$$

where D_r is a cylindrical rigidity of shell at the bend in circular direction; $w(x)$ is a function of the body displacement of the shell; k_w is a coefficient that could be defined by expression $k_w^4 = Et_w \big/ \left(D_w^2 D_r\right)$; E is a modulus of the material's elasticity; t_w is a thickness of the shell.

Calculation of the body sheets under the unsymmetrical semi-uniform load, to which the wind influence refer, has a certain features. First of all, accordingly to the momentless theory, determination of the deflected mode of a shell as in the case of axially symmetric load could be conducted by the simplified procedure. The more precise analytical solution is the use of equilibrium equations of the general theory, which allow considering the influence of internal bending moments in sections. But in practice these calculations are pretty difficult and demand a series of simplifying procedures.

Considering the wind influences, their action is schematized in the form of radially attached external load, which is changing in the cross-section accordingly to the diagram of the wind pressure. In the longitudinal direction wind load is considered as an equally distributed. The problem of calculation of cylindrical capacity could be simplified by decomposing the asymmetric load into the finite trigonometric series. For the wind load $W_k = A_k \cos(k\varphi)$ it will be cosine-series of the aerodynamic coefficient $C_{aer}(\varphi)$

$$\begin{aligned} C_{aer}(\varphi) &= a_0 + a_1 \cos(\varphi) + a_2 \cos(2\varphi) + \ldots \\ &\ldots + a_k \cos(k\varphi) + \ldots + a_m \cos(m\varphi) \end{aligned} \tag{3}$$

where A_k is an amplitude value of the k constituent; φ is an angle of incidence, which considers the wind flow; a_k are the coefficients of decomposition.

3.1. Momentless theory of shells

Accordingly to the momentless theory, for cylindrical shell loaded by asymmetric wind load, the equilibrium equation will be

$$\begin{aligned} &\frac{D_w}{2}\frac{\partial N_{p,k}(x)}{\partial x} + \frac{\partial S_k(x,\varphi)}{\partial \varphi} = 0, \\ &\frac{\partial N_{h,k}(\varphi)}{\partial \varphi} + \frac{D_w}{2}\frac{\partial S_k(x,\varphi)}{\partial x} = 0, \\ &N_{h,k}(\varphi) = \frac{W_k D_w}{2}, \end{aligned} \tag{4}$$

where $N_{h,k}$ and $N_{p,k}$ are the linear longitudinal efforts in radial and meridional (longitudinal) directions; S_k are the linear efforts of displacement.

Full efforts from the wind load influence will be expressed in the form of a sum of formulas (5)-(6) accordingly to the acceptable number of m members of the (3) trigonometric series

$$S_k(x,\varphi) = kA_k x \sin(k\varphi) . \tag{5}$$

$$N_{p,k}(x,\varphi) = k^2 A_k x^2 \frac{\cos(k\varphi)}{D_w} . \tag{6}$$

$$N_{h,k}(\varphi) = A_k D_w \cos(k\varphi) / 2 . \tag{7}$$

We will get maximum values of efforts for functions $N_p(x,\varphi)$ and $N_{h,}(\varphi)$, if $\varphi = 0$

$$N_{p,\max}(x) = \frac{w_p x^2 \xi_p}{D_w}, \qquad \xi_p = \sum_{k=0}^{m} k^2 a_k . \tag{8}$$

$$N_{h,\max} = w_p D_w \xi_h, \qquad \xi_h = \frac{1}{2}\sum_{k=0}^{m} a_k , \tag{9}$$

where w_p is a calculated value of the wind load without considering the coefficient C_{aer} .

For efforts $S_{\max}(x)$ the condition of maximum will be within the limits of $\varphi_{\max} = 37^\circ ... 41^\circ$

$$\begin{aligned} &S_{\max}(x) = w_p x \xi_s, \\ &\xi_s = \sum_{k=0}^{m} k a_k \sin(k\varphi_{\max}). \end{aligned} \tag{10}$$

The analysis of coefficients of the formulas (8)-(10) indicates that values ξ_h and ξ_s out of zone of the edge effect approach to some fixed value. It is explained by less dependence of the displacement effort and radial effort from the fluency of the load's changing. Coefficient ξ_p is changing in wide range depending on the acceptable number of terms of the m series (3). Growing number of terms of the trigonometric series is causing more abrupt transition on the diagram of loads, consequently, the search of border value of the given coefficient could not be made by methods of momentless theory.
We have to notice that conducted numeric analysis of normal strains in the body sheets from efforts $N_{h,\max}$ and $S_{\max}$ revealed quite a small part of influence at the general spectrum of strains in the capacity, consequently, they could be not considered in calculations.
In general case displacement of the capacity under the load W_k is determined when

$$\frac{\partial u(x,\varphi)}{\partial x}=\frac{k^2x^2}{Et_w}\frac{1}{D_w(1+\alpha_{wp})}A_k\cos(k\varphi),$$
$$\frac{\partial v(x,\varphi)}{\partial \varphi}+w(x,\varphi)=0, \qquad (11)$$
$$\frac{2}{D_w}\frac{\partial u(x,\varphi)}{\partial \varphi}+\frac{\partial v(x,\varphi)}{\partial x}=0,$$

where u, v and w are longitudinal, circular and radial displacements of the body; $\alpha_{wp}=A_p n_p/(\pi D_w t_w)$ is the relation of cross-sections' squares of all the vertical stiffeners and capacity's wall.
Full displacements under the influence of the wind load are evaluated by the way of addition accordingly to the isolated harmonicas. For maximum values of displacements (if $\varphi=0$) these formulas are valid

$$u_{\max}=\frac{G_H}{2\Delta_w}\xi_p,$$
$$w_{\Sigma,\max}=\frac{3G_H}{4}\xi_w, \qquad (12)$$
$$\xi_w=\sum_{k=0}^{m}k^4a_k,$$

where $G_H=\frac{2w_pH_w^2\Delta_w^2}{3(1+\alpha_{wp})Et_w}$ is a nominal deflection; w_p is a calculated value of the wind load.

3.2 The general theory

When applying general theory, it is worth to outline the main unknown constituents of the method, where will be also normal strains $\sigma_{p,k}$ in the capacity's cross-section, tangential $\tau_{p,k}$ in longitudinal and three main displacements, longitudinal u_k, radial w_k and circular v_k (fig. 3)

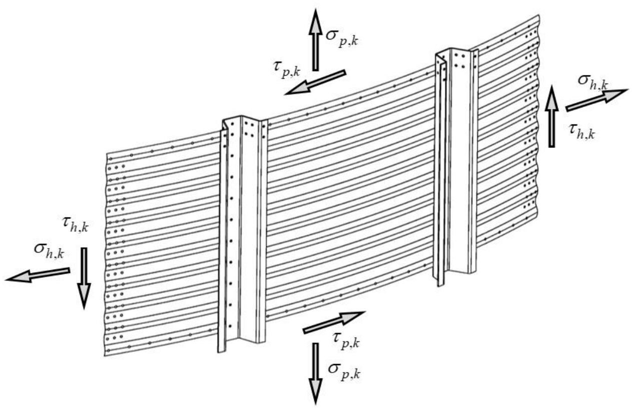

Fig. 3: The scheme of acting efforts in the body of the storage capacity

Considering generalities [10, 11], all unknown efforts and displacements from the load W_k could be expressed though a certain unknown function $\xi_k(x)$, which fulfill the linear differential equation

$$\frac{d^4\xi_k(x)}{dx^4}+4\eta_{w,k}^4\xi_k(x)=4\eta_{w,k}^4\xi_{k,0}, \qquad (13)$$

where $\eta_{w,k}=\frac{1.1k\sqrt{k^2-1}}{\delta_{Dt}D_w}\left(\frac{\Delta J_w}{1+\alpha_{wp}}\right)^{1/4}$ is a value, which considers rigid characterstics of the capacity within the given conditions of the k harmonicas of the load (while $k\geq 2$); k is a parameter, which is responsible for the fluency of changing the harmonious load around the perimeter of the capacity; ΔJ_w considers increased bending rigidity of the corrugated sheet of the body; $\delta_{Dt}=\sqrt{D_w/t_w}$ is the coefficient of geometric scale of the capacity.
Solution of equation (13) will be

$$\xi_k(y)=e^{-y\lambda_{\eta,k}}\left[C_{1,k}\sin(y\lambda_{\eta,k})+C_{2,k}\cos(y\lambda_{\eta,k})\right]+$$
$$+e^{y\lambda_{\eta,k}}\left[C_{3,k}\sin(y\lambda_{\eta,k})+C_{4,k}\cos(y\lambda_{\eta,k})\right]+\xi_{k,0}, \qquad (14)$$

where $\lambda_{\eta,k}=1.1k\sqrt{k^2-1}\frac{\Delta_w}{\delta_{Dt}}\left(\frac{\Delta J_w}{1+\alpha_{wp}}\right)^{1/4}$ is the function of non-dimension height $y=x/H_w$, for which we input the title of the reaction's coefficient.
The function $\xi_k(y)$ characterizes one of the reaction's parameters of the deflected mode of the capacity. For this purpose we could use the function of normal strains [11] or the function of the amplitude circular bending moments along the formative shell [10], which is more convenient for the mathematical transformations.
Constant integrations could be get from the border conditions of the function $\xi_k(y)$ for the capacity with one fortified and another free edge

$$\xi_k(y)=\xi_{k,0}[1-\psi_{m,k}(y)]. \qquad (15)$$

$$\psi_{m,k}(y)=M_{1,k}\,\mathrm{chs}(y\lambda_{\eta,k})+\cos(y\lambda_{\eta,k})\times$$
$$\times\left[M_{2,k}\exp(-y\lambda_{\eta,k})+M_{3,k}\exp(y\lambda_{\eta,k})\right], \qquad (16)$$

where $M_{1,k}$, $M_{2,k}$ and $M_{3,k}$ are the non-dimension functions of the capacity's reaction; $\xi_{k,0}$ is the coefficient of proportionality, which has the dimension of the $\xi_k(y)$ function and considers the character of harmonious load W_k.

We will get expressions for strains in cross-sections $\sigma_{p,k}(y,\varphi)$ and $\tau_{p,k}(y,\varphi)$

$$\sigma_{p,k}(y,\varphi)=\frac{E}{\Delta_w^2}\Lambda_w\frac{\psi_{\sigma,k}(y)}{k^2(k^2-1)^2}\lambda_{\eta,k}^2 a_k\cos(k\varphi) . \quad (17)$$

$$\tau_{p,k}(y,\varphi)=\frac{E}{\Delta_w^3}\Lambda_w\frac{\psi_{\tau,k}(y)}{k^3(k^2-1)^2}\lambda_{\eta,k}^3 a_k\sin(k\varphi) . \quad (18)$$

Radial $w_k(y,\varphi)$, circular $v_k(y,\varphi)$ and longitudinal $u_k(y,\varphi)$ displacements are determined accordingly to the functions

$$w_k(y,\varphi)=2D_w\Lambda_w\frac{[1-\psi_{m,k}(y)]}{(k^2-1)^2}a_k\cos(k\varphi) . \quad (19)$$

$$v_k(y,\varphi)=2D_w\Lambda_w\frac{[1-\psi_{m,k}(y)]}{k(k^2-1)^2}a_k\sin(k\varphi) . \quad (20)$$

$$u_k(y,\varphi)=\frac{D_w}{\Delta_w}\Lambda_w\frac{\psi_{u,k}(y)}{k^2(k^2-1)^2}\lambda_{\mu,k}a_k\cos(k\varphi) , \quad (21)$$

where $\Lambda_w=\frac{3\delta_{Dt}^6(1-\mu^2)}{8\Delta J_w}\frac{w_p}{E}$ is the coefficient of the capacity's compliance.

Functions $\psi_{u,k}(y)$, $\psi_{\sigma,k}(y)$ and $\psi_{\tau,k}(y)$ are analogous by mathematical nature $\psi_{m,k}(y)$ and determined by expressions of the first, second and third derivatives. Their introducing in formulas depends on the variant of the border conditions of fortification of the every edge (bounded by absolutely rigid circle, free or strengthened by elastic circle). Isometric function is illustrated on the fig. 4.

We have to notice, that the displacement strains $\tau_{p,k}(y,\varphi)$ are considerably less than normal $\sigma_{p,k}(y,\varphi)$, consequently in the engineering calculations we could neglect them, and consider only near the basis of the capacity or in the place of conjugation with the upper circle.

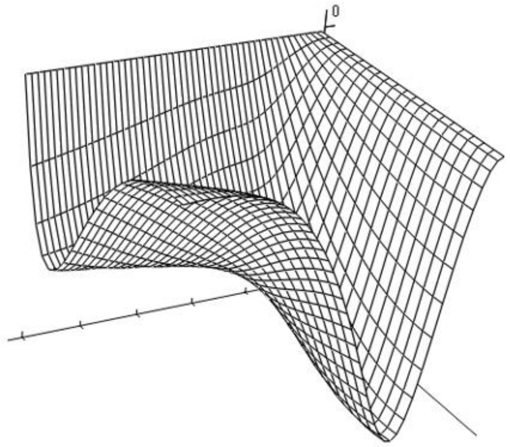

Fig. 4: Isometric introduction of the functions $\psi_{u,k}(y)$ and $\psi_{\sigma,k}(y)$ for capacities with absolutely rigid circle

Accordingly to the given functions, practical calculation of the capacity is complicated by calculation of the indicators of the functions $\psi_{m,k}(y)$, $\psi_{u,k}(y)$, $\psi_{\sigma,k}(y)$ and $\psi_{\tau,k}(y)$ by the given argument. Consequently, for their search it is convenient to use nomograms, which allow finding values of the mentioned functions if it is given $\lambda_{\eta,k}$ and y (fig. 5).

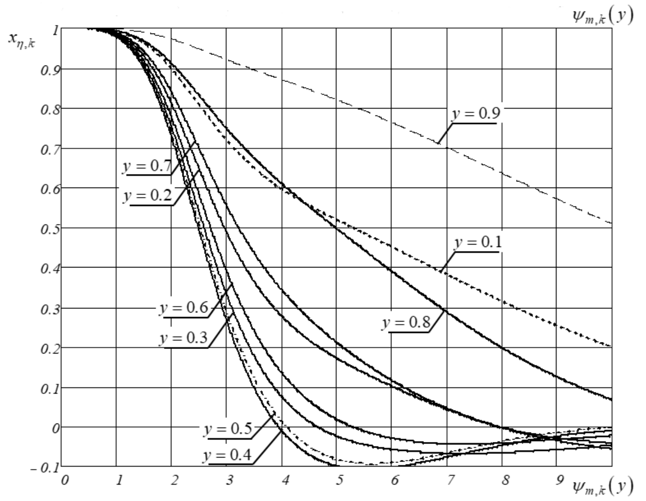

Fig. 5: Nomogam of the function $\psi_{m,k}(y)$ for capacities with absolutely rigid circle

4. Total Deflected Mode

In general case total mode is defined as a sum of deflected modes of the k influence, and depends on the number of coefficients of decomposition, which were put in calculations

$$R_D(y,\varphi)=\sum_{k=0}^{m}a_k R_k(y,\varphi) , \quad (22)$$

where R is a generalized parameter of the reaction $R=\sigma_p\vee\tau\vee w\vee v\vee u$.

Zero term of series (22) corresponds to the axially symmetric load, and, consequently, it does not cause any efforts in stiffeners.

The second constituent a_1R_1 causes usual bend of the capacity. Bending moment $M_0(x)$, transverse force $Q_0(x)$ and deflection $w_0(x)$ at the certain level x in this case are defined as

$$M_0(x)=w_p\pi D_w a_1 x^2/4, \qquad Q_0(x)=w_p\pi D_w a_1 x/2 . \quad (23)$$

$$w_0(x)=w_p\frac{a_1\Delta_w^2 H_w^2}{6Et_w\left(1+\alpha_{wp}\right)}\left(\frac{x^4}{H_w^4}-4\frac{x}{H_w}+3\right). \quad (24)$$

Other terms of the series (22) if $k\geq 2$ could be rewritten in the form of a sum of products of the certain value $r_{D,k}(y)$, which corresponds to the capacity's reaction at the point $(y;\varphi=0)$, at the cosine (sinus) of the function of the reaction's changing accordingly to φ

$$R_D(y,\varphi)=\sum_{k=1}^{m}r_{D,k}(y)\cos(k\varphi)\vee R_D(y,\varphi)=\sum_{k=1}^{m}r_{D,k}(y)\sin(k\varphi) . \quad (25)$$

Calculation of the capacity's reaction is done in the table (table 1), and for the visual image the given parameter is convenient to be made in the non-dimension polar coordinates (fig 6). Similar tables unambiguously determine deflected mode of the capacities and they are the basis for making further design solutions

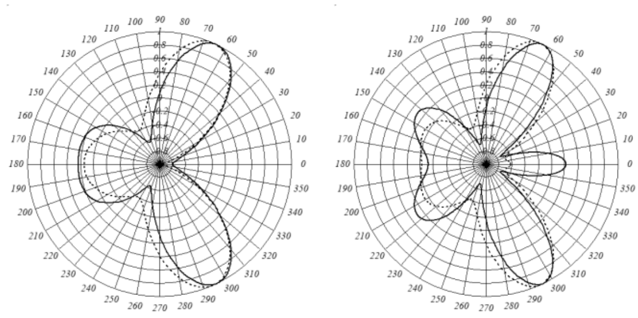

Fig. 6: Example of introducing the capacity's reactions in the non-dimension polar coordinates

Table 1: Example of the amplitude's calculation of the trigonometric series of reactions

y	1φ	2φ	3φ	...	7φ	8φ	9φ	10φ
Amplitude values for longitudinal efforts in the vertical stiffeners								
0.0	0	0	0	...	0	0	0	0
0.1	0.1	0.8	1.2	...	0	0.1	0	0
...	...	...	...	...	...	...	...	...
0.8	5.9	48.2	76	...	-1	-0.4	0	-0.1
0.9	7.5	61	96.3	...	-2.1	-1.7	-0.3	0.2
1.0	9.2	75.3	118.9	...	-3.6	-3.7	-0.7	0.9
Amplitude values for radial displacements of the body's points								
0.0	0.5	5	17.8	...	-1.3	-1	-0.1	0.1
0.1	0.4	4.3	15.4	...	-1.2	-0.9	-0.1	0.1
...	...	...	...	...	...	...	...	...
0.8	0	0.4	1.2	...	-0.2	-0.2	0	0.1
0.9	0	0.1	0.3	...	-0.1	-0.1	0	0
1.0	0	0	0	...	0	0	0	0

5. Conclusions

1. Wind influence at the storage capacities is characterized by unequal distribution around the perimeter of the cylindrical shell. In this case definition of the capacity's deflected mode by using the analytical way could be made if the load is decomposed into the finite trigonometric series and the calculation is done for every constituent separately. The number of the chosen terms of decomposition will affect both the precision, and the method of the further problem solution.
2. Accordingly to the momentless theory, the paper deals with convenient formulas for defining linear longitudinal efforts in radial and meridional (longitudinal) directions and efforts of displacements. Conducted numeric analysis of normal strains in the body sheets revealed quite a small part of influence of $N_{h,\max}$ and $S_{\max}$ at the general spectrum of strains in the capacity. Besides, the solution for maximum values of the three main displacements was formulated.
3. Accordingly to the general theory, the problem of the lateral deflection of the cylindrical shells was solved. In particular, it was given the expressions for normal strains in the capacity's cross-section and for the tangential strains in longitudinal direction and longitudinal, radial and circular deformations.
4. It was made graphical nomograph for the convenient calculation, which could be used as an alternative when searching for intermediate calculation functions. In addition, we need to consider the conditions of edges' fortification.
5. General assessment of the total deflected mode of the capacity's construction was made, depending on the number of decomposition coefficients, which were put in the calculations.

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