UDC 621.01.001

Zhyhylii S., PhD, Associate Professor
Reznichenko A., student
Poltava National Technical Yuri Kondratyuk University

CREATION AND RESEARCH OF MATHEMATICAL MODEL OF CONTROLLED MECHANICAL CENTRIFUGAL UNBALANCE VIBRATION EXCITER UVV-04

Kinematics scheme of controlled mechanical centrifugal unbalanced vibration exciter (CMCUVE) UVV-04 is presented and examined. Within using the vector shape of defining the motion elements of sketch model the mathematical model of CMCUVE UVV-04 was received. Construction method of dynamic analysis of unbalance to geometric method was used which is usually used in the theory of balancing of mechanisms. Mathematical model received in such a way determines the mechanical action of vibration machine equipped with CMCUVE UVV-04 on the work environment. An geometrical characteristics of unbalance of CMCUVE UVV-04 and its invariants of unbalance was received in the form. The conclusions were made according to those results: the UVV-04 generates vibration field angular structure and increasing of distance between the planes of symmetry moving unbalance ratably increases the amplitude of angular oscillation.

Keywords*: controlled mechanical centrifugal unbalanced vibration exciter (CMCUVE), movable balance, dynamic action, mathematical model, static and dynamic balanced condition, moment unbalance, vibration field of angular structure.*

Жигилій С.М., к.т.н., доцент
Резніченко А.О., студентка
Полтавський національний технічний університет імені Юрія Кондратюка

СТВОРЕННЯ ТА ДОСЛІДЖЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ КЕРОВАНОГО МЕХАНІЧНОГО ВІДЦЕНТРОВОГО ДЕБАЛАНСНОГО ЗБУДЖУВАЧА КОЛИВАНЬ УВВ-04

Наведено та розглянуто кінематичну схему керованого механічного відцентрового дебалансногозбуджувача коливань (КМВДЗК) УВВ-04. Із застосуванням векторної форми визначення руху елементів методом зведення динамічного аналізу незрівноваженості до геометричного, який зазвичай застосовують у теорії зрівноважування механізмів, отримано математичну модель, яка визначає та характеризує механічну дію вібраційної машини, оснащеної КМВДЗК УВВ-04, на оброблюване середовище. У відповідному вигляді отримано геометричні характеристики та інваріанти незрівноваженості, згідно з якими зроблено висновок про те, щоУВВ-04 генерує вібраційне поле кутової структури, а збільшення відстані між плоскостями симетрії рухомих дебалансів призводить до відповідного зростання амплітуд кутових коливань.

Ключові слова: *керований механічний відцентровий дебалансниййзбуджувач коливань, рухомий дебаланс, динамічна дія, математична модель, статична і динамічна зрівноваженості, моментнанезрівноваженність, вібраційне поле кутової структури.*

Introduction.Over the last thirty years by the department of theoretical mechanics of Poltava National Technical Yuri Kondratyuk University under the direction of professor L.I. Serdyuk made large in terms of volume scientific and research and design work connected with the study of dynamic action on the work environment by *controlled mechanical centrifugal unbalanced vibration exciter* (CMCUVE). The result of this work is highly developed various original designs and design schemes CMCUVE which are used as a drive of vibration machines of different technological purposes. At a certain stage of this activity a question arose to create CMCUVE catalogue with their design schemes, which would have been expressed dynamic and technological capabilities, advantages and some disadvantages of any of them, indicated limits and directions of desired application.

Analysis of last sources of research and publications. General principles of creation, construction and operation of all CMCUVE are the same and repeatedly were mentioned in scientific sources [1, 2]. Their defining design element is movable unbalance (or more movable unbalances), external control of which provides the ability to create vibration fields of necessary structure and manage them. CMCUVE main features are that their application: a) lets immediately to get rid of several significant disadvantages of traditional uncontrolled vibration exciter; b) leads to reduction of energy consumption and cost value per unit of produced technology production; c) provides a great resource of strength and reliability.In research papers [3, 4], which in their essence and content are the first steps toward creating a catalog of CMCUVE, peculiarities of dynamic action vibration exciter UVV-02 and UVV- 03 respectively are considered and detected.

Unsolved aspects of the problem. Construction scheme of CMCUVE UVV-04 differs from vibration exciters schemes UVV-02 and UVV-03. Its main feature and difference from vibration exciter UVV-02 [3], which is essentially a modification of it, is that in UVV-04 is absent motionless unbalance. Of course, that mechanical effect on the work environment and mathematical model that describes and defines this action is also different. This fact led to the necessity of creation and study CMCUVE UVV-04 mathematical model. Additionally, this article and its content is the next step in creating CMCUVE catalogue.

Statement of the problem. Consider in Figure 1 kinematics scheme of CMCUVE UVV-04. In unbalance shaft 1 symmetrically relative to its longitudinal axis are placed two geometrically and material identical controlled moving unbalance of mass *m* and with eccentricity *e*. Each unbalance is connected with shaft 1 by two ball dowels 5 (see Fig. 2b), placed symmetrically respectively to the cross-sectional unbalance shaft, each of which is in a hemispherical socket of unbalance 4 and has the ability to roll on the groove 6 unbalance of appropriate cut. All four of these grooves 6 made on the surface unbalance shaft. At each and at any time the position of ball dowels 5 determine the position of both unbalances 4 relatively unbalance shaft 1. Slid fit with needed clearance of both unbalances on shaft 1 allows to move them along unbalance shaft. These moving are rigidly synchronized via motion screw of control mechanism that is not showing Figure 1. At starting time both unbalances are in the middle of the shaft 1 in diametrically opposite positions 4′ that provides static balance of vibration exciter. As screw grooves 6′ for ball dowels 5′ on the left and right sides of the shaft have opposite direction, then while moving unbalances by motion screw of control mechanism in opposite directions from their original positions 4′ they will simultaneously return relative of shaft 1 in one direction at an angle θ. This is to determine a time and provides state of vibration exciter UVV-04 when moving unbalances are only in diametrically opposed to each other regulations 4″.

An important possible modification of CMCUVE UVV-04 is vibration exciter, which grooves 6 are parallel to the axis of the unbalance shaft 1 that simplifies the production technology of this shaft. In this case, both unbalances 4 moved from its original position 4′ without any rotation relative to the shaft 1.

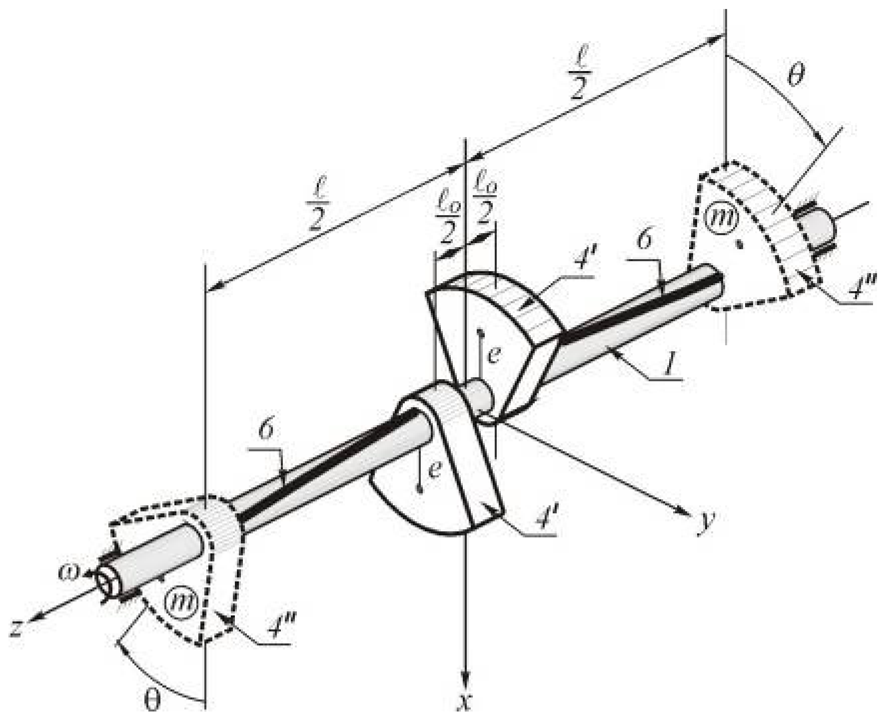

Figure 1 – Kinematics scheme of CMCUVE UVV-04

Consider and explore the dynamics of CMCUVE UVV-04, using the vector form of definition the motion of the objects, use the construction method of dynamic analysis of disbalance to geometric that usually applicable in theory of balanced mechanisms [5]. Thus mathematical model determines the mechanical effect of vibration machine (device, unit) equipped with appropriate vibration exciter on the work environment.

Basic material and results. Consider steady rotational motion debates unbalanced shaft 1, which defines the operating mode of controlled CMCUVE UVV-04. In general, the angular velocity of the shaft is determined by variable value

$$\omega_{work} \approx \omega + \omega_1 \cdot sin\, 2\omega t\,,$$

but the peak value of the variable ω_1 is less than 0,5% of average value ω. Assume that the shaft 1 rotates regularly with angular velocity

$$\omega_{work} = \omega = const\,.$$

Combine Cartesian axis z with rotation axis of unbalance shaft, axis x through its center of mass (through point O) and axis y – perpendicular to the axis x and z (see Fig. 2,a).

According to formula (15.7) [6] acceleration $\vec{a}_i$ of arbitory particle M_i of any of unbalances 4 are

$$\vec{a}_i = \vec{\varepsilon} \times \vec{r}_i + \vec{\omega} \times \vec{v}_i\,,$$

where according to formulas (15.7), (15.6) and (15.5) [6] $\vec{\varepsilon} = \frac{d^2\varphi}{dt^2} \cdot \vec{k}$ – vector of angular acceleration of the shaft *1*; $\vec{\omega} = \frac{d\varphi}{dt} \cdot \vec{k}$ – its vector of angular velocity and $\vec{v}_i = \vec{\omega} \times \vec{r}_i$ – vector of linear (angular or rotational) velocity of particle M_i respectively, and $\vec{r}_i = \vec{i}x_i + \vec{j}y_i + \vec{k}z_i$ – the radius vector defining the position of this particle relative the coordinate basic origin; $\vec{i}$, $\vec{j}$ and $\vec{k}$ – unit coordinate vector of Cartesian coordinate axes.

Since is assumed that $\omega_{work} = \omega = const$, then $d\varphi/dt = const$, $d^2\varphi/dt^2 = 0$, and the desired acceleration of particle M_i is determined only by centripetal component

$$\vec{a}_i = \vec{\omega}\times(\vec{\omega}\times\vec{r}_i).$$

By marking a mass of particle M_i as m_i, according to §37, p. 275 [7], we write the inertial force $\vec{\Phi}_i$ of this particle (which in this case is the centrifugal force of inertia)

$$\vec{\Phi}_i = -m_i\cdot\vec{a}_i = -m_i\cdot[\vec{\omega}\times(\vec{\omega}\times\vec{r}_i)] = m_i\cdot[(\vec{\omega}\times\vec{r}_i)\times\vec{\omega}] = m_i\cdot(\vec{\omega}\times\vec{r}_i)\times\vec{\omega} = (\vec{\omega}\times m_i\cdot\vec{r}_i)\times\vec{\omega}.$$

The main vector particles forces of inertia proposed unbalance (consider moving unbalance that in Fig. 2 is located right side) is determined according to formula (125) [7] as the vector sum of all forces $\vec{\Phi}_i$:

$$\vec{\Phi}_{right} = \sum_{i=1}^{\infty}\vec{\Phi}_i = \sum_{i=1}^{\infty}(\vec{\omega}\times m_i\cdot\vec{r}_i)\times\vec{\omega} = \left(\vec{\omega}\times\sum_{i=1}^{\infty}m_i\cdot\vec{r}_i\right)\times\vec{\omega}.$$

According to formula (70) [7] center of mass of considered unbalance as any other solid body, there is a point C, the position of which in space is defined by the radius vector

$$\vec{r}_c = \frac{\sum_{i=1}^{\infty}m_i\cdot\vec{r}_i}{m},$$

where m – unbalance mass, then writing this formula in the form of $\sum_{i=1}^{\infty}m_i\cdot\vec{r}_i = m\cdot\vec{r}_c$, we will get

$$\vec{\Phi}_{right} = (\vec{\omega}\times m\cdot\vec{r}_c)\times\vec{\omega}$$

and (after obvious transformations taking into account relevant properties of the vector sum)

$$\vec{\Phi}_{right} = m\omega^2\cdot(\vec{i}\cdot x_C + \vec{j}\cdot y_C) = \omega^2\cdot(\vec{i}\cdot mx_C + \vec{j}\cdot my_C).$$

Taking into account, that according to the formulas (17) and (18) [7] $mx_C = S_y$ and $my_C = S_x$, where S_x and S_y – static moments of considered unbalance regarding the axis x and y respectively (see Fig. 2,b), we will have

$$\vec{\Phi}_{right} = \omega^2\cdot(\vec{i}\cdot S_y + \vec{j}\cdot S_x).$$

In brackets of last expression is a vector whose projection on axis x is equal to S_y, and on axis y – S_x and that in theory of balancing mechanisms [5] is called vector $\vec{S}$ of static moment of body mass that rotating about an axis of rotation. On this account for proposed to unbalance

$$\vec{S}_{right} = \vec{i}\cdot S_y + \vec{j}\cdot S_x,$$

and its module

$$S_{right} = \sqrt{S_y^2 + S_x^2} = \sqrt{(mx_C)^2 + (my_C)^2} = \sqrt{m^2\cdot(x_C^2 + y_C^2)} = m\cdot\sqrt{(x_C^2 + y_C^2)} = m\cdot e,$$

where $e = x_C^2 + y_C^2$ –value, which determines the distance from the center of mass of unbalance to the axis of rotation of the unbalance shaft, which of course is the eccentricity of the unbalance.

Then

$$\vec{\Phi}_{right} = \omega^2\cdot\vec{S}_{right}$$

and

$$\Phi_{right} = \omega^2\cdot S_{right} = \omega^2\cdot me.$$

Because of the motion unbalances 4 of vibration exciter UVV-04:

- geometrically and materially are the same, then

$$S_{left} = S_{right} = me \quad \text{and} \quad \Phi_{left} = \Phi_{right} = \omega^2 \cdot me;$$

- at any given time they are diametrically opposed to each other regulations, then

$$\vec{S}_{left} = -\vec{S}_{right} \quad \text{and} \quad \vec{\Phi}_{left} = -\vec{\Phi}_{right}.$$

As mass static moment $\vec{S}$ and the main vector $\vec{\Phi}$ of the particles inertial forces of both unbalances shall be determined by vector sums, in this case,

$$\vec{S} = \vec{S}_{right} + \vec{S}_{left} = \vec{S}_{right} - \vec{S}_{right} = 0, \qquad \vec{\Phi} = \vec{\Phi}_{right} + \vec{\Phi}_{left} = \vec{\Phi}_{right} - \vec{\Phi}_{right} = 0$$

and

$$\Phi = S \cdot \omega^2 = 0.$$

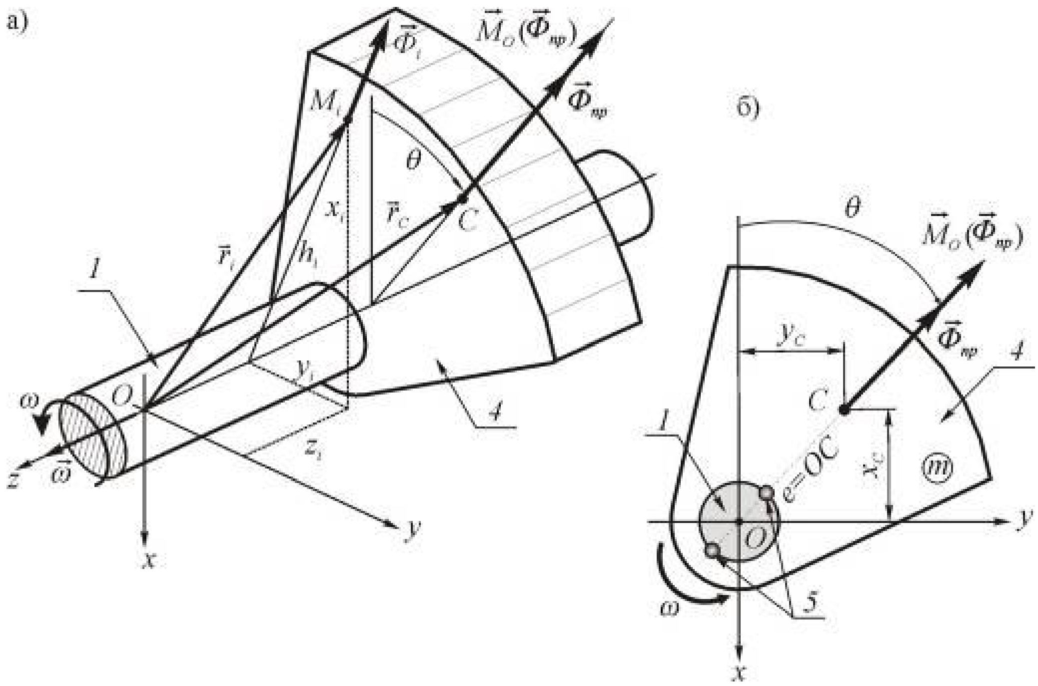

Figure 2 – To determine the mathematical model CMCUVE UVV-04

Now the formula (3.11) [6] define the moment $\vec{M}_O(\vec{\Phi}_i)$ of inertia force $\vec{\Phi}_i$ of particle M_i relative to zero point

$$\vec{M}_O(\vec{\Phi}_i) = \vec{r}_i \times \vec{\Phi}_i = \vec{r}_i \times [(\vec{\omega} \times m_i \cdot \vec{r}_i) \times \vec{\omega}].$$

Geometrically summing-up all the values $\vec{M}_O(\vec{\Phi}_i)$, detects the principal momentof force of inertia particles proposed unbalance relative to zero point

$$\vec{M}_O(\vec{\Phi}_{right}) = \sum_{i=1}^{\infty} \vec{M}_O(\vec{\Phi}_i) = \sum_{i=1}^{\infty} (\vec{r}_i \times \vec{\Phi}_i) = \sum_{i=1}^{\infty} \{\vec{r}_i \times [(\vec{\omega} \times m_i \cdot \vec{r}_i) \times \vec{\omega}]\}$$

and (after obvious transformations taking into account relevant properties of the vector product)

$$\vec{M}_O(\vec{\Phi}_{right}) = \omega^2 \cdot \left(\vec{j} \cdot \sum_{i=1}^{\infty} m_i \cdot x_i \cdot z_i - \vec{i} \cdot \sum_{i=1}^{\infty} m_i \cdot y_i \cdot z_i \right).$$

Taking into account that according to the formula (321) [7] $\sum_{i=1}^{\infty} m_i \cdot x_i \cdot z_i = J_{xz}$ and $\sum_{i=1}^{\infty} m_i \cdot y_i \cdot z_i = J_{yz}$, where J_{xz} – centrifugal moment of inertia of considered unbalance relatively axis x and z, and J_{yz} – its centrifugal moment of inertia relatively axis y and z, then

$$\vec{M}_O\left(\vec{\Phi}_{right}\right) = \omega^2 \cdot \left(\vec{j} \cdot J_{xz} - \vec{i} \cdot J_{yz}\right).$$

In brackets of last expression was vector who se projection on the axis x is equal J_{yz} (apparently from the recording that is mentioned projection is negative), and on the axis y – J_{xz}; the theory of balancing mechanisms [5] it is called generalized vector $\vec{J}_{Oz}$ centrifugal moment of inertia of body mass that is rotating, relatively axis of rotation. With all this for the proposed unbalance

$$\vec{J}_{Oz|_{np}} = \left(\vec{j} \cdot J_{xz} - \vec{i} \cdot J_{yz}\right),$$

and its module

$$J_{Oz|_{np}} = \sqrt{\left(J_{xz}\right)^2 + \left(J_{yz}\right)^2} = \sqrt{\left(\sum_{i=1}^{\infty} m_i \cdot x_i \cdot z_i\right)^2 + \left(\sum_{i=1}^{\infty} m_i \cdot y_i \cdot z_i\right)^2}.$$

Then

$$\vec{M}_O\left(\vec{\Phi}_{right}\right) = \omega^2 \cdot \vec{J}_{Oz|_{right}}.$$

This formula shows that the vector of principal moment $\vec{M}_O\left(\vec{\Phi}_{right}\right)$ of inertia forces $\vec{\Phi}_i$ of particles of proposed unbalance relative to zero point and generalized vector $\vec{J}_{Oz|_{right}}$ centrifugal moment of inertia mass of the proposed unbalance relatively the rotating axis of unbalance shaft are in a plane perpendicular to that axis.

According to the design features vibration exciter UVV-04 is easy to prove and clear that:

$$J_{Oz|_{left}} = J_{Oz|_{right}} \quad \text{and} \quad M_O\left(\vec{\Phi}_{left}\right) = M_O\left(\vec{\Phi}_{right}\right);$$

$$\vec{J}_{Oz|_{left}} = \vec{J}_{Oz|_{right}} \quad \text{and} \quad \vec{M}_O\left(\vec{\Phi}_{left}\right) = \vec{M}_O\left(\vec{\Phi}_{right}\right).$$

As generalized vector $\vec{J}_{Oz}$ of centrifugal moment of mass of inertia and vector of the principal moment $\vec{M}_O$ of inertia forces $\vec{\Phi}_i$ of particles of both unbalances are determined by respective vector sums, in this case, after appropriate additions and legitimate necessary changes we come to the expressions:

$$\vec{J}_{Oz} = \vec{i} \cdot me\ell \cdot \cos\theta + \vec{j} \cdot me\ell \cdot \sin\theta = me\ell \cdot \left(\vec{i} \cdot \cos\theta + \vec{j} \cdot \sin\theta\right); \qquad J_{Oz} = me\ell;$$

$$\vec{M}_O = \omega^2 \cdot \vec{J}_{Oz}; \qquad M_O = \omega^2 \cdot J_{Oz} = me\omega^2\ell,$$

where $\ell = \frac{\ell}{2} + \frac{\ell}{2}$ – linear parameter, which determines arm of couple $\left(\vec{\Phi}_{right}, \vec{\Phi}_{left}\right)$.

Then according to work [8] dimensionless geometric characteristics of controlled mechanical centrifugal oscillation vibration exciter UVV-04

$$S^* = 0, \quad J^* = 1,$$

and its invariants of disbalance

$$in1 = \vec{S} = 0, \quad in2 = \vec{S} \cdot \vec{J}_{oz} = 0, \text{ but } \quad J^* \neq 0.$$

It is easy to see that in this case the value J_{oz} is independent of the choice on the axis of the shaft of center construction (zero point). Generalized centrifugal moment of inertia for such kinematics scheme is invariant of vibration exciter unbalance.

Start of vibration exciter UVV-04 are performed in statically balanced state with minimum moment disbalance, which is defined by initial distance ℓ_0 between the plane of symmetry of unbalances (see Fig. 1). After switching unbalance shaft 1 in to kinematics state of steady uniform rotation with angular velocity ω_{work} specified distance is increased by control mechanism of move able unbalance by moving them from their original position, which leads to the increase of moment disbalance to necessary given value.

An important feature is that in a balanced state unbalances are in the middle of the shaft, while increasing disbalance they move to the bearings, which are the pillars of the shaft 1, which determines and optimizes its bend.

With equally-spaced location of unbalances relative bearings the value of centrifugal moment of inertia is determined by dependence $J_{Oz} = S_1 \cdot \ell / 2$, where S_1 – static moment of unbalancemass. The axis of rotation of vibration exciter UVV-04 in working conditions carries angular oscillation movements and describe the surface in space, called the second-order cone (Fig. 3).

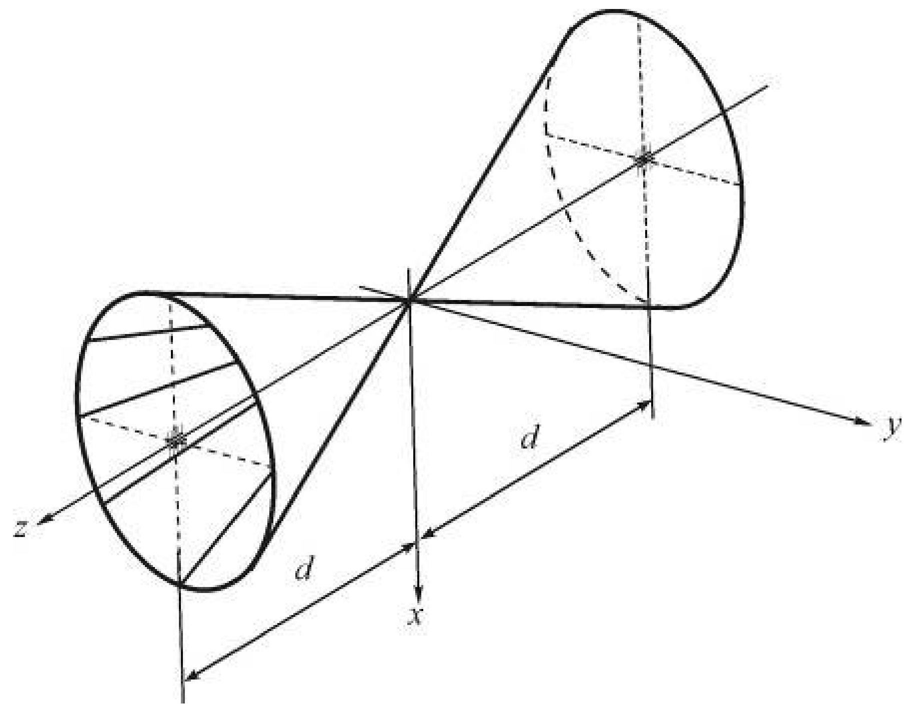

Figure 3 – Surface (second-order cone) that creates rotational axis UVV-04

Conclusions. CMCUVE UVV-04 generates vibration field of angle structure. Increasing the distance ℓ leads to increase of value J_{oz}, which leads to an adequate increase of the amplitude of angular oscillation. From Figure 3 it is clear that the amplitude of the points of longitudinal axis of unbalance shaft is different. This can be seen as a disadvantage of UVV-04, but that allows in the case of a technological need to use different amplitudes of oscillations for different intensity of vibration action along the axis of rotation of the unbalance shaft. As CMCUVE UVV-04 is a modification of vibration exciter UVV-02, the disadvantages of the last one which were identified in work [3], inherent in UVV-04.

References

1. *Serdyuk L. The controlled vibromachines / L. Serdyuk // Jubilee scientific conferens. University of architecture, civil ingineering and geodezy. – Sofia, 2007. – P. 43 – 48.*
2. *Сердюк Л. И. Повышение надежности и долговечности вибрационной техники с использованием управляемых вибрационных машин / L. I. Serdyuk, V. A. Onishchenko // The improvement of the quality, reliability and long usage of technical systems, and technological processes. – Hurghada, 2007. – P. 47 – 51.*
   Serdyuk L. I. Povyshenie nadezhnosti i dolgovechnosti vibracionnoy tehniki s ispolzovaniem upravlyaemyh vibracionnyh mashin / Л. И. Сердюк, В. А. Онищенко // The improvement of the quality, reliability and long usage of technical systems, and technological processes. – Hurghada, 2007. – P. 47 – 51.
3. *Жигилій С. М. Динаміка дебалансного вала керованого вібрappробуджувача УВВ-02 / С. М. Жигилій, К. С. Дяченко // Збірник наукових праць. Серія: Галузеве машинобудування, будівництво. – Полтава : ПолтНТУ, 2012. – Вип. 1 (31). – С. 186 – 193.*
   Zhyhylii S. M. Dynamika debalansnoho vala kerovanoho vibrozbudzhuvacha UVV-02 / S. M. Zhyhylii, K. S. Diachenko // Zbirnyk naukovykh prats. Seriia: Haluzeve mashynobuduvannia, budivnytstvo. – Poltava : PoltNTU, 2012. – Vyp. 1 (31). – S. 186 – 193.
   http://znp.pntu.edu.ua
4. *Жигилій С. М. Дослідження динаміки дебалансного вала керованого вібрoзбуджувача УВВ-03 / С. М. Жигилій, К. С. Дяченко // Збірник наукових праць. Серія: Галузеве машинобудування, будівництво. – Полтава : ПолтНТУ, 2012. – Вип. 2 (32). – С. 159 – 164.*
   Zhyhylii S. M. Doslidzhennia dynamiky debalansnoho vala kerovanoho vibrozbudzhuvacha UVV-03 / S. M. Zhyhylii, K. S. Diachenko // Zbirnyk naukovykh prats. Seriia: Haluzeve mashynobuduvannia, budivnytstvo. – Poltava : PoltNTU, 2012. – Vyp. 2 (32). – S. 159 – 164.
   http://znp.pntu.edu.ua
5. *Петров Г. Н. К синтезу колеблющихся систем балансировочных устройств. В кн.: Уравновешивание машин и приборов / Г. Н. Петров. – М. : Машиностроение, 1965. – С. 17 – 45.*
   Petrov G. N. K sintezu koleblyuschihsya sistem balansirovochnyh ustroystv. V kn.: Uravnoveshivanie mashin i priborov / G. N. Petrov. – M. : Mashinostroenie, 1965. – S. 17 – 45.
6. *Vector mechanics for engineers: statics and dynamics, tenth edition / F. Beer, E. Russell Johnston, D. Mazurek, P. Cornwell. – New York: McGraw-Hill Companies, Inc., 2013. – 1360 p.*
7. *Timoshenko S. Engineering mechanics, fourth edition / S. Timoshenko, D. Young. – New York: McGraw-Hill book Company, Inc, 1960. – 508 p.*
8. *Жигилий С. М. Управляемая вибрационная машина для подготовки металлической фибры: автореф. дис. канд. техн. наук / С. М. Жигилій / Полтавский государственный технический университет. – Полтава, 1997.– 16 с.*
   Zhigiliy S. M. Upravlyaemaya vibracionnaya mashina dlya podgotovki metallicheskoy fibry: avtoref. dis. kand. tehn. nauk / S. M. Zhigiliy / Poltavskiy gosudarstvennyy tehnicheskiy universitet. – Poltava, 1997.– 16 s.

Надійшла до редакції 31.03.2016