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Research paper

Approximational Projection of a Network with an Arbitrary Structure

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Abstract

Improving the efficiency of the functioning of the engineering networks of cities involves solving the issues of expanding and deepening the process of studying the interconnection of its components.
An approximate definition of the property of the network structure's connectivity belongs to the conceptual class of diminishing the dimension of the multiplicity of system parameters. It studies the structures of networks with arbitrary reliability of its constituent parts. The reflection of the reliability values of the components of a redundant engineering network structure is appropriate for large-scale networks. The multiplicity of the network is projected onto the subspace of its parameters with variable values of the argument of the function of the integrity of the connectivity.

Keywords: *Structural modeling, geometric modeling, connectivity probability*

1. Introduction

Improving the efficiency of functioning of various complex systems involves solving issues of expanding and deepening the process of knowing the interconnection of its components. It is necessary to detect and investigate the properties of systems and its constituents [1]. The analysis of the structure and life of the system is realized in two main directions: the study of the features of the system as whole, individual elements, as well as the relationship between them [2].
The probability of structural connectivity is a significant aspect of designing and restoring engineering networks of cities: water, gas, electric, networks of cellular operators, etc. [3]. The level of reliability of existing water supply systems in cities is constantly decreasing. Failure to perform the necessary maintenance and replace the components of the system manner can cause serious accidents [4]. Such complex systems of engineering infrastructure, which are water supply networks of cities, need to be optimized [5].
The risks in the management of supply chains lead to negative consequences in the structure of the distribution network components [6]. Distribution of possible failures of the chain of supply in the network depends on the nature of violations, the structure and the adoption of managerial decisions. The reliability of the functioning of the system is influenced by the factors: the ratio of risks, different effects of compactification, closed communication in the structure, etc. [7].
In the publication [3] the reliability of systems with the structure, modeled by the graph of the network type, was worked out. The study of the reliability of structurally complex systems uses logical probabilistic methods [8]. Simulation of reliability uses the correspondence of dependences of the algebra of logic to probabilistic functions [9].

2 Features of the Study of the Probability of the Connectivity of Network Structures

An unsuccessful project of the engineering network leads to the formation of inappropriate redundancy and unwarranted increase in the cost of construction and operation. The construction and refinement of research methods of complex, especially multidimensional, structures greatly expands the possibilities of structural approximation. In many cases, networks of very large dimension and arbitrary shape, accurate methods for determining the properties of the probability of connectivity are too laborious, leading to significant time expenditures and information resources for solving problems. In another approach to addressing the probability of network connectivity, it is necessary to limit the topology of the investigated structures. For example, the possibilities of applying structures differing in some dependencies and peculiarities of its formation are considered: fractal, topologically symmetric, recurrent. Recurrent structures are obtained by successive transformations from each other. Each subsequent variant of structures is obtained according to a certain rule from the previous one.
The value of the structural reliability of the system is a continuous n-dimensional function, which is differentiated by all variable parameters of the elements reliability. This is a nonlinear dependence on the reliability values of the components of the research object. The function does not have an analytical description, and this has the finest complexity in the calculations, which is to determine its partial derivatives.
The first stage of the research is the structural approximation of the investigated network. At this stage, the necessary restrictions

in the structure are established. The structural approximation involves identifying the class of the structure of the analyzed network and choosing its analog. Technical networks have the property of homogeneity. They are divided into similar components and this helps to effectively use structural approximation. The main obstacle to its application is not a large list of samples of network structures. Due to the complicated form of structure and arbitrary values of the reliability of its elements, probabilistic analysis is complicated enough.

3 Approximations of Network Structures by Projection into Spaces of Smaller Dimensions

Suppose that on the set of spaces of smaller dimensions n-1, ..., n-m it is necessary to consistently find systems of projections corresponding to the dependences of n variables (Fig. 1). Let's establish the correspondence of the form of the structure and its some projection in a space of smaller dimension. This match must be provided in a given series of specified node objects. An analysis of a multidimensional object simulating a system of functions of n variables is carried out in an h-dimensional projection system. Other n-h variable values set certain steel values, which corresponds to the formation of multidimensional sections of the multidimensional model as a model of multiparameter dependence. Derivatives (finite differences) to the h-level, including the modeling dependence of many variables, and its projective correspondence must coincide.

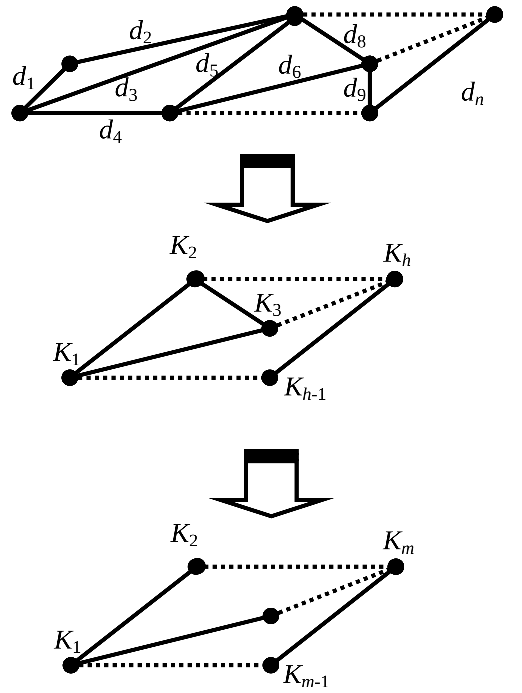


Fig. 1: Projection of a system of dependencies on spaces of smaller dimensions

It is necessary to find sets of values in which the mapping of the modeled dependence of many variables coincides with its corresponding projection. The problem is significantly simplified under the condition of a binary expression of the probability of the connectivity of the components of the network structure and the reliability of its elements. Then a structural approximation is applied, which reduces the level of dimensionality of the network and the complexity of the task. It is accepted that the reliability of all nodes in the system is absolute: $r_{v_1} = r_{v_2},..., r_{v_n} = 1$.

The given structure of the investigated network denote $M(n)$. Assume that the vector h of the variables of the probability dependence of components and the reliability of the elements $M(n)$ is given. It is necessary to find $R(n)$ - the function of the structural reliability of the network. The value of the probability of the connection of the structure of the network $R(n) = R(r_1, ..., r_n)$ is a function of n variables r_i, $i = 1, 2, ..., n$, where $R(r_1, ..., r_n)$ - initial vector of variables. The reliability of an element or a component (group of elements) of the network is denoted by r_i. $R(h)$, $R(m|h)$ - is the value of the probability of coupling, respectively, for the vector h with variables parameters and vectors of smaller dimension $m|h$. The set of variables $m \leq h$ is formed from the set of h, by setting constant values of another part of the variables. The dependence of n variables $R_h(n) = R_h(r_1,...,r_n)$ is obtained by approximating the h-level for the function $R(n)$. For a particular vector h, the values of functions (projection nodes of approximation) $R_h(h) \equiv R(h)$ must coincide.

We execute the consistent projection of the desired dependence of n variables into spaces of smaller dimension (Fig. 2). Structural projection corresponds to the removal of elements with a zero value of its reliability r_i=0. Nodes together with its respective sites, the reliability of which is equal to zero, turn into degenerate projections. To two absolutely reliable nodes v_i, v_j and their common area d_i, a structural projection operation is used. As a result, the 0-dimensional elements v_0 (nodes) coincide $v_i \equiv v_j$, and the one-dimensional element d_1 (plot) joining them is projected into a null-dimensional element $v_i \equiv v_j$.

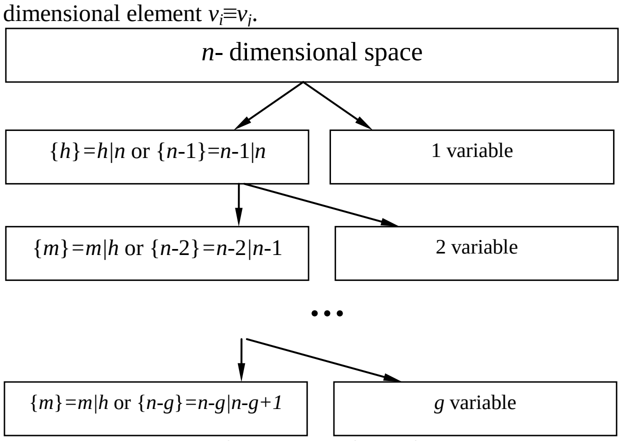


Fig. 2: The process of distribution of sets of variables by its values.

The created object is a projection of the h-th level of approximation of the initial structure $M(n)$ to a space of lesser dimensionality, and it is denoted by $M(h)$. In this case, h - the variables of the given vector $h|n$, and $\{h\}$ - are the set of different h-vectors of variables. Part of the values of the h-vector with variable parameters denote $m|h$, and the value $\{m|h\}$ - is the set of different $m|h$ components.

The parameters $h_1<h$ of the desired function and its approximation also coincide: $R_h(h_1) \equiv R(h_1)$, since the vector h_1 is taken by the vector h with a certain set of constant values. In the projection node, $r_i = const$, i=1,2,...,n, the derivatives of any level of the sought $R(n)$ and the approximation dependence of $R_h(n)$ coincide with the number of differentiation parameters.

The vector value of the r_i component of the structure reduces the dimension of the problem by reducing the total number of variables in the studied system. However, the complexity of the modeling of the h-structure of the network increases due to the growth in its number of random components. When the resulting h-structures of the network are special, then the reliability of $R(h)$ is determined quite simply. For example, for a structurally reserved network $M(n)$, n=43, $D_n \subset \{d_1, d_2, ..., d_n\}$, (in Figure 3 n=43), $Vs \subset \{v_1, v_2, ..., v_s\}$, s=30 (Fig. 3) it is necessary to find $R(n)$ - the structural reliability of the network $M(n)$.

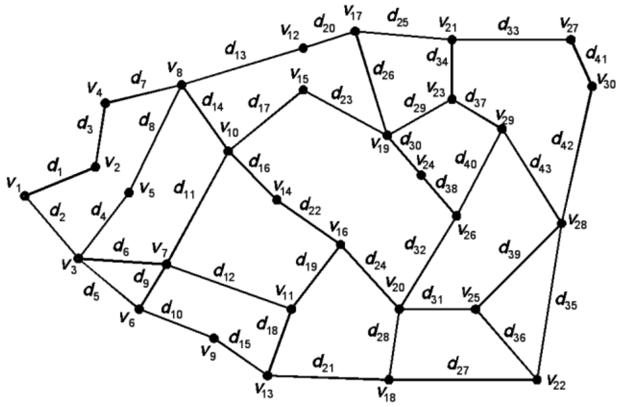
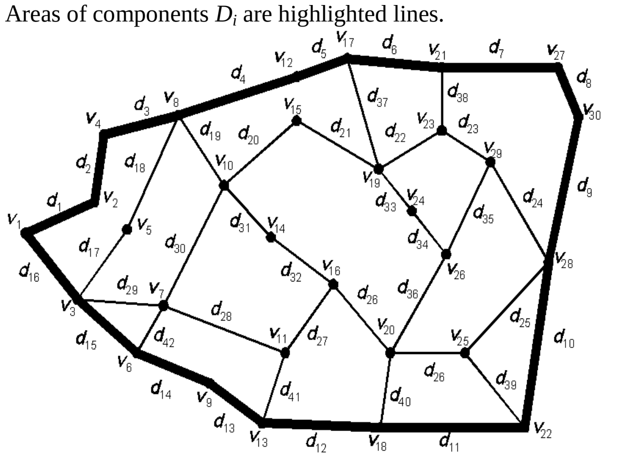
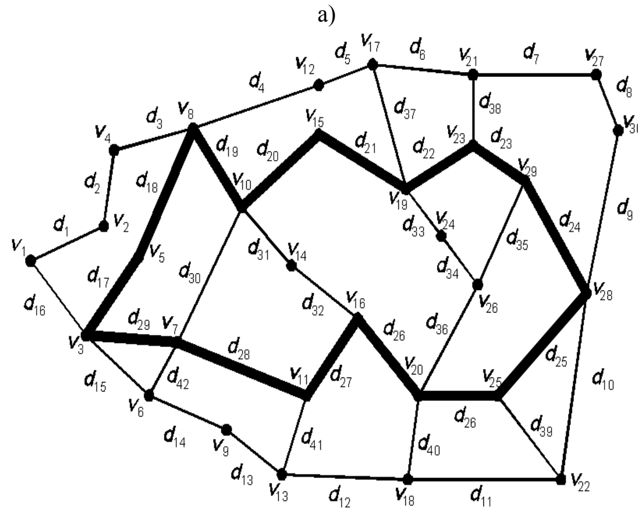


Fig. 3: Structurally reserved network $M(n)$.

The nodes of the network $M(n)$ are assumed to be absolutely reliable. From the set of nodes of the structure of the network V we select three subsets V_1, V_2, V_3, (Fig. 4, a), which are included in some circular chains of sites. Define an approximation of the first level for the three components $R_1(3)$ with sets of nodes V_i, i=1,2,3. The first component $D_1 \subset \{d_1, ..., d_{16}\}$ we accept a closed set of sites and nodes on the outer perimeter $M(n)$ (Fig. 4, a). The second component $D_2 \subset \{d_{17}, ..., d_{29}\}$ is a closed formation in the form of a single ring that includes the sections $M(n)$ passing at a distance of one site (or node) from D_1 and do not belong to D_1 (Fig. 4, b). Areas of components D_i are highlighted lines.



а)



б)

Fig. 4: Two closed network components $M(n)$ of the 1st level of approximation: a) D_1, б) D_2.

Assign a vector value r and a set of values of the probability of the connectivity of the components formed by sections of one set D_i, i=1,2,...,5, r_5 – the probability of the connectivity of other sections of the network $\{d_{37}, ..., d_{42}\}$. The probability of connectivity is sought for $R(3) = R(r_1, r_2, ...,r_5)$.

Perform the structural transformation of the V_i component. We project four nodes v_1,...,v_4 into one v_i (Fig. 5, b). The plurality of sites between objects K_1 and K_2, K_2 and K_3, K_2 and K_4 is approximated by one section d_i, i=1,2,...,4. The 1-components of the network are formed: $M(r_1)$,...,$M(r_5)$. These components are represented by a single- or 2-pole radial-ring form (Fig. 4, a, b, 5, a, b). Some component $M(0)$, which is an approximation of the level 0 level of the network $M(n)$, is formed by a connection from the sites d_1,..., d_m 4 of various objects v_1,...,v_4. Plots $d_1[v_1, v_2]$, $d_1[v_1,v_2]$, components D_3 and D_4 are combined in series. Subset 1|h = 1|h

$$R_0(3)=1,\ R_1(3)=\prod_{i=1}^{3} r_i = R_2(3) \tag{1}$$

Approximation D_4 of level 4 is equal to

$$R_0(4)=1,\ R_1(4)=\prod_{i=1}^{4} r_i = R_2(4) = R_3(4). \tag{2}$$

a) b)

Fig. 5: Branched components of the network $M(n)$ level 1 approximation: a) D_3, b) D_4

Approximation probability of connectivity is defined as the sum of some values of the h-components of structures. The approximation of the h-th level of the desired function $R(n)$ is given by the expression [13]:

$$R_h^+(n) = R_{h-1}^+(n) + \sum_{\{h\}} (R(h) - r_{h-1\,|\,h}^+)\,, \tag{3}$$

where, $R(h)$ is the probability of the structure of the structure component of the network of the h-th level of approximation, $R_{h-1}^+(n)$ is its additive approximation of h-1 level.

The choice of the form of approximation depends on the features of the object is being studied. Form (3) is expedient in the analysis of the linear dependence of the change in structural reliability with the increase in the dimension of the structure of the system. For example, the mathematical expectation of the number of normally functioning objects. The approximation $R^*_h(n)$ prevails in the calculation of reliability values that exponentially depend on the network dimension [13]:

$$R_h*(n) = R*_{h-1}(n) + \frac{\prod_{\{h\}} R(h)}{r*_{h-1|h}}\,. \tag{4}$$

In this expression, the product in all possible vectors h of variables ri is calculated.

The values $r_{h-1}|h$, $r*_{h-1}|h$, which express the connections of vectors h, are determined for each vector by h equations:

$$r^+{}_{h-1|h} = \sum_{m=0}^{h-1} (-1)^{h-1-m} \sum_{\{m|h\}} R(m|h) \tag{5}$$

$$r^{*}_{h-1|h} = \sum_{m=0}^{h-1} \left(\prod_{\{m|h\}} R(m|h) \right)^{(-1^{h-1-m})} \tag{6}$$

Consider the definition of the probability of network connectivity, where qi is the probability of a communication violation. Determination of the quantities $r^{+}_{h-1}|h$, $r^{*}_{h-1|h}$ is based on the following provisions. If $R(h) = 1\text{-}q_{i1}, q_{i2} \ldots q_{in}$, then $r_{h-1}|h = r^{*}_{h-1}|h = 1$.
For example, the structure of the network $M(n)$, n=3 is formed by three nodes v_i, i=1,2,3 and three sections d_j, j=1,2,3 in the form of a triangle. There are three variants of the network $M(3)$ having one working area d_j, j=1,2,3. But these states of the system are not interconnected, since one of the three nodes remains isolated. There can be only three unique variants of the states of this system, which corresponds to the components of the structure with two sections: $d_1 \cap d_2$, $d_1 \cap d_3$, $d_2 \cap d_3$. And there is one state of the system, which is simulated by the structure of the network with three working areas $d_1 \cap d_2 \cap d_3$. The number of variables (subspaces) of the approximation level h=3, the number of variables of the approximation level m=h-1=2. The values $r_{h-1|h}$=$r_{2|3}$, taking into account the mutual influence of different h-sets (components), for each h-set are calculated by the equation:

$$r_{2|3} = \sum_{m=0}^{2} (-1)^{0} \sum_{\{2|3\}} R(2\,|\,3) \tag{7}$$

were $R(2|3)$=(r_i, r_j) with (r_i, r_j, r_h) – is desired reliability of the structure of a subset of two variables, m = 2 is the number of variables of the approximation level, h is the number

$$R_2(n) = 2 \sum_{\{2|3\}} R(2\,|\,3) \tag{8}$$

Function of approximation of 3-th level n=3 variables is equal

$$R_3(n) = R_2(n) + \sum_{\{3\}} (R(3) - r_{2|3}) \tag{9}$$

The value of ri may have a vector form that describes the properties of a particular component - a set of components of the structure of the network.
As a result of approximation, the component of the network $M(h)$ with the weighted structure objects is formed. The weight of an individual object is determined by the weight of the degenerate (recovered) components and taken unchanged in the structure of the network $M(h)$. For example, in determining the average number of workable nodes in the network $M(n)$, the weight of the node in the network $M(h)$ is equal to the number of nodes that is projected to this node. The accuracy of the approximation is influenced by the form of structure and the set of reliability values of the components of the research object.
The approximation necessary to facilitate the simulation of the probability of the network structure's connection is advisable in the direction of simplifying the determination of the values of $r_{h-1|h}$, taking into account the mutual relationships between sets of variables and reducing the number of vectors h.
Due to the high level of visibility, computer graphic modeling of various dependencies is widely used in the branches of science and production [14], [15]. In solving many theoretical and practical problems, special images of figures of multidimensional space are used.
Structural approximation reduces the dimension of the initial set (selection) in the geometric modeling method and includes projective representations [16] (Fig. 6). This is an interface geometric model of Mint of structural transformation. It changes the geometric model of the form M_F and the algorithmic model of the M_A.

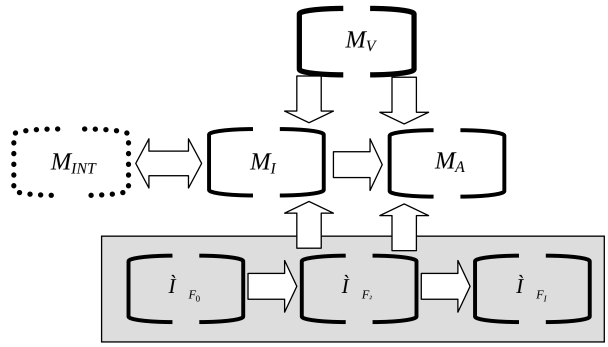


Fig. 6: Interaction of the interface geometric model

Geometric models of the form correspond to the first level of complexity of the hierarchy - this is the initial description of the form, the result of shaping, or the intermediate state of a certain model of the object.
The algorithmic model of the M_A system implements the construction of the model M_F. It may include data systems of the model M_F: the initial M_{F0}, the intermediate M_{FI} and the final M_{FK}. M_F is the initial representation of the form, the intermediate state of the object model and the result of the formation. The geometric model Mint is part of the geometric modeling of the morphological level of classification.
The geometrical model M_v provides computer forming, analysis, and display of models M_A, M_F, M_{int}.

4 Conclusion

To achieve a smaller dimension of the structures of communication networks, it is expedient to apply the properties of the desired dependence of many variables in the nodal objects. Accordingly, the task is to analyze the set of h random components (elements) where h of the parameters with n are variables, and the rest of the others get a constant value.
The method of geometric mapping of the reliability values of the components of the structure of the reserved engineering network is appropriate for networks of a large dimension. The multiplicity of the network is projected onto the subspace of its parameters with variable values of the argument of the function of probability of connectivity.
An approximate definition of the property of the network structure's connectivity belongs to the conceptual class of lowering the dimension of the initial set of system parameters. It studies the structures of networks with arbitrary reliability of its constituent parts.
Due to the regular structure in the form of lattice, triangulation networks, fractal formations, the complexity of modeling reliability can decrease to linear dependence on the number of its constituent parts. These structures are often used in redundant networks of different purposes.

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