

# Mathematical Model of the Dynamic Action of the Controlled Vibration Exciter on the Processed Medium of Mixer with Toroidal Working Container

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## Abstract

Vibration mixers are technological machines that are meant for mixing of different processed medium. A driving force in such machines is realizing by oscillation exciter. In this article, the constructive scheme of the vibration mixer is introduced. Such a mixer has the toroidal working container and is equipped with controlled mechanical centrifugal unbalanced exciters of oscillations with a vertically located unbalanced shaft. The work principle of one of the possible configurations in this exciter is considered and provided. One inflexible and two mobile unbalances are strengthened on its unbalanced shaft. The mobile unbalances by means of independent external action, that is caused by mechanism for managing of mobile unbalances, have an opportunity to change synchronously its positions on the unbalanced shaft directly in time of mixer's work. The centrifugal inertia forces of inflexible and mobile unbalances make the dynamic wrench that consists of the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$  and rotates with unbalanced shaft. It is determined that the value of the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$  evaluate the dynamic action of this vibration exciter to the mixer's working container. The mathematical model of the dynamic action of oscillator exciter on the processed medium of mixer with the toroidal working container is received. Depending on the value of turn angle of mobile unbalances from its starting positions exciter: a) staying in the dynamic balance state; b) is generating the translation force field; c) generates the wrench force field of this or that direction. These opportunities of controlled vibration exciter firmly provide anfractuous circulative motion of the processed medium on the volume of the mixer's working container. Using of controlled exciter also leaves out the transfers through intermediate resonance frequencies. As its starting and stopping happen in a dynamic balance state, so it leaves out the possibility of manifestation of the "Sommerfeld's effect" that is harmful for the driven motor, improves the constructive availability of the vibration mixer and increases its efficiency and life duration.

**Keywords:** controlled mechanical centrifugal unbalanced exciters of oscillations, mobile unbalance, dynamic action, mathematical model, dynamic balanced and unbalanced states of vibration exciters, vibration force fields of wrench or translation structure.

## 1. Introduction

A vibratory mixer with a mechanical exciter of oscillations of any technological purpose is a complex electromechanical system. All constituent elements of this system are characterized by its parameters and are in a certain interaction. Let's consider as shown in the Fig. 1 the conventional structural scheme of the vibratory mixer, as a specified complex electromechanical system, and establish the main characteristic features of this interaction.

Directly the vibratory mixer consists of (see also Fig. 3 and Fig. 4):

- 1 – working container,
- 2 – exciter of oscillations,
- 3 – driven electric motor,
- 4 – external spring supports.

The working container 1 is connected with a fixed base with a help of these spring supports. The electric motor 3 consumes electrical energy from its external source of energy and leads to rotational movement the oscillation excitation elements. The exciter housing 5 is rigidly fixed, for example, by bolted connection 6 to the working container 1.

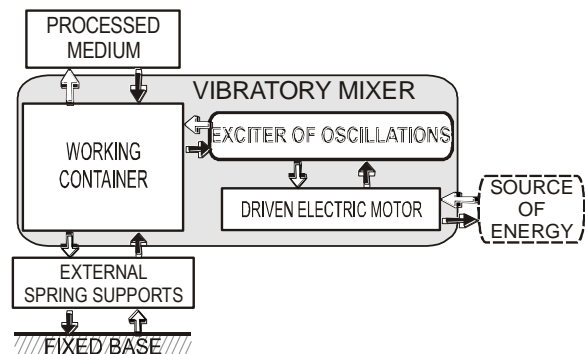


Fig. 1: Structural scheme of the vibratory mixer

The main elements of the exciter of oscillations are an unbalanced shaft 7 and unbalances (one or few). While the unbalanced shaft is rotating the overwhelmed force  $\vec{\Phi}_i$  is generated by unbalances. Physically such a force is a centrifugal force of inertia of the unbalance. According to [1] force  $\vec{\Phi}_i$  the module is:

$$\Phi_i = m_i \cdot e_i \cdot \omega^2 \quad (i=1 \dots n) \quad (1)$$

where  $m_i$  – the mass of the  $i$ -th unbalance,  $e_i$  – the eccentricity of the  $i$ -th unbalance relatively to axis of rotation of unbalanced shaft 7,  $n$  – number of unbalances,  $\omega$  – angular velocity of rotation of unbalanced shaft 7.

When the force  $\vec{\Phi}_i$  is rotating with unbalance with a cyclic frequency  $\omega$ , at any time in every position of unbalance, this force is directed from the axis of rotation of the unbalanced shaft 7 and appropriately operates on it. The total action of inertia forces of all unbalances from unbalanced shaft is transmitted through the support bearings 11 on the body 5 of oscillator exciter. Also, the force is transmitted to the working container 1 of the vibratory mixer through the connection 6. As a result, the working container is in a continuous motion. It causes the corresponding deformations  $\Delta l_j$  ( $j=1 \dots k$ ) of an external spring supports 4, where  $k$  – a number of these supports. The values of deformations of all spring supports are constantly changing. They are staying in a certain functional time dependence and they are not the same in the general case, i.e.

$$\Delta l_j = \phi_j(t) \neq const$$

and

$$\Delta l_1 \neq \Delta l_2 \neq \dots \neq \Delta l_j \neq \dots \neq \Delta l_k.$$

Deformations  $\Delta l_j$  of external spring supports 4 predetermine occurrence of corresponding reactions. They are physically the forces of elasticity whose values are determined by Hooke's law:

$$F_{sp,j} = K_j \cdot \Delta l_j$$

where  $K_j$  – spring stiffness of the  $j$ -th support.

As a rule, the value of the spring stiffness of all supports 4 differs from each other a little and with high accuracy  $K_1 = K_2 = \dots = K_j = \dots = K_k = K$ . But all the deformations of external spring supports are different and variables in time, so the reactive forces:

$$\vec{F}_{sp,1} \neq \vec{F}_{sp,2} \neq \dots \neq \vec{F}_{sp,j} \neq \dots \neq \vec{F}_{sp,k}$$

and

$$\vec{F}_{sp,j} = \psi_j(t) \neq const.$$

Forces  $\vec{F}_{sp,j}$  that act on the working container 1 continuously shifted it in the appropriate directions.

The oscillating movement of the working container that caused by forces' action  $\vec{\Phi}_i$  ( $i=1 \dots n$ ) and  $\vec{F}_{sp,j}$  ( $j=1 \dots k$ ) is transmitted from its walls to the particles of the processed medium 12. As a result, the technological operation of vibrating mixing the desired mixture happens.

The cyclic mechanical interactions arise in the process of mixing between a separate piece of the medium, surrounding its other particles and walls of the working container 1. Such cyclic interactions:

- depend on gravity, inertia and elasticity properties of moving particles of the processed medium,
- lead to formation of dissipative processes in the tillable mixture,
- affect the movement of the working container too.

As the container 1 is tightly bound with the exciter 2, so particles' movement of processed medium probably changes the kinematic characteristics of the rotational motion of the unbalanced shaft 7 with unbalances. It determines the electrodynamic state of the driven electric motor 3. In turn this electric motor produces

periodic effects on the external source of energy. Such interaction between the energy source and the electric motor 3 leads to an appropriate change of the magnitude of engine power coefficient  $\cos\varphi$ .

All considered electrical and mechanical interconnections between constituent elements of system that is shown on the Fig. 1 depend on many factors (from the constructive schemes of the vibration exciter 2 and the vibration mixer, the physical and mechanical properties of the processed medium 12, etc.) and are changed in time for more or less complicated laws. One of the important features of the vibratory mixer is connected with the complexity of theoretical study of its dynamics.

But the primary source of functioning of the vibratory mixer is rotating of unbalance shaft 7 with unbalances of exciter 2.

As pointed in [2] the parameters and characteristics of the exciter of oscillations completely determine structure, value and efficiency of the dynamic action of mixer on the processed medium.

## 2. Analysis of Recent Research and Publications

Vibrating machines of any technological appointment equip exciters with unbalanced shafts that located either horizontally or vertically. According to [3] "exciters of oscillations with vertically located unbalanced shafts have some dynamic and consumer benefits, namely:

- At such arrangement of unbalanced shaft for each period of fluctuations perturbed forces  $\vec{\Phi}_i$  do not perform work that is connected with overcoming the force of gravity of unbalances. Largely these forces do useful work concerning creating and overcoming of forces in the system that is fluctuated. Herewith a driven electromotor is laden during the period of oscillation more uniformly. It contributes to a decrease of its installed power that is necessary for the implementation of the technological process.
- The exciter with vertical unbalanced shaft needs comparatively less time interval and starting power for the acceleration to the frequency of constant oscillations. So in the power supply starting current jump that exceeds the nominal value in several times will be less lasting".

As the motion of walls of working container of vibratory mixer determines the motion of particles of processed medium, so an important influence on the efficiency of the mixing process and its technological parameters have sizes and geometric form of working container, that can be: a)  $U$ -like in the section and rectangular in a plan; b) toroidal; c) other. As usual, a cross section of working containers has an  $U$ -like form. According to [4] "the working container often determines features of vibration process. One can influence the process flow and its effectiveness changing form of working container and its orientation as for oscillator".

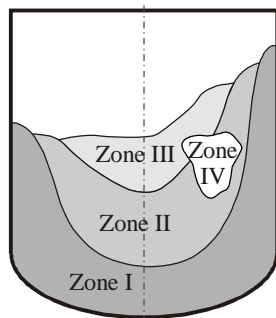
During each cycle of the particle's vibration action of processed medium, that located directly near the walls of working container, are in two different kinetic states:

- While staying in mechanic contact with walls of container, interact with them with appropriate forces. Its size and direction defines Newton's third law.
- By getting power impulse from walls, separated and removed from them and interact with other particles of the processed medium.

In result, particles of the processed mixtures come into directional complex motion on certain trajectories. Such particles' motion is conveniently considered dividing it into:

- fluctuation motion that determines and characterizes the process of mixing particles of processed medium,
- transportation motion that determines and characterizes the process of circulation of processed medium on working container.

Few zones (see Fig. 2) are formed in a volume of the working container of mixer while the processed medium is mixing. In these zones, mechanical interrelations between separate particles have different dynamic. The zone I is the zone of mechanical interrelations, low dynamic dispersion of medium's particles and small transportation particles' velocities of the processed mixture. It's located near the walls of a working container. The zone II is characterized by decreasing of values of mechanical interrelations and rising of dynamic dispersion and transportation velocities. Particles of this zone contact only with particles from zone II. The zone III is characterized by intensive vibratory transportation. The zone IV or dead zone is the zone where the motion of particles of the processed mixture and its mechanical interrelation are lack. Such a zone is always formed in some parts of the formation in mixing session of the processed mixture. A birth, a number, sizes and location places of the dead zones are not determined now.



**Fig. 2:** Mechanical interrelations' zones in the working container of the vibratory mixer

Availability of different zones I-IV degrades quality of mixing of constituent components and often serves to contrary effect, for example, solidification of the processed mixture or cracking it to constituent components and independent transportation of them on working container. For prevention or for decreasing these negative events different approaches are used:

1. A place of location of vibration exciter is changed comparatively to mass center of the working container of mixer.
2. An angle of location of unbalanced shaft's vibration exciter axle is changed comparatively to mixer's working container.
3. Geometric form and sizes of working container are optimized to important individual conditions of mixing the concrete mixture.
4. Additional constructive elements in the working container of mixer are strengthened freely or hardly that disperse the mechanical motion from the walls of container to particles of the processed medium. They are located in its formation.
5. A few vibration exciters are installed in different angles on the working mixing container. They generate forces of different directions and sizes.
6. Other approaches.

According to [5] "...vibration machines with toroidal container spread over and do fluctuation motions." In this paper, the research results of dynamic vibration machines with toroidal working container with vertically located unbalanced shaft are explained. On this shaft "2 unbalances are strengthened. One of them is located higher than mass center of system, the other one is lower located. Such a constructive version of the vibration machine allows managing the motions' trajectories of the working container and correspondingly managing the circulation motion of the technological batch. It happens when the location of unbalances according to each other and to mass center changes." From results of research that are explained in the article [5] one can see that the change of location of unbalances happens with a help of dismantlement and its strengthening (or other unbalances due to weight and sizes) in new positions on the non-working vibration machine. Only after this, change of the dynamic action of the vibration exciter on the processed medium takes place. It makes possible appropriate change of circulation motion.

It is obvious that in such a case it would be true to speak about a discrete change of the dynamic action that is generated by the vibration exciter. This action could be realized only when the technological process stops. As the one or the other individual regime of the vibrations (frequency and amplitude of fluctuations, structure of the vibration field etc.) depends on mass, granulometric composition, water-cement balance, rheological and other characteristics of the processed mixture, so in fact it's necessary to change constructive elements of the oscillator when the processed mixture's characteristics are also changed.

Jian Ruan [6] offers the electrohydraulic vibrational exciter that changes characteristics by means of 2D Valve, for controlling the dynamic action. Disadvantage of such a construction is a problem of the working liquid leakage that is inherent to all of hydraulic exciters.

### 3. Basic Material and Results

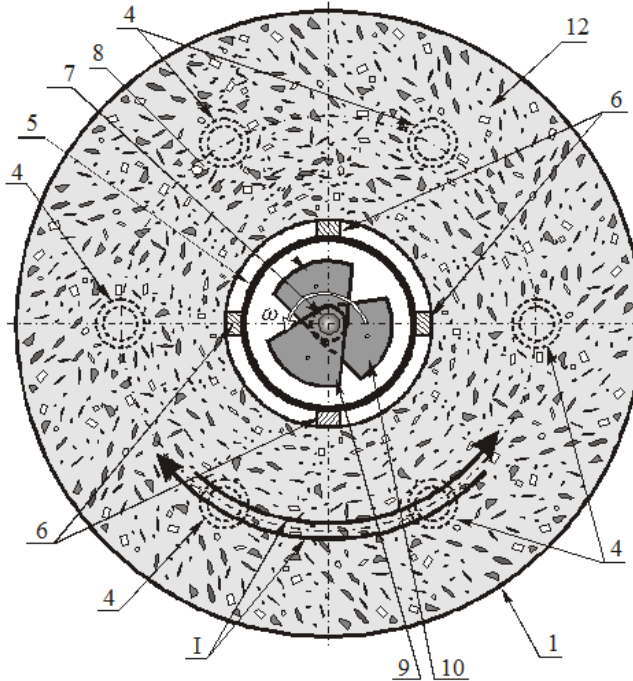
Let's consider as shown in the Fig. 3 the constructive scheme of the vibratory mixer with the toroidal working container and find out the principles and features of the vibration mixer, equipped with an oscillator exciter with a vertically arranged unbalanced shaft. The toroidal working container 1 is connected to the upper ends of the outer spring supports 4 that are uniformly arranged in a circle and secured by its lower ends to the fixed base. The lower part of container 1 has a form of truncated tor. External and internal cylinders limit the upper part and rely on the truncated tor. In the internal part of the working container 1 the exciter 2 is rigidly fixed by connection 6. Herewith, the axis of the unbalanced shaft 7 of the oscillator coincides with the vertical axis of the working container 1.

The unbalanced shaft 7 is fixed by supporting bearings 11 in the body 5 of the vibration exciter 2 (see also Fig. 4). Also, unbalanced shaft is connected with the shaft of the driven electromotor 3 by the deaf hub coupling 13. Such a shaft of the driven electromotor is connected to the container 1 by the bracket 14. A moveless unbalance 8 is rigidly fixed on the unbalanced shaft 7. Two moving unbalances 9 and 10 are fixed relatively to the shaft 7.

Definitive constructive property of this exciter of oscillation is an availability of opportunity for external control of moving unbalances' 9 and 10 positions. During last years such exciters are a subject study of a scientific school that was founded and developed by Professor L.I. Serdiuk. Different original highly effective constructions of controlled mechanical centrifugal unbalanced exciters of oscillations (CMCUEO) were designed, created and researched during this time. They are used as drives' vibration machines of different technological appointment, including vibration mixers.

The main feature of the CMCUEO is that start and stop of the driven motor 3 are realized in equilibrium state of the unbalanced shaft 7 with unbalances 8, 9 and 10. As a result, the rotating of these constructive elements during start and stop is identical to rotating of some equilibrium flywheel. Surely, it is possible to bring in rotation appropriate elements of CMCUEO with driven motor 3. Its power  $N$  is essentially lower than the power of motor that is necessary for start the traditional exciter of oscillations. The entrance of the driven motor 3 and the unbalanced shaft 7 of the CMCUEO on the working detrimental frequency of rotating occurs in the absence of any fluctuation motion of the working container 1. It occurs when its angular velocity grows from zero to the necessary value  $\omega_{work} = \omega$  for the technological process. Also it completely excludes the possibility of manifestation of the Sommerfeld's effect that is harmful for the driven motor 3. The fluctuation exciter 2 is transferred into the unbalanced state by the mechanism with moving unbalances 9 and 10 after the entrance to the working angular velocity  $\omega_{work} = \omega$ . It leads to the emergence and subsequent change of the fluctuation motion of working container's 1 mixer and particles of the processed medium 12.

During the required time interval technological operation of mixing the needed mixture is executed.



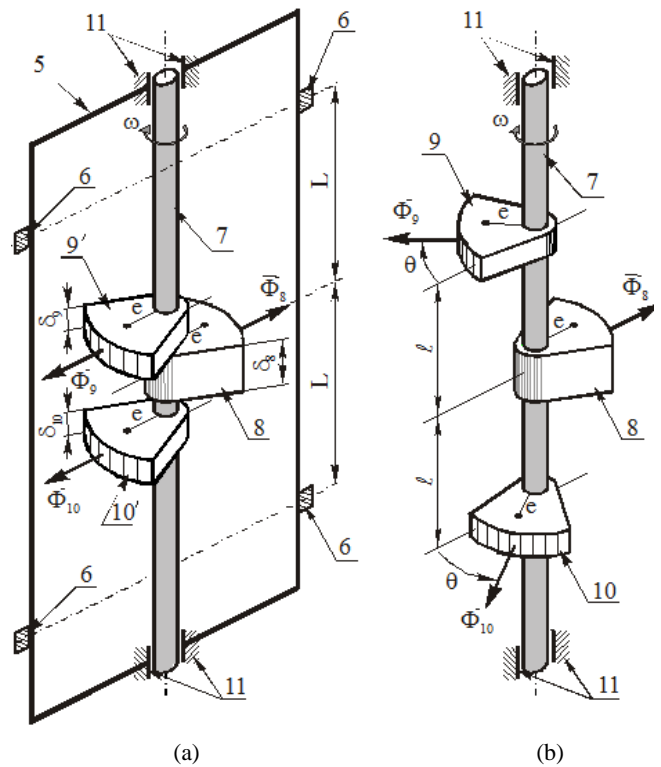
**Fig. 3:** Constructive scheme of the vibratory mixer with toroidal working container

Stop of the driven motor 3 and ending of mixing the processed medium happen in such a sequence:

- exciter 2 is driven to equilibrium state with a help of mechanism for managing moving unbalances 9 and 10; herewith fluctuations of the working container 1 on external spring supports 4 are decreasing under the law of damped oscillations until complete cessation movement of container,
- power supply of the driven motor 3 is disconnected,
- the angular velocity of the rotating rotor of driven motor and equilibrium unbalanced shaft 7 with unbalances 8, 9 and 10 falls down from value  $\omega_{work.} = \omega$  to zero; herewith the reverse transition through intermediate resonance frequencies does not change at all the resting state of the working container 1 and the processed medium 12.

As the article is not devoted to questions of elements' kinematic interactions of the controlled mechanical centrifugal unbalanced exciter of oscillations and elements of mechanism for managing moving unbalances, so we can get acquainted with the kinematic and constructive schemes and the principle of work of one of the possible options for both of these mechanisms in work [7].

To receive the mathematical model of the dynamic action of controlled mechanical centrifugal unbalanced exciter of oscillations (CMCUEO) on the processed medium 12 of mixer with toroidal working container 1, let's consider as shown in the Fig. 4 the schematic model CMCUEO. Equilibrium unbalance 8 and two moving unbalances 9 and 10 are located on the unbalanced shaft 7. All unbalances are made from the same constructive material and have the same geometric sizes except for their thicknesses; thickness  $\delta_8$  of equilibrium unbalance twice as big as thicknesses  $\delta_9$  and  $\delta_{10}$  of each moving unbalance:  $\delta_8 = 2 \cdot \delta_9 = 2 \cdot \delta_{10}$ . In such a case  $e_8 = e_9 = e_{10} = e$ , that is, eccentricities of all unbalances relatively to axis of its rotating are the same and  $m_9 = m_{10} = \frac{m_8}{2} = \frac{m}{2}$ , where  $m_9$ ,  $m_{10}$  and  $m_8 = m$  are masses of unbalances 9, 10 and 8 in accordance.



**Fig. 4:** The schematic model CMCUEO

Then, while unbalanced shaft 7 is rotating with angular velocity  $\omega_{work.} = \omega$ , appropriate centrifugal force of inertia is generated by unbalances according to formula (1):

$$\Phi_8 = m_8 \cdot e_8 \cdot \omega^2 = m \cdot e \cdot \omega^2 = \Phi,$$

$$\Phi_9 = m_9 \cdot e_9 \cdot \omega^2 = \frac{m}{2} \cdot e \cdot \omega^2 = \frac{m \cdot e \cdot \omega^2}{2} = \frac{\Phi}{2},$$

$$\Phi_{10} = m_{10} \cdot e_{10} \cdot \omega^2 = \frac{m}{2} \cdot e \cdot \omega^2 = \frac{m \cdot e \cdot \omega^2}{2} = \frac{\Phi}{2}.$$

In the Fig. 4.a CMCUEO is illustrated in state when its moving unbalances are located diametrically opposite to moveless unbalance 8 in its starting positions 9' and 10' (in this state of CMCUEO and the start and stop of the drive engine are carried out). As at such an arrangement of unbalances in every time moment in each position of the unbalanced shaft 7, that rotates in external ring bearings 11, forces  $\vec{\Phi}_8$ ,  $\vec{\Phi}_9$  and  $\vec{\Phi}_{10}$  are collinear, so, as it is known from theoretical mechanics, system of parallel forces  $\{\vec{\Phi}_8, \vec{\Phi}_9, \vec{\Phi}_{10}\}$  is reduced to one force – ancestral, that is denoted  $\vec{R}_1$ . As directions of forces' actions  $\vec{\Phi}_9$  and  $\vec{\Phi}_{10}$  are opposite to direction force's  $\vec{\Phi}_8$  action, so in that case, ancestral of inertia forces CMCUEO

$$\vec{R}_1 = \vec{\Phi}_8 - \vec{\Phi}_9 - \vec{\Phi}_{10} = \vec{\Phi} - \frac{\vec{\Phi}}{2} - \frac{\vec{\Phi}}{2} = 0.$$

Accordingly, on the Fig. 4.a:

- total dynamic action of unbalanced shaft 7 with unbalances 8, 9 and 10 is equivalent to zero,
- CMCUEO is in a dynamically balanced state,
- any motions of working container 1 and particles of the processed medium 12 are absent.

After the entrance on working angular velocity, moving unbalances 9 and 10, that are rotating with unbalanced shaft 7, are driven to additional motions by the mechanism of managing. In

result, these unbalances are synchronize moved from its starting positions 9' and 10' along the unbalanced shaft 7 in opposite directions on the same distances and are simultaneously rotated in opposite directions to identical angles.

Let's define the dynamic action that CMCUEO produces on the working container 1 mixer, in position, that is illustrated in the Fig. 4.b, where each of moving unbalances 9 and 10 is offset from moveless unbalance 8 on the distance  $\ell$  and is returned from its starting position to appropriate angle  $\theta$ . If forces  $\vec{\Phi}_9$  and  $\vec{\Phi}_{10}$  have laid out to its orthogonal components  $\vec{\Phi}'_9, \vec{\Phi}''_9$  and  $\vec{\Phi}'_{10}, \vec{\Phi}''_{10}$ , as its shown on the power scheme of CMCUEO on the Fig. 5, we will get a new force's system  $\{\vec{\Phi}_8, \vec{\Phi}'_9, \vec{\Phi}'_{10}, \vec{\Phi}''_9, \vec{\Phi}''_{10}\}$  (that is equivalent to output force's system  $\{\vec{\Phi}_8, \vec{\Phi}_9, \vec{\Phi}_{10}\}$ ), where modules of appropriate forces:

$$\Phi'_9 = \Phi_9 \cdot \cos\theta = \frac{\Phi}{2} \cdot \cos\theta,$$

$$\Phi'_{10} = \Phi_{10} \cdot \cos\theta = \frac{\Phi}{2} \cdot \cos\theta,$$

$$\Phi''_9 = \Phi_9 \cdot \sin\theta = \frac{\Phi}{2} \cdot \sin\theta,$$

$$\Phi''_{10} = \Phi_{10} \cdot \sin\theta = \frac{\Phi}{2} \cdot \sin\theta.$$

As, in every time moment in each position of the unbalanced shaft 7, that rotates, forces  $\vec{\Phi}_8, \vec{\Phi}'_9$  and  $\vec{\Phi}'_{10}$  are collinear, and directions of forces' actions  $\vec{\Phi}'_9$  and  $\vec{\Phi}'_{10}$  are opposite to direction force's  $\vec{\Phi}_8$  action, so the module of ancestral of such forces, that is denoted  $\vec{R}_2$ , determines the dependence:

$$\begin{aligned} R_2 &= \Phi_8 - \Phi'_9 - \Phi'_{10} = \Phi - \frac{\Phi}{2} \cdot \cos\theta - \frac{\Phi}{2} \cdot \cos\theta = \\ &= \Phi \cdot \left(1 - \frac{1}{2} \cdot \cos\theta - \frac{1}{2} \cdot \cos\theta\right) = m \cdot e \cdot \omega^2 \cdot (1 - \cos\theta). \end{aligned}$$

In every time moment in each position of the unbalanced shaft 7, two forces  $\vec{\Phi}''_9$  and  $\vec{\Phi}''_{10}$  that are left, have: a) identical modules; b) parallel lines of forces that are not laid on the same straight line; c) opposite directions. Therefore, these forces are formed a couple of forces  $(\vec{\Phi}''_9, \vec{\Phi}''_{10})$ . Its action's area is perpendicular to line of force's action  $\vec{\Phi}_8$  and the moment is:

$$M(\vec{\Phi}''_9, \vec{\Phi}''_{10}) = \Phi''_9 \cdot 2\ell = \frac{\Phi}{2} \cdot \sin\theta \cdot 2\ell = m \cdot e \cdot \omega^2 \cdot \sin\theta \cdot \ell.$$

From the force's scheme as shown in the Fig. 5 its obviously that lines of actions and directions of vectors  $\vec{R}_2$  and  $\vec{M}(\vec{\Phi}''_9, \vec{\Phi}''_{10})$  coincide with appropriate characteristics of force  $\vec{\Phi}_8$ .

As a result of executed legitimate transformations it turned out that outgoing force's system  $\{\vec{\Phi}_8, \vec{\Phi}'_9, \vec{\Phi}'_{10}\}$  is equivalent to aggregate the one force  $\vec{R}_2$  and one couple of forces  $(\vec{\Phi}''_9, \vec{\Phi}''_{10})$ . According to the theory of the construction of different systems of forces to its canonical form of theoretical mechanics (see in [8] sections 3.17 ... 3.21.), let's call the force  $\vec{R}_2$  the main vector of unbalance's inertia forces  $\vec{\Phi}^*$  and the vector  $\vec{M}(\vec{\Phi}''_9, \vec{\Phi}''_{10})$  of the

moment's couple of forces is the main vector of unbalance's inertia forces  $\vec{M}$ .

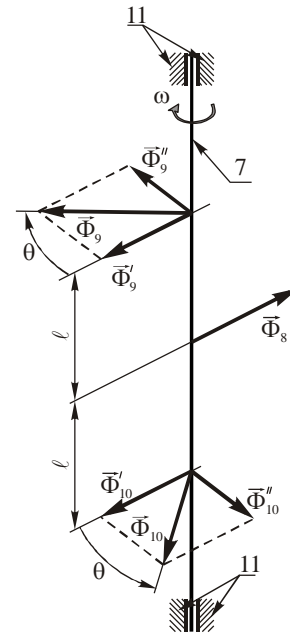


Fig. 5: Power scheme of CMCUEO

So, the centrifugal forces  $\vec{\Phi}_8, \vec{\Phi}'_9$  and  $\vec{\Phi}'_{10}$  of unbalance's inertia form the dynamic power wrench (see in [8] section 3.21), that consists of the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$ . Parameters of its dynamic power wrench that rotates with unbalanced shaft 7, determine the vibratory action of CMCUEO to the working container 1 and the processed medium 12.

Let's set the value of the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$ .

As  $(1 - \cos\theta) = 2 \cdot \sin^2 \frac{\theta}{2}$ , so the module of the main vector:

$$\Phi^* = 2me\omega^2 \cdot \sin^2 \frac{\theta}{2}. \tag{2}$$

According to [9]  $\ell = \frac{d}{2 \cdot \text{tg}\gamma} \cdot \theta$ , where  $d$  is a diameter of

unbalanced shaft 7,  $\gamma$  is an angle of inclination of wrench grooves to longitudinal axis of the shaft 7 (wrench grooves are elements of mechanism for managing the moving unbalances, that provide synchronize additional motion of unbalances 9 and 10). The module of the main moment:

$$M = me\omega^2 \frac{d}{2 \cdot \text{tg}\gamma} \cdot \theta \cdot \sin\theta. \tag{3}$$

In formulas (2) and (3), that are the mathematical model of the dynamic action of CMCUEO on the processed medium 12 mixer with the toroidal working container 1, values  $m, e, d$  and  $\gamma$  are appropriate material-geometric parameters of concrete mixer's model,  $\omega = \omega_{\text{work}}$  is the working angular velocity rotating of the unbalanced shaft 7; all these values are constant in the conditions of a specific process of mixing. Whether conditionally marked

$$2me\omega^2 = C_1 = \text{const}$$

and

$$me\omega^2 \frac{d}{2 \cdot \text{tg}\gamma} = C_2 = \text{const},$$

so formulas (2) and (3) are:

$$\Phi^* = C_1 \cdot \sin^2 \frac{\theta}{2}$$

and

$$M = C_2 \cdot \theta \cdot \sin \theta,$$

where it is clear that the main vector and the main moment are in the functional dependence from an angle  $\theta$ , and the kind of changing  $\Phi^*$  and  $M$  are determined by functions  $f_1 = \sin^2 \frac{\theta}{2}$

and  $f_2 = \theta \cdot \sin \theta$  appropriately. Graphics are on the Fig. 6.

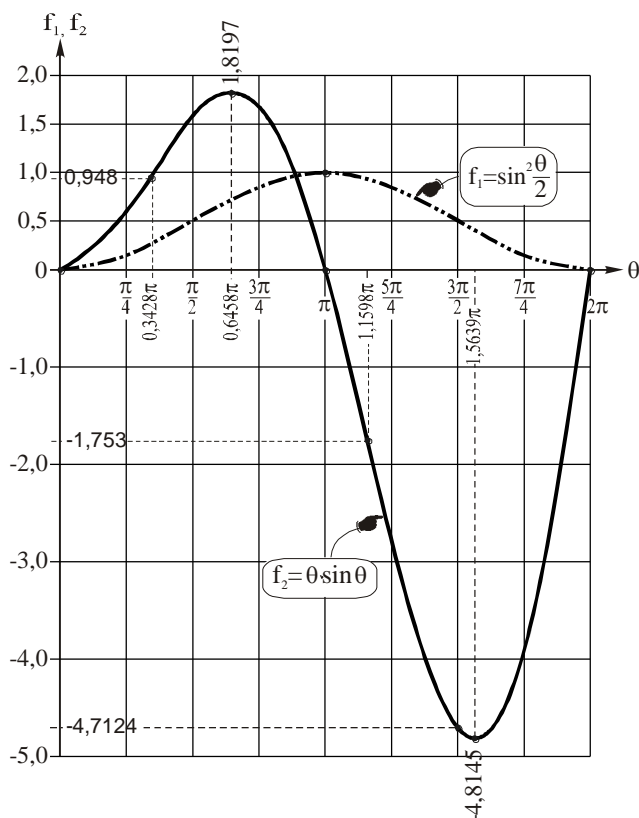


Fig. 6: Graphics of functions  $f_1$  and  $f_2$

One can see from graphics that function  $f_1$  is symmetric. It is growing on the segment  $0 < \theta < \pi$  from zero to its maximum value  $f_{1max} = 1$  and it comes down on the segment  $\pi < \theta < 2\pi$  that causes appropriate character of changing the module of the main vector  $\Phi^*$ . This vector acquires on  $\theta = \pi$  the maximum value

$$\Phi_{max}^* = C_1 = 2me\omega^2.$$

The function  $f_2$  is changing another way. From simple mathematical analysis it goes without saying that, function  $f_2 = \theta \cdot \sin \theta$  on the segment  $0 < \theta < 2\pi$  has two extremes and two points of an overhang. Such points match the solution of the equation  $\frac{d^2}{d\theta^2}(f_2) = 2 - \theta \cdot \text{tg} \theta = 0$  that has on such segment two roots  $\theta_1 \approx 0,3428\pi = 61,70^\circ$  and  $\theta_2 \approx 1,1598\pi = 208,76^\circ$ . At first, it grows and acquires the maximum value  $f_{2max} \approx 1,8197$  on  $\theta \approx 0,6458\pi = 116,24^\circ$ , and then it comes down and acquires zero value on  $\theta = \pi$ , and minimum value  $f_{2min} \approx -4,8145$  on  $\theta \approx 1,5639\pi = 281,50^\circ$ ; after this function  $f_2$  grows again and

acquires the zero value on  $\theta = 2\pi$ . In addition, the module of the main moment  $M$  experiences the corresponding changes: on the segment  $0 < \theta < 0,6458\pi$  it grows up from zero to extreme value

$$M_{max1} = 1,8197 \cdot C_2 = 1,8197 \cdot me\omega^2 \cdot \frac{d}{2 \cdot \text{tg} \gamma},$$

on the segment  $0,6458\pi < \theta < \pi$  it comes down to zero, then on the segment  $\pi < \theta < 1,5639\pi$  it grows again and achieves the next extreme

$$M_{max2} = 4,8145 \cdot C_2 = 4,8145 \cdot me\omega^2 \cdot \frac{d}{2 \cdot \text{tg} \gamma},$$

and on the segment  $1,5639\pi < \theta < 2\pi$  it comes down to zero.

In this way, modules of the main vector  $\Phi^*$  and the main moment  $M$  are changed when the angle  $\theta$  of rotation of moving unbalances 9 and 10 changes from its starting positions  $9'$  and  $10'$ . The rotating of unbalanced shaft 7 causes the constant changing of vector's  $\vec{\Phi}^*$  and  $\vec{M}$  directions. According to established character of parameters'  $\vec{\Phi}^*$  and  $\vec{M}$  changes of the dynamic wrench, the vibratory action of CMCUEO on the working container 1 and the processed medium 12 changes too. Let us observe all aggregate of forces, that are generated by the exciter 2 and are dispatched through the working container 1 to the particles of the processed medium 12, as the vibrational force field of the exciter.

Consider the work of a vibratory mixer. The processed medium 12 is loaded to the mixer's working container 1. The starting of the balance CMCUEO is realized: a power supply is discharged to the driven motor 3 from the external source of energy; a rotation is discharged from shaft to the unbalanced shaft 7 through the hub coupling 13, unbalances 8, 9 and 10 are in positions, that are illustrated on Fig. 4,a. The flywheel 15 (see Fig. 3) of controlling mechanism is rotated after the entrance of the unbalanced shaft 7 on the conventional angular velocity  $\omega_{work}$  of the working regime of the mixing process. It causes additional synchronous motions of unbalances 9 and 10, that are connected with an angular  $\theta$  change. Herewith, the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$  are changed acquiring values that are appropriate to angle's  $\theta$  value (see Fig. 6). Particles of the processed medium 12 simultaneously take part in two related fluctuation motions: vertical and horizontal. Directions, amplitudes and other parameters of its fluctuation motions depend on:

- values  $\vec{\Phi}^*$  and  $\vec{M}$  (on angle's value  $\theta$ ),
- physical and mechanical peculiarities of the processed medium 12,
- fullness of working container 1 by its medium.

In result of these fluctuation motions, the technological process of mixing is realized and characterized by means of two transportable motions of each particle of the processed medium:

1. By transportable motion I along the ring axis of the toroidal working container 1.
2. By transportable motion II around the ring axis of the toroidal working container in the plane that is perpendicular to its axis.

Transportable motions I and II of each mixture's particle cause the spiral circulatory motion of the processed medium 12 by working container 1 volume. Possible directions of such motions are listed on Fig. 3 by appropriate arctic lines I and II.

It is extremely cumbersome and complex problem – to mathematicise the described multi-factor mixing process. Nevertheless, in case of application of CMCUEO independently from physical and mechanical peculiarities of the specific processed medium and from fullness of mixing working container by its medium it is easy to choose experimentally the angle's

value  $\theta$ . With what, appropriate the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$  in aggregate provide the high intensity of the dynamic action on the processed medium and transportable motions I and II that are appropriate to specific technological process. The all given factors cause improving the quality of the final product while simultaneously reducing the mixing time.

The most important peculiarity of application of CMCUEO is that parameters changings  $\vec{\Phi}^*$  and  $\vec{M}$  are realized without any "wrecking-installation" of elements of the exciter and (or) mixer.

A mixer BIO-T with the toroidal working container, as shown on the Fig. 7, was developed, researched and then successfully implemented in the technological process that was ordered for one of the industrial enterprises of Poltava. The controlled oscillator with the unbalanced shaft realizes driving force of the mixer's BIO-T working camera, as you can see on the Fig. 4. A construction of an applied CMCUEO provides the synchronous turn of mobile unbalances from its start positions  $9'$  and  $10'$  to the maximum angle  $\theta_{\max} = 270^\circ$ .

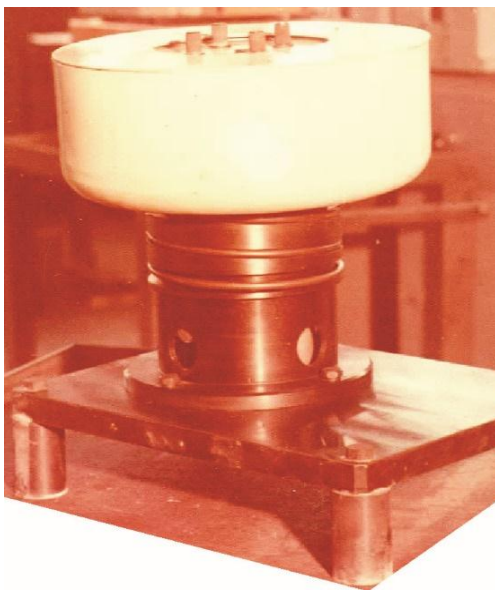


Fig. 7: Photo of mixer BIO-T with the toroidal working container

Herewith, according to graphic  $f_2 = \theta \cdot \sin\theta$  on Fig. 6, the main moment acquires an appropriate value

$$M_{270^\circ} = 4,7124 \cdot me\omega^2 \frac{d}{2 \cdot \text{tg}\gamma}.$$

Such a construction of exciter, that according to catalogue of controlled mechanic centrifugal unbalanced vibration exciters is called UVV-05-270°, provides that on mobile unbalances' 9 and 10 turn on angle  $\theta_{\max} = 270^\circ$ ,  $M_{270^\circ} \approx 0,9788 \cdot M_{\max 2}$  (i.e. is almost 98% from maximum possible value  $M_{\max 2}$ ). It allows using the unbalanced shaft of less length that causes decreasing of CMCUEO and appropriate reduction of material capacity.

From the function  $f_1$  and  $f_2$  (see at Fig. 6) analysis, it is obvious that oscillator UVV-05-270° is able to be in some dynamic states that causes appropriate mechanic action on the working container 1 and the processed medium 12:

1. When  $\theta = 0$  parameters  $\Phi^* = 0$  and  $M = 0$ . Vibration exciter is in dynamic balance state and any action on the processed medium is absent.
2. When  $0 < \theta < \pi$  parameters  $\Phi^* \neq 0$ ,  $M \neq 0$  and have one sign. Exciter generates the wrench force field of one direction.

3. When  $\theta = \pi$  parameters  $\Phi^* \neq 0$  and  $M = 0$ . Exciter generates the translation force field.
4. When  $\pi < \theta \leq \frac{3\pi}{2}$  parameters  $\Phi^* \neq 0$ ,  $M \neq 0$  and have different signs. Exciter generates the wrench force field of other direction.

So, the vibration exciter UVV-05-270° allows to control not only intensity of the dynamic action, but also the structure of the force vibration field that contributes the increasing of mixer's effectiveness and expands the limits of its application.

Long-term research, testing and application in a productive technological process of mixer BIO-T have witnessed its high reliability, simplicity of maintenance and ability to reconfiguration of vibration exciter UVV-05-270° depending on requirements' one or another technological process that are connected with physical and mechanical properties of a processed medium 12 and fullness of working container 1. While a flywheel 15 of controlling mechanism is rotating and positions of mobile unbalances 9 and 10 are changing, we can choose one or another necessary regime of the vibration action on the processed medium and change the intensity and directions of transport motions I and II.

On the mixer's BIO-T base, the vibration mixer with  $0.1 m^3$  volume of the finished mixture for the preparation of fiber-reinforced concrete products [10] was designed and created. The fiber-concrete products in comparison with products of ordinary reinforced concrete are much cheaper, as instead of the expensive rod valves, the metal fiber from waste steel ropes is used. Nevertheless, for sufficient fiber grip with concrete they must be pre-prepared. The developed technological processes allow combining operations of preparation of fibers with operations of mixing fibro-concrete mixture. It reduces energy spending without reduction of physical and mechanical products' properties.

## 4. Conclusions

The considered mixer BIO-T that is equipped with the controlled oscillator UVV-05-270°, allows realizing different technological regimes without any changes of constructive schemes of mixer and exciter. The centrifugal inertia forces of unbalances 8, 9 and 10 form the dynamic force wrench that rotates with unbalanced shaft 7 of vibration exciter 2 and consists of the main vector  $\vec{\Phi}^*$  and the main moment  $\vec{M}$  with appropriate modules

$$\Phi^* = 2me\omega^2 \cdot \sin^2 \frac{\theta}{2}$$

and

$$M = me\omega^2 \frac{d}{2 \cdot \text{tg}\gamma} \cdot \theta \cdot \sin\theta.$$

The constituent elements of the dynamic force wrench determine the vibratory action of the considered CMCUEO on the working container 1 and the processed medium 12. Depending on the angles'  $\theta$  turn value of mobile unbalances from its starting positions  $9'$  and  $10'$ , the vibratory exciter UVV-05-270°: a) is in the dynamic balance state, b) generates the progressive force field, c) generates the wrench force fields of one or another direction.

The applying of the controlled oscillator completely excludes passes through the intermediate resonance frequencies. As its starting and stopping happens in the dynamic balance state, so it completely excludes the opportunity of manifestation of the Sommerfeld's effect that is harmful for the driven motor 3, greatly increases the constructive reliability of vibration mixer, increases its durability and efficiency.

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