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Reinforced concrete section analysis based on design strength of reinforced concrete

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The paper presents the simplified deformation model for the section analysis of reinforced concrete members, which is considered as composite material formed of concrete and reinforcement. On this basis, it is proposed to use the design strength of reinforced concrete. This allows to reduce the section analysis of reinforced concrete members to the methodology of calculations introduced in the classical “strength of materials”, which contributes to a significant simplification and acceleration of the design process of both single members and structures in general. This approach to the section analysis of reinforced concrete members makes it universal for all deformation types of reinforced concrete members. The paper demonstrates and confirms the possibility of using the developed methodology in the bearing capacity calculations of bended reinforced concrete members.

Keywords: concrete, reinforcement, reinforced concrete, design strength, analysis

Розрахунок міцності залізобетонних елементів на основі розрахункового опору залізобетону

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У роботі представлена спрощена деформаційна модель для розрахунку залізобетонних конструкцій, в якій залізобетон розглядається як композитний матеріал, утворений з арматури та бетону. Модель реалізована із використанням поняття розрахункової міцності залізобетону. Це дозволяє звести розрахунок міцності залізобетонних елементів до методики розрахунків, запровадженої в класичному «опорі матеріалів», що сприяє значному спрощенню та прискоренню процесу проектування конструкцій в цілому. Основною перевагою методу розрахунку міцності з використанням поняття опору залізобетону є те, що він залишається незмінним при зміні діаграм напруження-деформації арматури та бетону, введенні нової інформації про властивості цих матеріалів, технологічні параметри, навантаження тощо. У запропонованому методі розрахунку змінюється лише один параметр – розрахунковий опір залізобетону, який синтезується за всіма перерахованими вище факторами. Такий підхід до розрахунку залізобетонних елементів робить його універсальним для всіх видів деформацій залізобетонних елементів. Для зручності використання розробленого методу сформовані таблиці значень розрахункового опору залізобетону для різних класів бетону й арматури при визначених відсотках армування. Крім того, отримані прості аналітичні формули, що можуть бути застосовані при отриманні проміжних значень цієї характеристики. Встановлено граничну умову визначення розрахункового опору залізобетону, що характеризується настанням текучості розтягнутої арматури. На основі запропонованої умови визначені граничні значення розрахункового опору залізобетону для нормально армованих залізобетонних елементів, тобто для таких, в яких у момент руйнування напруження розтягнутої арматури досягають межі текучості. У роботі демонструється та підтверджується можливість використання розробленої методики при розрахунках несучої здатності згинальних залізобетонних елементів.

Ключові слова: бетон, арматура, залізобетон, розрахунковий опір, розрахунок.

Introduction

Reinforced concrete is composite material formed of two materials namely: concrete and steel reinforcement. Each of these materials has its own physical and mechanical characteristics, which are used in strength analysis separately. However, the combination of this two materials in one, create possibility of deriving general characteristic of reinforced concrete as design strength of reinforced concrete. In this way strength analysis of reinforced concrete section becomes simple as in classical strength of materials.

Review of the research sources and publications

Today, there is a tendency to widely implement the nonlinear analysis and the finite element methods in the design of reinforced concrete structures. A large number of proposals have been developed to realize these methods for section analysis of reinforced concrete members [1 – 11]. Meanwhile, in most real-world design situations, there is no need to create complex models to design ordinary structures. In such cases, it will be appropriate to use simple and convenient methods for determining the strength of members. One of such methods – the reinforced concrete strength concept – was proposed and considered in [12 – 13] provide using of nonlinear stress-strain diagrams for materials. The general theory of reinforced concrete strength was widely descripted in [14 – 15] on the basis of deformation model. Deriving formulas in those publications is a complex mathematical process and developed tables not always meet characteristics of materials, which are used for the calculation.

Definition of unsolved aspects of the problem

To get simple engineering formulas it is expedient using simplified deformation model with rectangular stress distribution in the concrete compressed area. The design strength of reinforced concrete may be obtained by using this approach for the most common rectangular sections with reinforcement A400C and A500C.

Problem statement

The purpose of this work is to simplify the structural analysis of reinforced concrete members by introducing the synthesized concept of the design reinforced concrete strength, taking into account the strength characteristics of concrete and reinforcement, the reinforcement ratio and the location of reinforcement in the cross-section.

Basic material and results

The following prerequisites for structural analysis are accepted [16]:

– plane sections remain plane;

– the strain in bonded reinforcement, whether in tension or in compression, is the same as that in the surrounding concrete;

– the tensile strength of the concrete is ignored;

– a rectangular stress distribution in compressed concrete area, as given in Figure 3.5 [16], is assumed;

– the stresses in the reinforcing steel are derived from the design curve with a horizontal top branch without strain limit according to Figure 3.8 [16].

The ultimate concrete compressive strains $\varepsilon_{cu3,cd}$ and the yield strength of the tensile reinforcement are achieved in the cross-section at the moment of failure.

For the above conditions, the deformation mode of the section of the flexural reinforced concrete member is realized in Figure 1.

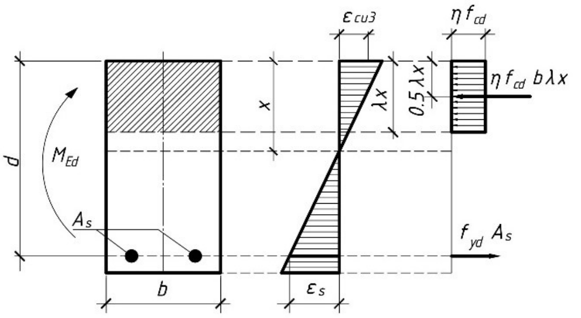

Figure 1 – Deformation mode of a single reinforced concrete beam

The equations of equilibrium are:

$$\Sigma X = 0 : f_{yd}\, A_s - \eta f_{cd}\, b\, \lambda x = 0; \tag{1}$$

$$\Sigma M_0 = 0 : M_{Ed} - f_{yd} A_s (d - x) - f_{cd}\, b\, \lambda x \left(x - \frac{\lambda x}{2} \right) = 0, \tag{2}$$

where f_{yd} – design yield strength of reinforcement; A_s – is the cross sectional area of tensile reinforcement bars; η – the coefficient, η = 1 for concrete grade from C12/15 to C50/60; f_{cd} – design value of concrete compressive strength; b – breadth of the beam; λ – the coefficient, λ = 0.8 for concrete grade from C12/15 to C50/60; x – the neutral axis depth; d – the effective depth of the beam; M_{Ed} – design value of the applied external bending moment.

After simple transformations, taking advantage of the prerequisites, it is received from (2)

$$\frac{M_{Ed}}{6W} - f_{yd} \rho + \frac{f_{yd}^2\, \rho^2\, \lambda}{2 \eta f_{cd}\, \lambda} = 0, \tag{3}$$

where $W = bd^2/6$ – elastic resistance moment of the section; $\rho = A_s/bd$ – the longitudinal reinforcement ratio.

The general condition of flexural reinforced concrete members bearing capacity has the form

$$M_{Ed} \le M_{Rd} = f_{zM} W \tag{4}$$

where M_{Rd} – the bending moment, which may be perceived by the beam; $f_{zM} = f\,(C, \rho_l, f_{yd})$ – the design value of reinforced concrete strength in flexural members.

From equation (3) and (4) taking $fz_{M1} = M_{Ed}/W$ for single reinforced section f_{zM} is obtained as:

$$f_{zM1} = 6 f_{yd} \rho \left(1 - \frac{f_{yd}\, \rho}{2 \eta f_{cd}} \right). \tag{5}$$

In order to use the concept of reinforced concrete strength in calculations of reinforced concrete members bearing capacity, tables of its values are compiled by using formula (5) depending on the reinforcement ratio, concrete and reinforcement classes (Tables 1, 2, 3).

Table 1* – Design strength values of reinforced concrete f_{zM1} for flexural members with single reinforcement (A400C)

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 364 MPa							
C12/15	1.080	9.751	17.164	19.993	23.899	–	–	–
C16/20	1.083	10.056	18.384	21.899	27.635	29.854	32.997	–
C20/25	1.085	10.235	19.099	23.017	29.825	32.715	37.467	40.848
C25/30	1.086	10.335	19.502	23.647	31.059	34.327	39.986	44.477
C30/35	1.087	10.410	19.802	24.115	31.977	35.526	41.860	47.174
C32/40	1.087	10.468	20.033	24.477	32.687	36.453	43.308	49.259
C35/45	1.088	10.523	20.250	24.816	33.351	37.320	44.663	51.210
C40/50	1.088	10.559	20.395	25.042	33.793	37.898	45.566	52.511
C45/55	1.089	10.589	20.515	25.230	34.162	38.380	46.319	53.595
C50/60	1.089	10.619	20.635	25.418	34.531	38.862	47.072	54.679

Table 2* – Design strength values of reinforced concrete f_{zM1} for flexural members with single reinforcement (Ø6 – 22 A500C)

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 435 MPa							
C12/15	1.288	11.380	19.421	22.190	–	–	–	–
C16/20	1.293	11.816	21.164	24.912	30.558	–	–	–
C20/25	1.295	12.071	22.185	26.508	33.685	36.540	–	–
C25/30	1.297	12.215	22.761	27.407	35.449	38.843	44.380	–
C30/35	1.298	12.322	23.189	28.076	36.760	40.555	47.055	52.100
C32/40	1.299	12.405	23.520	28.593	37.773	41.879	49.123	55.077
C35/45	1.299	12.482	23.829	29.077	38.721	43.117	51.058	57.864
C40/50	1.300	12.534	24.036	29.400	39.353	43.943	52.348	59.722
C45/55	1.300	12.577	24.208	29.668	39.880	44.631	53.423	61.270
C50/60	1.301	12.620	24.380	29.937	40.407	45.319	54.499	62.818

Table 3* – Design strength values of reinforced concrete f_{zM1} for flexural members with single reinforcement (Ø25 – 40 A500C)

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 417 MPa							
C12/15	1.288	11.380	19.421	22.190	–	–	–	–
C16/20	1.293	11.816	21.164	24.912	30.558	–	–	–
C20/25	1.295	12.071	22.185	26.508	33.685	36.540	–	–
C25/30	1.297	12.215	22.761	27.407	35.449	38.843	44.380	–
C30/35	1.298	12.322	23.189	28.076	36.760	40.555	47.055	52.100
C32/40	1.299	12.405	23.520	28.593	37.773	41.879	49.123	55.077
C35/45	1.299	12.482	23.829	29.077	38.721	43.117	51.058	57.864
C40/50	1.300	12.534	24.036	29.400	39.353	43.943	52.348	59.722
C45/55	1.300	12.577	24.208	29.668	39.880	44.631	53.423	61.270
C50/60	1.301	12.620	24.380	29.937	40.407	45.319	54.499	62.818

***Notice.** Values of fzM1 in tables 1-3 are obtained for nonoverreinforced sections, in which reinforcement achieves yield stress at the time of failure.

The value of the design strength of concrete is limited by the value, which corresponds to the yield stress of reinforcement. To determine this value, a well-known notation is used

$$\alpha_m=\lambda\xi(d-0.5\lambda\xi), \quad (6)$$

where $\xi = x / d$ – relative neutral axis depth.

Using (6), equation (2) is written in the form

$$\Sigma M_A=0: M_{Ed}-\alpha_m f_{cd}\, bd^2=0. \quad (7)$$

Using the right-hand side of condition (4) it is received

$$f_{zM}=6\alpha_m f_{cd}. \quad (8)$$

Ultimate value of coefficient α_m, which corresponds to the yield point of tensile reinforcement is determined by (7) using the value of relative neutral axis depth

$$\xi_R=\frac{1}{1+\frac{f_{yd}\varepsilon_{cu3}}{E_s}}. \quad (9)$$

where ε_{cu3} – the ultimate values of compressive concrete strains, ε_{cu3} = 3.5 ‰ for concrete classes from C12/15 to C50/60 according to Table 3.1 [16].

Thus, for nonoverreinforced members, the design strength of reinforced concrete is limited by the following dependence

$$f_{zM1}\le f_{zM,R}=6\alpha_R f_{cd}. \quad (10)$$

The ultimate values of the design strength of reinforced concrete for members with single reinforcement are given in Table 4 for different classes of concrete and reinforcement.

Table 4 – Ultimate values of design strength of reinforced concrete $f_{zM1,R}$ for single reinforced members

Strength classes of concrete	Classes of reinforcement		
	A400C	A500C (Ø6 – 22)	A500C (Ø25– 40)
C12/15	24.984	23.988	24.235
C16/20	33.802	32.454	32.788
C20/25	42.619	40.920	41.342
C25/30	49.968	47.976	48.470
C30/35	57.316	55.031	55.598
C32/40	64.664	62.086	62.725
C35/45	73.482	70.552	71.279
C40/50	80.830	77.607	78.407
C45/55	88.178	84.663	85.535
C50/60	96.996	93.129	94.088

For above conditions the deformation mode of the double reinforced section of the flexural reinforced concrete member is realized in Figure 2.

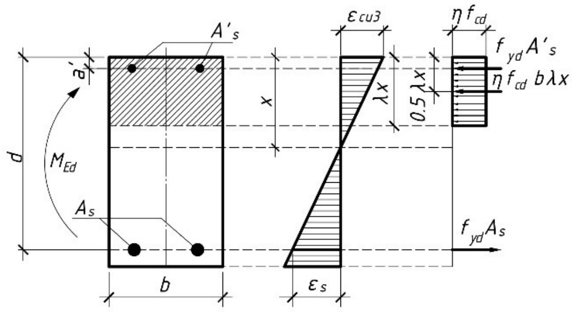

Figure 2 – Deformation mode of double reinforced concrete beam

The equations of equilibrium are:

$$\Sigma X=0: f_{yd}A_s-f_{yd}A_s'-\eta f_{cd}\, b\lambda X=0; \quad (11)$$

$$\Sigma M_A=0: M_{Ed}-f_{yd}A_s'\left(d-a'\right)-$$
$$-\eta f_{cd}\, b\lambda X\left(d-\frac{\lambda X}{2}\right)=0, \quad (12)$$

where A_s' – is the cross-sectional area of compressed reinforcement; a'– distance from the most compress edge of the section to the center of gravity of compressed reinforcement.

After simple transformations, taking advantage of the prerequisites, it is received from equation (12)

$$\frac{M_{Ed}}{6W}-f_{yd}\rho\left(1-\frac{ca'}{d}\right)+\frac{f_{yd}^2\rho^2 d^2(1-c)^2}{2\eta f_{cd}}=0, \quad (13)$$

where $c=A_s'/A_s$ – the ratio between the compressed and tensile reinforcement areas.

From equation (13) and (4) taking $f_{zM2} = M_{Ed}/W$ for double reinforced section f_{zM2} is obtained as:

$$f_{zM2}=6f_{yd}\,\rho\left(1-\frac{ca'}{d}-\frac{f_{yd}\rho(1-c)^2}{2\eta f_{cd}}\right). \quad (14)$$

In order to use the concept of reinforced concrete strength in calculations of reinforced concrete members bearing capacity tables of its values are compiled by using formula (14) depending on the reinforcement ratio, concrete and reinforcement classes taking c = 0.5, a'/d = 0.1 (Tables 5, 6, 7).

For double reinforced members, the design strength of reinforced concrete is limited by the following dependence

$$f_{zM2}\le f_{zM,R}=6\alpha_R f_{cd}. \quad (15)$$

Table 5 – Design strength values of reinforced concrete f_{zM2} for flexural members with double reinforcement (A400C)**

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 364 MPa; c = 0,5; a' / d = 0.1							
C8/10	1.033	9.960	–	–	–	–	–	–
C12/15	1.034	10.082	19.579	24.108	–	–	–	–
C16/20	1.035	10.158	19.884	24.585	–	–	–	–
C20/25	1.036	10.203	20.063	24.864	34.210	38.755	–	–
C25/30	1.036	10.228	20.163	25.022	34.519	39.158	48.217	–
C30/35	1.036	10.247	20.238	25.139	34.748	39.458	48.685	57.658
C32/40	1.036	10.261	20.296	25.229	34.926	39.689	49.047	58.179
C35/45	1.036	10.275	20.351	25.314	35.092	39.906	49.386	58.667
C40/50	1.036	10.284	20.387	25.370	35.202	40.051	49.612	58.992
C45/55	1.037	10.291	20.417	25.417	35.295	40.171	49.800	59.263
C50/60	1.037	10.299	20.447	25.464	35.387	40.291	49.988	59.534

Table 6 – Design strength values of reinforced concrete f_{zM2} for flexural members with double reinforcement (Ø6 – 22A500C)**

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 435 MPa; c = 0,5; a' / d = 0.1							
C8/10	1.234	11.806	–	–	–	–	–	–
C12/15	1.236	11.980	23.125	–	–	–	–	–
C16/20	1.237	12.089	23.561	29.066	–	–	–	–
C20/25	1.237	12.153	23.816	29.464	–	–	–	–
C25/30	1.238	12.189	23.960	29.689	40.835	–	–	–
C30/35	1.238	12.216	24.067	29.857	41.162	46.679	–	–
C32/40	1.238	12.236	24.150	29.986	41.416	47.010	–	–
C35/45	1.238	12.256	24.227	30.107	41.653	47.319	58.440	–
C40/50	1.238	12.268	24.279	30.187	41.811	47.526	58.762	–
C45/55	1.239	12.279	24.322	30.255	41.942	47.698	59.031	70.127
C50/60	1.239	12.290	24.365	30.322	42.074	47.870	59.300	70.514

Table 7 – Design strength values of reinforced concrete f_{zM2} for flexural members with double reinforcement (Ø25 – 40A500C)**

Strength classes for concrete	Longitudinal reinforcement ratio ρ_l, %							
	0.05	0.5	1	1.25	1.75	2	2.5	3
	f_{yd} = 417 MPa; c = 0,5; a' / d = 0.1							
C8/10	1.183	11.341	–	–	–	–	–	–
C12/15	1.185	11.501	22.235	–	–	–	–	–
C16/20	1.186	11.601	22.635	27.939	–	–	–	–
C20/25	1.186	11.660	22.870	28.306	38.841	–	–	–
C25/30	1.187	11.693	23.002	28.513	39.246	44.469	–	–
C30/35	1.187	11.717	23.100	28.666	39.548	44.863	–	–
C32/40	1.187	11.736	23.176	28.785	39.780	45.167	55.717	–
C35/45	1.187	11.754	23.247	28.896	39.998	45.451	56.162	–
C40/50	1.187	11.766	23.295	28.970	40.143	45.641	56.458	67.039
C45/55	1.187	11.776	23.334	29.032	40.264	45.799	56.705	67.394
C50/60	1.187	11.786	23.374	29.094	40.385	45.957	56.952	67.750

****Notice.** Values of f_{zM2} in tables 5 – 7 are obtained for nonoverreinforced sections, in which the tensile reinforcement achieves yield stress at the time of failure.

The application of the developed method for section analysis of reinforced concrete members is considered on examples.

Example 1. Given: a beam of rectangular profile with cross-sectional dimensions b = 200 mm. h = 400 mm; concrete class C30/35 (f_{cd} = 19.5 MPa. E_{cd} = 27 GPa. $\varepsilon_{c1.cd}$ = 1.70‰); reinforcement class A400C (f_{yd} = 364 MPa. E_s = 210 GPa) is located at a distance from the lower face of the section a = 30 mm, reinforcement area A_s = 1256 mm^2.

It is necessary to determine the bending moment M_{Rd} that may be perceived by the beam.

Solution: determination of the effective depth of the beam $d = h - a = 400 - 30 = 370$ mm.

Resisting moment of the section

$W = bd^2/6 = 200 \times 370^2/6 = 4.5\times10^6$ mm^3.

Longitudinal reinforcement ratio

$\rho = A_s/bd \times 100\% = 1256 / (200 \times 370) \times 100\% = 0.017\times 100\% = 1.7\%$.

From Table (1) f_{zM1} was determined using interpolation method, f_{zM1} = 31.077 MPa < $f_{zM,R}$ = 49.968 MPa (Table 4).

$$M_{Rd} = f_{zM} W = 31.077 \times 4.5 \times 10^6 = 139.85 \text{ kNm}.$$

The bending moment, which may be perceived by the beam, is M_{Rd} =139.85 kNm.

Example 2. Given: beam from example 1 with compressed reinforcement A_s'= 628mm^2 located at a distance a'=30 mm.

It is necessary to determine the bending moment M_{Rd} that may be perceived by the beam.

Solution: determination of the effective depth of the beam $d = h - a = 400 - 30 = 370$ mm.

Resisting moment of the section

$W = bd^2/6 = 200 \text{ x } 370^2/6 = 4.5\text{x}10^6$ mm.

Longitudinal reinforcement ratio

$\rho = A_s/bd \times 100\% = 1256 / (200 \times 370) \times 100\% = 0.017\times 100\% = 1.7\%$;

$c = A_s'/A_s = 628 / 1256 = 0,5$;

$a' / d = 30 / 370 = 0.1$.

From table (5) f_{zM2} was determined using interpolation method, f_{zM2} = 33.787 Mpa < $f_{zM,R}$ = 49.968 MPa (Table 4).

$$M_{Rd} = f_{zM} W = 33.787 \times 4.5 \times 10^6 = 152.04 \text{ kNm}.$$

The bending moment, which may be perceived by the beam, is M_{Rd} =152.04 kNm

Conclusion

The implementation of the method of section analysis of reinforced concrete members based on the concept of design strength of reinforced concrete using a simplified deformation model is clear and understandable. The limiting condition for determining the design strength of reinforced concrete, characterized by the onset of yielding of tensile reinforcement, has been established.

On the basis of the proposed condition, the ultimate values of the design strength of reinforced concrete are determined for normally reinforced concrete member, that is, for those in which at the moment of failure, the stress of the tensile reinforcement reaches the yield point. Derived formulas are simple and may be easily used when using materials with properties different from tabular data.

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