

The Statistical Characteristics and Calculated Values for Air Temperature in Building's Cladding Design

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Abstract

The article demonstrates the methodology and the results of statistical characteristics determination and territorial distribution as well as free air winter temperature calculated values required for the designing the building's cladding. Daily temperatures of free air have been described by a probabilistic model of quasi-stationary differentiable random process. The expectation function is set by the Fourier series or sequence of monthly temperatures, the standard and asymmetry are determined by the mathematical expectation. The Ordinates Distribution Law is described by a linear combination of Gauss and Gumbel distributions. The frequency structure is determined by the time-constant value of the effective frequency. The required statistical characteristics are determined by the results of systematic measurements of average daily air temperature at 485 observation points in Ukraine. The calculated values of winter air temperature are determined by using probabilistic approach with accounting for return period equal to building working life, as well as temperature averaging interval, reflecting building's cladding thermal inertia. The map of territorial distribution of Ukraine has been created based on free air temperature values and dependencies which can lead to calculated values taking into account given building operating life, temperature averaging interval and altitude in mountainous regions. The developed methodology ensures the possibility of minimal calculated values determination for free air temperatures including all the factors mentioned above, and the suggestions as for introduction of norms enabling accounting for building's cladding thermal inertia within the building working life when making thermotechnical characteristics calculations.

Keywords: air temperature, probabilistic specification; statistical characteristics; the calculated values.

1. Introduction

Problem of energy saving holds one of the leading position in the scientific sphere of modern construction. Realized resistance in [2] raising standards of heat transfer of cladding, as well as the introduction of effective walling and facade insulation systems with different thermal inertia requires clarifying thermal calculations. The most variable parameter of these calculations is the temperature of free air. The determining factor while designing the building's cladding is the calculated values of temperature in the cold season. Standards of Ukraine [3] provide only four calculated temperature values: temperature of the coldest days and the coldest five days with provision of 0.92 and 0.98 that corresponds to an average return period of the values - 12.5 and 50 years. The modern requirements and the increased capabilities of software systems used for thermal calculations provide the necessity of more detailed evaluating of air calculated values taking into account the expected service life and the value of thermal inertia of building's cladding. Another drawback of standards [3] is a discrete representation of the calculated values defined only for a limited list of settlements. In case of other areas it is recommended to take the calculated values based on the nearest point of observation from that number specified in the standards. However, the geographical location does not always guarantee the identity of the climatic conditions and these results may lead to possibility of a

temperature wrong choice calculated by the designer. One can clarify the choice by mapping the territorial zoning, as it is done in the loads norms for structures [1].

2. Article Aims and Objectives

The article deals with foundations of the valuating air temperature methodology, designed as a result of years' research carried by the author. The giving methodology is illustrated by the results of valuation made by [8] in the territory of Ukraine, although the methods of probabilistic describing of the temperature, determining, synthesis and territorial zoning for calculated values are quite applicable to other areas.

3. Main Body of the Article and Results

The principles of temperature valuation for designing the thermal protection cladding focus on solving these problems. The calculated values of air temperature are to meet the following requirements:

- to be determined by probabilistic methods under long-term results of temperature systematic measurements on a representative network of observation points;

- to take into account the return period that corresponds to the specified service life of the building, as well as the averaging interval of temperature, reflecting the thermal inertia of cladding;
- to be determined clearly and accurately for each geographic area of studied area, including mountain areas.

The calculated air temperature values are determined according to developed in [3 – 5] probabilistic model of quasi-stationary differentiable random process that, due to its universal nature, allows the determination of different temperature characteristics required for solving thermotechnical and static design project tasks.

The probabilistic model of the temperature has been developed on the basis of statistical analysis results of measuring the daily temperatures at weather stations in Ukraine. According to the proposals [6] and in [8], seasonal changes and day to day variability of air temperature are presented in the form of a quasi-stationary differentiated random process. The function of the mathematical expectation is rather accurately described by Fourier series:

$$M(t) = a_0 + a_1 \sin(0,01745t) + b_1 \cos(0,01745t), \tag{1}$$

where a_0, a_1, b_1 are parameters calculated by the method of least squares.

Standard functions $S(t)$ and asymmetry coefficient $A(t)$ can be defined by mathematical expectation function $M(t)$ by the formulas:

$$S = 5,31 - 0,15M + 0,0075M^2 - 0,00024M^3, \\ A = -0,36 + 0,045M - 0,002M^2 + 0,00003M^3, \tag{2}$$

that greatly simplify the probabilistic model of air temperature.

The Distribution Ordinates Law of the random process of the daily average air temperature is unimodal, but in winter and autumn months have a significant left-sided asymmetry. Experience histogram describes the Gauss - Gumbel mixed distribution with density:

$$f(x) = \frac{C}{0,78S} \exp[y - \exp(y)] + \frac{1-C}{S\sqrt{2\pi}} \exp\left[-\frac{(x-M)^2}{2S^2}\right], \tag{3}$$

where M and S are mathematical expectation and distribution standard;

$C = 0,8775 |A|$ – is weigh multiplier;

$y = \frac{M-x}{0,78S} \text{Sign}(A) - 0,577$ is Gumbel distribution argument;

$|A|$ и $\text{Sign}(A)$ – is module and the asymmetry ratio.

You can also use the normal distribution for approximate calculations.

The frequency structure of a random process of average daily air temperature is set by time-constant value of the effective frequency ω . According to the method [6], the effective frequencies have been calculated as a ratio of the derivative's standard to a standard of this process. Taking in account the absence of predominate seasonal variability and minor territorial variability ($0,55 \leq \omega \leq 0,63$ 1/a day), it is recommended to set the total effective frequency value = 0.6 1/a day for whole territory of Ukraine.

Parameters of a probabilistic model of daily average temperatures are determined by the results of term measurements of air temperature on a network of 485 observation points, published in [4] and other specialized meteorological publications. Network of observation points adequately and evenly covers the territory of Ukraine. 91% of weather stations are located in the plain areas, and 45 observation points are in the Carpathian and Crimean mountains at an altitude of 500 m above the sea level. As shown in [5, 6], the duration of climatic series in 10 - 60 years provides the obtaining reliable values of statistical characteristics.

Mapping is a generally accepted method of describing climatic parameter changes that allows generalising the data available and making them more convenient for practical use. As geographical map permits showing the territory change of one single factor, the issue of choosing the method for generalisation of mathematical expectation annual function values from different meteorological stations occurs. This task has been performed in [6] in the following way. Each of functions $M_k(t)$, that is equal to k meteorological station, is normed through its division by its average of maximum value m_k . The proximity of normed functions allows their averaging within all the territory, having obtained the generalised standardised function of mathematical expectation $M_0(t)$:

$$M_0(t) = \frac{1}{N} \sum_{k=1}^N \left[\frac{M_k(t)}{m_k} \right]. \tag{4}$$

Having approximated (4) data by suitable analytic dependence and created the map of territorial distribution of m_k values, one can determine the function of mathematical expectation for an unspecified geographical region according to the formula

$$M(t) = m \times M_0(t), \tag{5}$$

where m is representative value from map of zoning.

The possibility of using the procedure described for regional division of mathematical expectation functions for air temperature is verified according to data provided by 485 observation points in Ukraine. The research has indicated that the generalisation according to formula (4) does not provide results which are accurate enough. Subtraction of average annual temperatures from functions $M_k(t)$ for every meteorological station is more reliable method. In this case generalised normed function of mathematical expectation is calculated according to the formula

$$M_0(t) = \frac{1}{N} \sum_{k=1}^N [M_k(t) - m_k] \times N_k, \tag{6}$$

where m_k is average annual air temperature at k – meteorological station;

N_k is the number of years observed at k - meteorological station;

$N_\Sigma = \sum_{k=1}^N N_k$ is total sum of years observed at all meteorological stations.

The usage of formula (6) allows determining generalised normed function of mathematical expectation $M_0(t)$ taking into account the reliability of data from each meteorological station that is characterised by the duration of observation N_k . The generalisation result of annual function of mathematical expectation of air temperature calculated according to the formula (6) using data from 485 meteorological stations in Ukraine are given in Table 1.

Table 1: The generalised functions of mathematical expectation of air temperature

Months of the year	Time since the beginning of the year t, days	$M_0(t)$, calculated according to (6)	Approximation of functions (1)	Approximation inaccuracy Δ , °C
January	15	-12.28	-12.44	-0.16
February	45	-11.79	-11.09	0.70
March	75	-7.16	-6.76	0.40
April	105	-0.12	-0.62	-0.50
May	135	6.46	5.68	-0.78
June	165	9.94	10.46	0.52
July	195	12.13	12.44	0.31
August	225	11.39	11.09	-0.30
September	255	6.52	6.77	0.25
October	285	0.59	0.63	0.04
November	315	-5.60	-5.67	-0.07
December	345	-10.08	-10.46	-0.38

Generalised functions $M_0(t)$ possess sinusoidal nature and are best rendered by the Fourier series (1) with one couple of coefficients. Coefficients a_0, a_1, b_1 in dependence (1) are obtained by least square method and parameter $0.01745=2\pi/360$ is equal to year duration of 360 days.

As it is seen from the table, function approximation error (3) does not exceed $\pm 0.8^\circ\text{C}$ at coefficient value $m=0, a=-11.86, b=-3.82$.

The transition from generalised normed function (4) to real function of mathematical expectation of quasi-stationary differentiable random process of air temperature at certain meteorological station is made by adding M_p to (1) average annual temperature at this station.

$$M(t) = M_p + M_0(t) = M_p - 11.86 \cos(0.01745t) - 3.82 \sin(0.01745t) \quad (7)$$

Probable approximation errors of average monthly temperatures from changing function of mathematical expectation $M(t)$ into generalised approximations $M_0(t)$ (7) are determined by error statistical processing.

$$\Delta = M(t) - M_p - M_0(t).$$

For 95% of meteorological stations the error from generalised function of mathematical expectation does not exceed 2.1°C .

According to generally accepted methodology formula (7), annual air temperature AM at all the territory of Ukraine equals to 24.88°C , and average temperatures for the coldest M_1 and the hottest M_7 months of the year are equal to

$$M_1 = M_p - 12.44^\circ\text{C};$$

$$M_7 = M_p + 12.44^\circ\text{C}.$$

The average annual value for air temperature M_p required for statistical calculations is determined by using the regional division map of Ukraine, given in Figure 1. The map gives characteristics for temperatures at flat parts of Ukraine, and the temperature for mountainous regions at the altitude more than 500 m above sea level is reduced for 6°C at increasing altitude for 1 km.

Map in Fig. 1 is made according to the methodology and by software, developed in [3]. The input data for creating the map are the location coordinates of all 440 meteorological stations, the number

of observation years as well as corresponding values of average annual temperature. The software calculates the random field function of mathematical expectation for average annual temperature by data smoothing onto rectangular grid knots.

Having taken into account standards of this random field and the necessary zoning, upper threshold is determined. The level lines, made by the software according to upper threshold set in such a way, are the sought-for limits for particular region or zone. Technique used allows selecting the optimal increment for territorial zones and running their boundary lines on the map with given probability, which ensures the low chances for ascribing the meteorological station to the "lower" zone.

Taking into account the necessity of calculated values norming both for the hottest and the coldest months of the year, the map of zoning according to annual average air temperature given in figure is made for probability around 0.5, thus reflecting the average temperatures for particular region, 7 regions with average air temperature limits, given in Table 2, are given on the map.

Table 2: Regional values of average annual air temperature

Regions	1	2	3	4	5	6	7
$M_p, ^\circ\text{C}$	5-6	6-7	7-8	8-9	9-10	10-11	11-12

The alternative method for generalising the function of mathematical expectation of random process of air temperature is based on that fact that the Fourier series (1) with one couple of coefficients can be substituted by one sine function with displaced phase and AM changed accordingly which is equal to average monthly temperature amplitude. The fact that the coldest and the hottest months are distributed in time within half a year interval (half period of sine function) also speaks in favour of this method. Formula free term (1) equal to the average annual temperature with a small error can be substituted by average values for temperature in January and July. Then the function of mathematical expectation of air temperature in geographical point given should be described by the dependence

$$M(t) = \frac{M_1 + M_7}{2} - \frac{M_7 - M_1}{2} \cos\left(\frac{t-15}{57.3}\right), \quad (8)$$

where M_1 i M_7 are average temperatures in January and July; t is time, counted in days since January, 1.

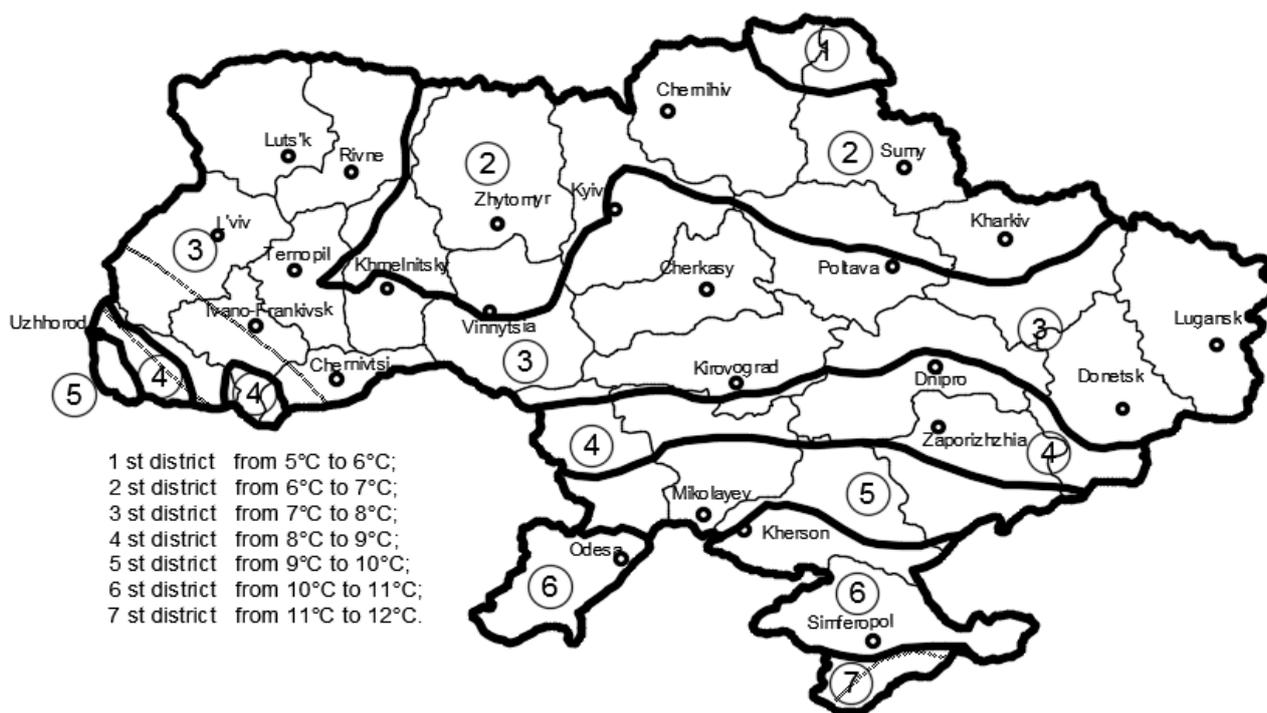


Fig. 1: Regional division of Ukraine according to average annual air temperature

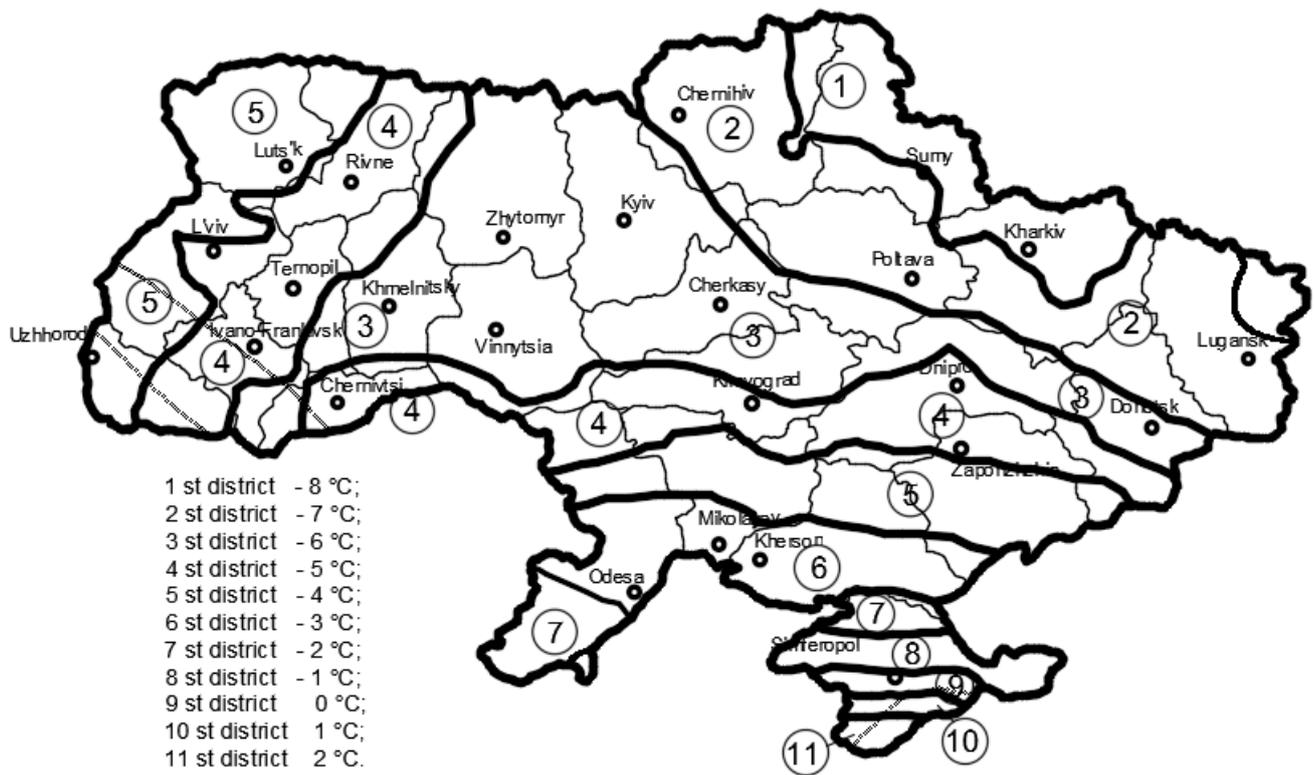


Fig. 2: Regional division of Ukraine according to average air temperature in January

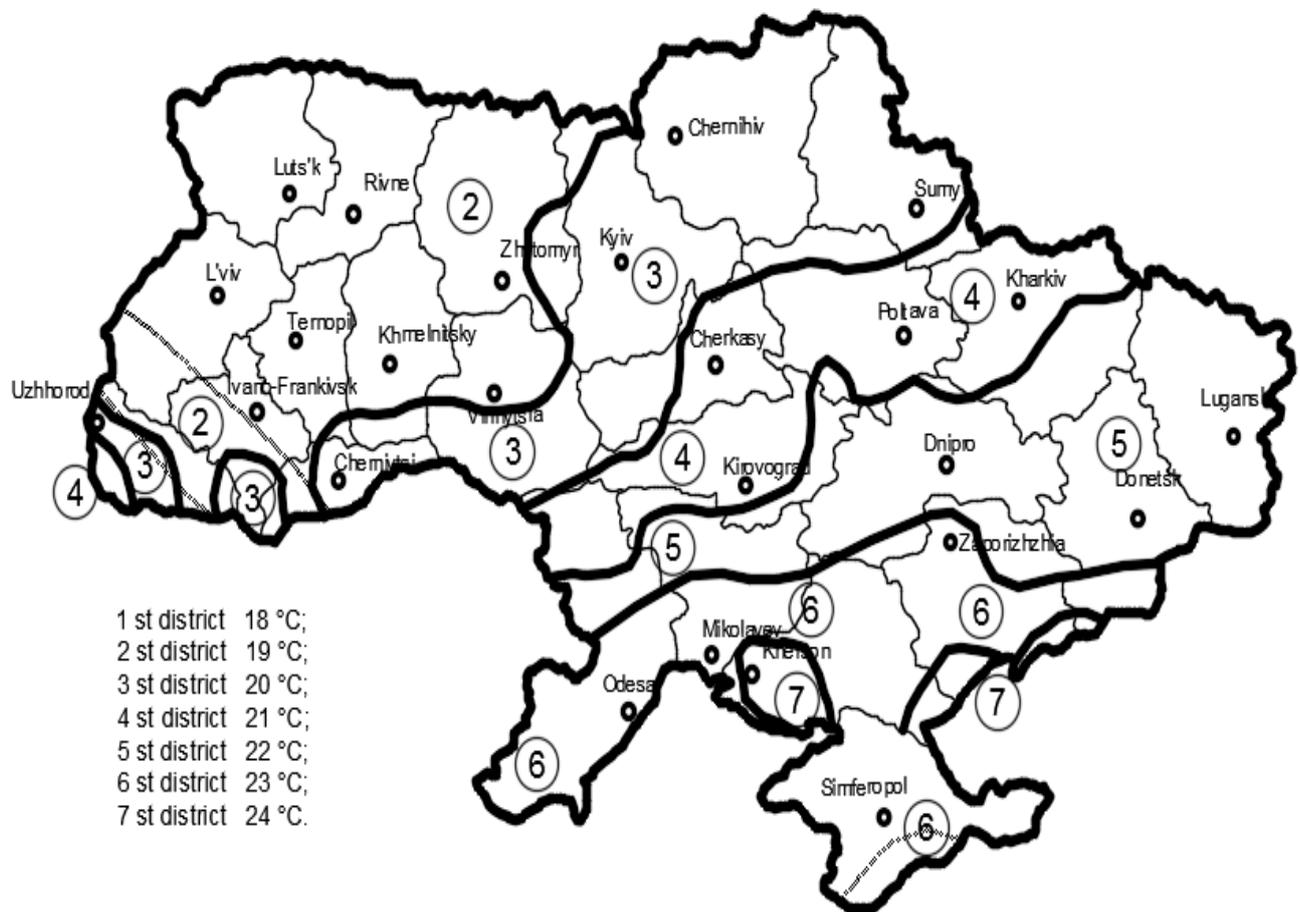


Fig. 3: Regional division of Ukraine according to average air temperature in July

The approximation inaccuracy expression (8) of generalised functions of mathematical expectation from Table 1 does not exceed 0.9°C and for most of meteorological stations it is within 1.1°C. So, this generalisation method provides more accurate function representation $M(t)$, though it still requires the creating of two maps for regional zoning of Ukraine: according to average temperature in January and July.

The maps required are made using the same way as the annual average air temperature value map in Fig.1.

Territorial zoning maps for temperatures in January and July given in Figure 2 and Figure 3 are made with probability around 0.9. It means that for approximately 90% of meteorological stations the actual values of average temperature in January are higher and in July are lower than the values for the particular region. Such an approach helps to ensure necessary content when determining calculated values for the hottest and the coldest months of the year.

Regional division of temperatures in January and July are performed with the increment of 1°C. Alongside with that, on the map of temperatures in January (Fig. 2) there are 11 territorial zones with calculated temperature values from -8°C to +1°C. The map in Fig. 3 contains 7 territorial zones with calculated temperature values in July within the range of 18°C - 24°C. Consequently, winter temperatures are of greater regional variety than summer ones.

The obtained results allow determining the statistical characteristics of the quasi-stationary random process of daily air temperature changes for any geographic area in Ukraine in the following order:

- under designed map of territorial zoning [2, 5] it is determined the average annual temperature M_T , which for the mountains with altitudes of over 500 meters is increasing for 6° C with increasing altitude up to 1 km;
- according to formula (7) or (8) with the usage of corresponding zoning maps the function of mathematical expectation of

air temperature is presented as a sequence of 12 monthly values,

- which correspond to standard value and asymmetry ratio, calculated according to formula (2) for each month of the year;
- the Ordinates Distribution Law is accepted as in the formula (3) or as normal;
- effective frequency is equal to = 0.6 1/a day.

The characteristics, obtained by this methodology or given in [8] for certain Ukrainian meteorological stations, can be used both for calculating of thermal reliability designing the building's cladding or envelope structure according to approach in [7, 9], and for standardising the calculated values of air temperature.

Practical calculating methods of the estimated values of air temperature, needed for designing the thermal protection envelope of building, have been designed and implemented in [8] on the basis of the mathematical apparatus [6]. With the goal of taking into account the thermal inertia of cladding the calculated values of air temperature are calculated on the characteristics of the smoothed (averaged over the interval Z) random process of the temperature:

$$Y(t) = \frac{1}{Z} \int_{t-Z}^t X(\tau) d\tau, \text{ for which } \omega_Y = \sqrt{\frac{2}{Z} \left[1 - \exp\left(-\frac{Z}{3}\right) \right]},$$

$$M_Y(t) = M_X(t); S_Y(t) = \frac{S_X(t)}{Z} \sqrt{6,6Z + 20 \exp(-\alpha Z) - 20}. \quad (9)$$

In formulas (9) $M_X(t)$ and $M_Y(t)$ are the mathematical expectation functions for random process of daily average and smoothed temperature; $S_X(t)$ and $S_Y(t)$ are the functions of the same processes for standards; Z is averaging interval of random process of air temperature depending on the thermal inertia of the structure.

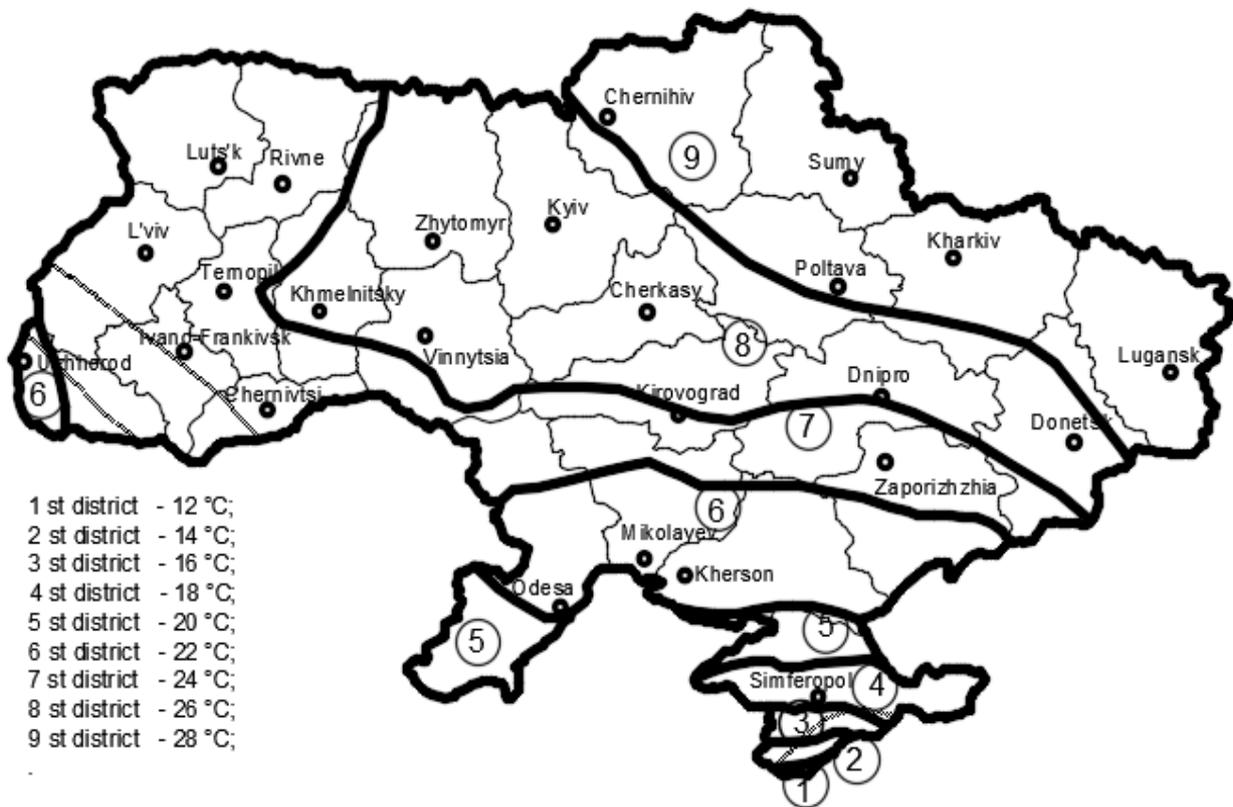


Fig. 4: Zoning of Ukraine on the characteristic values of minimum air temperature X_0

Considering the random process of smoothed temperature $Y(t)$ as normal and quasi-stationary with the statistical characteristics (9), given in the form of tabular sequences from 12 monthly values based on the Rice formula, there have been obtained the formula for calculating the average number of exceeds of the deterministic level Y (emissions) for a year:

$$\lambda_p = \int_0^{12} \lambda(t) dt = 4,8\omega_y \sum_{i=1}^{12} \exp\left[-\frac{(Y - M_i)^2}{2S_i^2}\right], \quad (10)$$

where M_i and S_i are function values $M_Y(t)$ and $S_Y(t)$ for the i -th month of the year according to (5);

ω_y is effective frequency of smoothed process (5), described as 1 for a day;

4.8 – ration taking into account 30 days in a month and some of the parameters of the Rice formula.

Equating (10) to the allowable frequency of exceeds, we obtain the non linear equation, the numerical solution of which gives the minimum calculated air temperature in the area, depending on a given return period T and the averaging interval Z , which takes into account the thermal inertia of the building envelope (cladding). At approximating the ordinate's distribution by more accurate mixed Gauss-Gumbel distribution in formula (6) must be replaced by the density (3) instead of the normal distribution.

The minimum calculated values of temperature have been obtained in [8] by the presented methodology and the data contained 485 observation points in Ukraine for return periods from $5 \leq T \leq 200$ years and averaging intervals from $1 \leq Z \leq$ days. According to the data of 117 observation points located at different altitudes of the Carpathian and Crimean Mountains, it has been found that the minimum calculated temperature reduces for 1 °C at rise up per each 100 m above 500 m above sea level.

Generalizing dependences of the calculated values of the air temperature on return period T (in years) and the averaging interval Z (in days) has been carried out by the introduction of the relative return period $L=365T/Z$. For the whole territory of Ukraine the averaged characteristic values dependence of air temperature on the relative return period is described by a logarithmic function, as is shown in [1, 6, 10, 11] for atmospheric loads and effects on building structures.

Having used the methodology and software described above, the territorial zoning map of Ukraine on the characteristic values of minimum air temperature (as characteristic ones have been taken the calculated values of temperature corresponding to the period of relative frequency $L = 10,000$) has been developed. Shown in Figure 4 the zoning map of Ukraine has 9 districts with the characteristic values of minimum air temperature of -12 °C up to -28 °C, which rise from the north-east to the south and the west of Ukraine. At designing a heat protective cladding of buildings the minimum calculated values of air temperature are determined by taking into account the set lifetime of the building T and the specified interval Z of temperature averaging as it is shown below:

- according to the task of the project and the massiveness of the building envelope, it is set the smoothing interval Z (in days) and return period T (in years) of calculated values of air temperature;
- under the map of the territorial zoning (Fig. 4) the characteristic value of temperature X_0 is determined, which corresponds to the relative frequency of period $L = 10000$;
- determined by the height of the construction area above sea level (at $H < 500$ m should be $H = 500$ m);
- the calculated temperature value is calculated by the formula:

$$X(T, Z) = (X_0 - 36,5) \times \left[0,74 + 0,07 \log\left(\frac{365T}{Z}\right) \right] + \frac{4150 - H}{100}, \quad (11)$$

designations of which are given above.

4. Conclusion

1. The developed zoning map of Ukraine allows the determination of average annual temperature for certain geographical region including the mountainous region of the Carpathians and the Crimea.

2. Function of mathematical expectation, norm and asymmetry ratio of quasi-stationary differentiable random process of average daily air temperature is accurately given through average annual value for air temperature.

3. Using the proposed kinds of ordinate distributive law, known frequency values and data given, quasi-stationary differentiable random process of average daily air temperature at a certain geographical region of Ukraine can be characterised.

4. Obtained statistical characteristics can be used both for calculating the reliability of load carrying structure and building's envelope (cladding) and for norming the necessary calculated values for air temperature.

5. The developed methodology allows determining the minimum calculated values of air temperature with specified return period and temperature averaging interval.

6. The statistical characteristics of the random process of air temperature, obtained from data of observation points of Ukraine, allowed calculating, compiling and zoning the calculated values of air temperature on territory that are required for thermal calculating the cladding of buildings.

7. The given recommendations can extend the capabilities of design organizations concerning the accounting thermal calculations of the thermal inertia of the claddings and the deadlines of their operation.

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