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TEXTBOOK

BUILDING STRUCTURES

(reinforced concrete and masonry structures)

Teaching aid

for students on specialty 192 "Construction and Civil Engineering"

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In order to facilitate the perception of theoretical material, it is accompanied by drawings. Methods of calculating the bearing capacity of reinforced concrete structures meet the current regulatory framework of Ukraine DBN B.2.6-98: 2009 and DSTU B B.2.6-156: 2010. Examples of calculation of bearing capacity of reinforced concrete flexural members in normal and inclined sections are offered.

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FOREWORD

The textbook is written in accordance with the program of the course "Building Structures" (reinforced concrete and masonry structures) for the preparation of bachelor's specialty 192 "Construction and civil engineering".

The purpose of the textbook is to promote the formation of students' skills in calculation and design of separate reinforced concrete members.

In order to facilitate the perception of theoretical material, it is accompanied by drawings. Methods of calculating the bearing capacity of reinforced concrete structures meet the current regulatory framework of Ukraine DBN B.2.6-98: 2009 and DSTU B B.2.6-156: 2010. Examples of calculation of bearing capacity of reinforced concrete flexural members in normal and inclined sections are offered.

The textbook consists of seven chapters.

Chapters 1 – 3, 7 are written by Dovzhenko O.O., chapters 4 – 6 are written by V.V. Pohribnyi.

1 REINFORCED CONCRETE AND MASONRY STRUCTURES AND THEIR ROLE IN CONSTRUCTION OF BUILDINGS AND STRUCTURES

1.1. The essence of the reinforced concrete

The concrete, like any other stone material, has a different compressive strength f_c and tensile strength f_{ct}. The tensile strength f_{ct} is approximately 15 – 20 times less than the compressive strength f_c. Concrete is a fragile material, its ultimate relative tensile strain ε_{ctu} is approximately 13 times less than the ultimate relative compressive strain ε_{cu}. In many cases it is fragility that doesn't allow to use the tensile strength of concrete even at small stresses due to the formation of initial cracks in concrete, which may be caused by fluctuation in temperature, the uneven drying, random dynamic loads. Therefore, only axially compressed elements (walls, massive foundations, columns, retaining wall, etc.) were made of concrete. Tensile, eccentric compressed**,** flexural members in the past were made of deficient steel or short-lived wood.

Let's look the behavior of the concrete beams under the load (fig. 1.1, a).

With increasing of load from 0 to F_1 stress and strain of concrete in both compressed and tensile zones are related by a linear dependence. With further increasing of load in concrete tensile zone an inelastic strain appears, which at F_2 reaches ε_{ctu}, and the first normal crack is formed. It also causes beam fragile failure. In this case, the bearing capacity of concrete compressed zone is used no more than for 5 – 7 %.

If steel bars are placed in the tensile zone of the beam, then the behavior and the failure of such a reinforced concrete structure is changed. In the initial period of loading, concrete and reinforcement work together. Gradually, in the tensile concrete zone small inelastic strains develop, which, with an increasing load, reach the ultimate values. This is the reason of first cracks formation. The load value at this moment is only 15 – 25% of ultimate load. In the section with cracks, tensile concrete is excluded from work. The reinforcement takes all tensile stresses. And the load can be increased. Further its growth causes an increase of the crack width, its development on cross section height and the formation of new cracks.

If the amount of reinforcement is not large, the failure begins, when in one of the sections with the crack the strain in reinforcement corresponds yield point (fig. 1.1, b). It is a plastic failure, which is accompanied by large deflections and ends by crushing of the concrete compressed zone. Therefore, the reinforcing of concrete beams makes it possible rationally to use the strength of steel reinforcement in tension and the strength of concrete in compression. Bearing capacity of reinforced concrete beams in comparing with concrete increase up to 10 times.

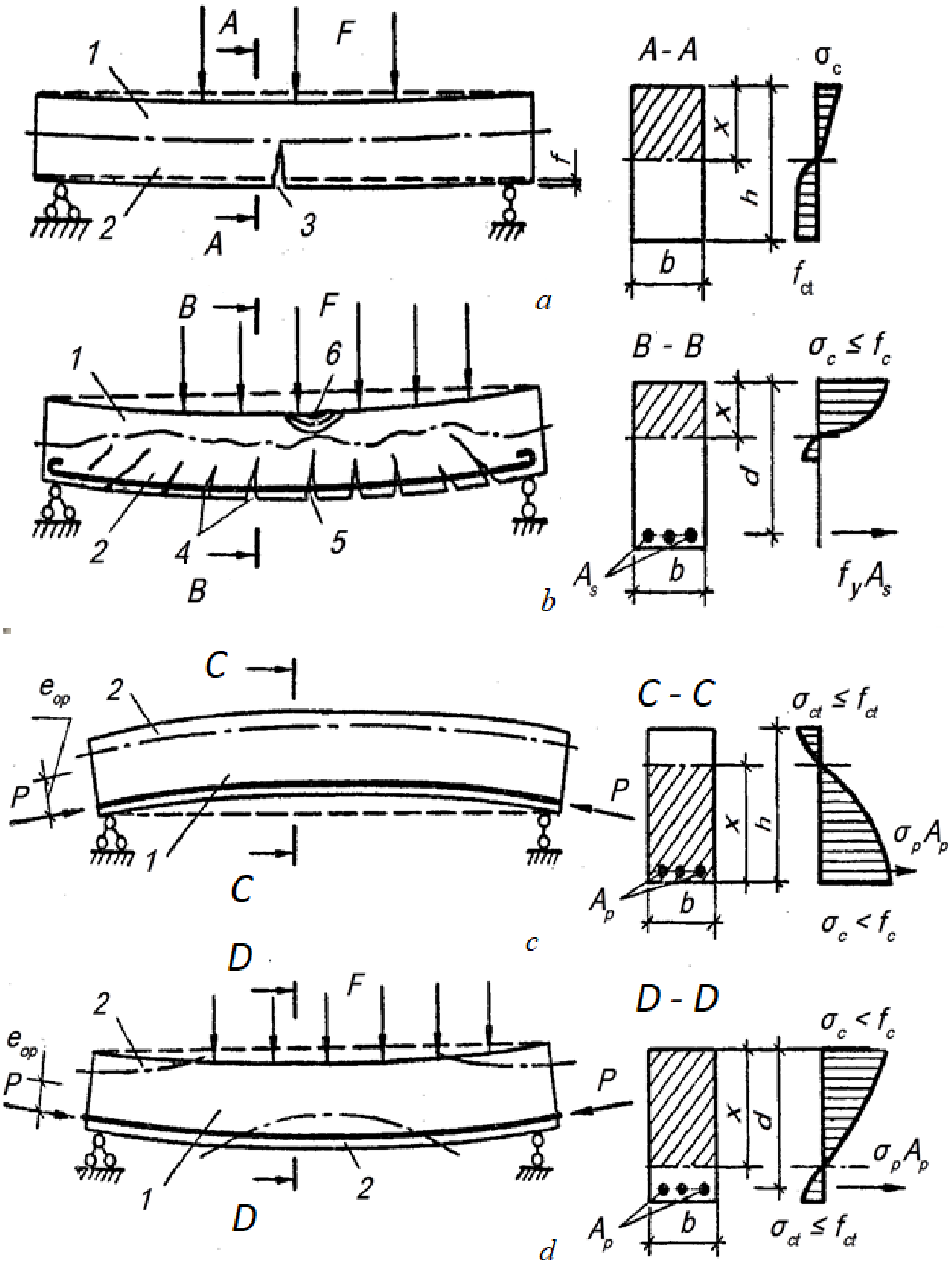


Fig. 1.1 The behavior of the concrete and reinforced concrete beams under the load: a – failure of the concrete beam; b – failure of the reinforced concrete beam; c – stress-strain state of the pre-stressed beam at the stage of manufacture; d – the same in the operation phase; 1 – the compressed zone; 2 – the tensile zone; 3 – the crack, which causes failure of the beam; 4 – cracks in tensile concrete; 5 – the yielding zone of the reinforcement; 6 – the concrete crushing

Reinforced concrete is complex building material, in which concrete and steel reinforcement, connected by mutual bond, work under the load as a monolithic body.

Cooperation of concrete and steel reinforcement is caused by an optimum combination of physical and mechanical properties of these materials, namely:

- during the concrete hardening between it and reinforcement there is considerable strength of bond, as a result, materials are deformed under the load together;

- the dense concrete protects steel reinforcement from corrosion and direct action of fire;

– steel and concrete have similar coefficients of linear temperature expansion, therefore when the temperature is smaller than 100°C in both materials there are unimportant initial stress and slippage of reinforcement in concrete is not observed (for concrete $\alpha'_{c(t)}$ = 0.00001 C^{-1}, for reinforcement $\alpha'_{s(t)}$ = 0.000012 C^{-1});

– reinforcement compensates disadvantages of concrete, changing its behavior in reinforced concrete structures.

1.2. A short historical overview of the appearance of reinforced concrete structures

The appearance of reinforced concrete structures (RCS) belongs to the second half of the XIX century and connects with the development of industry, transport, trade, that required the construction of new factories, ports and other capital structures. By this time, cement industry and metallurgy were developed, centuries of experience in the field of construction of stone, concrete, wood and metal were gained. Today it is not possible to name a date and discoverer. So, let's try to mention the discoverers.

In 1832 an Englishman M. Brunel built the brick arch on cement mortar, which was reinforced by "ribbon" iron, and determined the bond between them. This can be considered the first study of this combination, which is significantly outpaced the others, and then was forgotten.

In 1849 – 1850 a Frenchman I. Lambo planned to replace the wooden boat to boat with reinforced cement. Its sidewalls and bottom thickness were 40 – 50 mm. The boat was made of square cross section bars, connected to each other. Then the mesh was coated with cement mortar. In 1855 the boat was demonstrated at the World Exhibition in Paris, and in 1902 it was seeing afloat. In 1855 I. Lambo received a patent for a new wood substitute – the steel system of wires and bars, which then were coated by the hydraulic cement. Both during constructing the boat and the preparation of patent description I. Lambo provided the decisive importance to iron cage. He considered cement mortar as aggregate and protector for iron from external influences.

In 1855 F. Quanier described his experiments with unreinforced concrete. He built a house in Lyons with a flat roof with thickness 270 mm, bearing structure which was made of concrete, steel beams with T-sections and mesh between them. In 1861 a book by F. Quanier "Reinforced concrete blocks used in construction" was published. The author firstly suggested that the iron bars work together with concrete, increasing its bearing capacity, concrete reliably protects iron from moisture and corrosive environment.

In 1854 an Englishman W. Wilkinson received a patent for the structure of fire-resistant floor, he described the necessity of placing the iron ropes, wires or ribbon in tensile zone. Apparently, this patent may be considering as the first description of reinforced concrete in the modern sense of cooperation of its components – concrete and reinforcement. In 1865 C. Wilkinson built a small house, where the walls, floors, stairs and even the smokestack were made of reinforced concrete. The structure of reinforced concrete caisson floor was particularly interesting. Its thin slab (37.5 mm) was reinforced by a mesh with rectangular cross section bars. Beams of two directions were reinforced with rope. Unfortunately, in 1955 this building was destroyed, but the measurements and tests, that took place at the same time, confirmed the high quality of performance all rebar and concrete works.

American T. Hiat began in 1855 his experiments to improve the fire resistance of brick floor which reinforced by iron bars. Bricks were joined with cement mortar. In 1877 T. Hiat described special study of reinforced concrete beams in the book. He found out: in a reinforced concrete beam, iron takes the tensile forces that are necessary to balance the compressive forces in concrete; the synchronicity that exists between the temperature expansions of both materials allows to consider concrete and reinforcement as a whole.

Only after remembered studies the Frenchman J. Monier received the patent for portable flower garden pots of iron and cement. In 1868 he built a small reinforced concrete pool and sent an application for reservoirs and pipes. In 1869 he made the reinforced concrete floor and sent an application for slabs and partition walls. In 1873 J. Monier received the patent for reinforced concrete bridge. In 1875 he made the same for stairs. After that J. Monier began to patent RCS in different countries, including Russia. Despite this activity, the idea by J. Monier about reinforced concrete was at the level of knowledge of gardener. He believed, that "firstly iron cage of certain shape and size is made, then it is coated with cement, which gives it greater strength and protects it against oxidation". Iron cage in all structures and on the drawings of patents was located in the middle of the member height section. Although J. Monnier was not the first who invented reinforced concrete, he persistently used and promoted it. He was the first who intuitively understood the prospects and profitability of reinforced concrete, which was named as "Monier´s system" during long time. In 1880 – 1884 J. Monier’s patents were purchased in Germany and Austria, there the serious research of reinforced concrete began, scientific and technological side of which was entrusted to I. Baushynher, M. Kennen, E. Mersh.

In 1897 in the French state school of pathways and bridges Sh. Rabyu started to read the first course of reinforced concrete. From 1892 the French State Commission of reinforced concrete began to work. From 1901 the first specialized German magazine "Concrete and iron" had been published. The first German (1904), French (1906), Russian (1908) standards were adopted.

There is some information, that in 1852 in Nikolaev’s palace in St. Petersburg "fire-proof floor was performed with a lime concrete and cage of iron bars".

A military engineer Kaldevin in the 1880 at the exhibition in Tbilisi built bridge, in which he placed an iron cage in a concrete mass. The first mention of a new material in the press belongs to 1859, here the "Engineering Journal" placed the note "About the structure from cement and iron". The author talked about the structure of the I. Lambo’s boat. He wrote: "The boat is mentioned here only as an example. RCS can be used much more effectively in the construction, because it can be used in field of fire-resistant and water-proof walls". In 1898 the first Russian book on reinforced concrete by S. Rudnitsky was published.

A large number of fundamental tests and researches were conducted in the period from 1886 to 1905 and they proceeded to the widespread use of reinforced concrete in Russia. In 1886 at Moscow slaughterhouses the trading house "Julius Hook and K" applied reinforced concrete slabs and vaults that were tested. In 1890 the construction department of the trading house established corporation to implement concrete and other building work. In 1891 reinforced concrete structures, manufactured in full size, were tested. M. Belelyubsky developed the program of experiments. The program included: three pairs of slabs with span of 1, 1.5, 2 m (paired one slab was reinforced, other wasn’t); two vaults with span of 4 m; two pipes with a diameter of 0.71 m, one of which was tested under a mound, and another – as a beam on two supports with span of 1.42 m; a 500 buckets capacity reservoir; hexagon bin of elevator from prefabricated elements, which was loaded with water; a bridge with span of 17 m. During the 1 – 5 of November 1891 all these structures were tested, data of experiments – the load value, deflection, formation of the first cracks and failure – were carefully recorded. Experiments have shown absolute advantages of the new material. The bearing capacity of reinforced concrete structures was up to 3.5 – 6 times higher than, that of concrete.

Described experiments 1886 – 1896 contributed, that in the Moscow region at that time all new structures had at least one reinforced concrete element.

1.3. Reinforced concrete structures properties and the fields of application

The main positive properties of concrete include: practically inexhaustible reserves of raw materials for the manufacture of binders and aggregates; environmental expediency of using waste products as raw materials for concrete components; high reliability and durability of reinforced concrete structures, its resistance to high temperatures and aggressive environments, resistance to dynamic loads; possibilities to satisfy various requirements of construction, including the creation of underground and underwater structures; low energy process of manufacture reinforced concrete; relatively simple manufacture technology, possibility to create any shape; structural compatibility of concrete with many building constructional and decoration materials; low operating costs of maintaining buildings.

At the same time reinforced concrete structures have such disadvantages: cracking due to shrinkage and force effects (load at the time of formation of cracks usually is between 15 – 25% from the ultimate), which is not always permissible for normal operation; increased deformability (active process of development of cracks quickly reduces the neutral axis depth, that rapidly (up to 5 times) reduces the second moment of area I_{red} of the reduced section and causes a rapidly increase of deflection *f*); impossibility of rational using of reinforcement with yield strength more than 500 MPa (experimentally established, that at the time, when the crack width and deflection of structure reaches the admissible limit values, strength in plane reinforcing bar is not more than 300 MPa, and in the deformed reinforcing bar is not more than 500 MPa); inexpediency of using of modern high strength concrete due to the low strength of reinforcement, the low crack resistance and stiffness of reinforced concrete without pre-stressing; high density that leads to excess weight and limits values of spans, which overlapping (when spans are 12 m reinforced concrete beams without pre-stressing become self-supporting structure and these beams are not cost-effective); high thermal and sound conductivity; complexity of alterations and strengthening; the necessity of additional time to acquire a predetermined strength concrete; low resistance to dynamic loads.

RCS are the basis of the modern construction industry; they are widespread in (fig. 1.2):

– industrial and civil construction (elements of the floor and roof, columns, crane beams, foundations, wall panels);

– agricultural construction;

– transport construction (bridges, tunnels, underground, runways of airfields);

– energy construction (dams, gateways, structures power plant and atomic reactors);

– hydro melioration construction (irrigation systems);

– vessel building (floating docks, pontoons);

− mining (fixing of rocks, on over mine constructs);
– extraction of mineral resources (floating oil platform);
– sports and cults buildings.

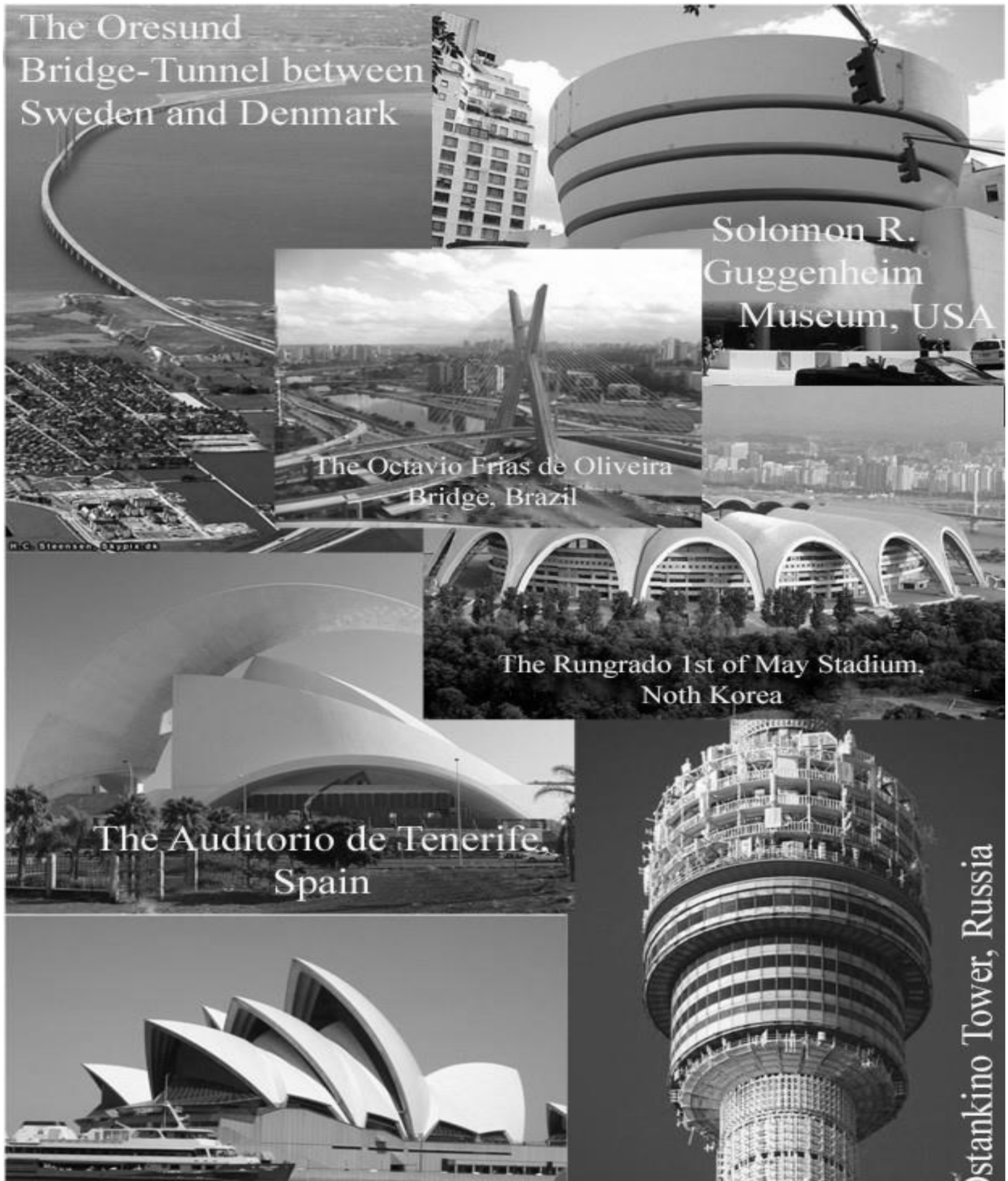


Fig. 1.2. Examples of application of reinforced concrete structures

1.4. The concept of pre-stressed structures

In *pre-stressed reinforced concrete structures* in the manufacturing process in accordance with the calculation the initial stresses of tension in the part of (or for all) reinforcement and compression for all (or part of) concrete are created artificially. Compression of concrete (usually tensile zone under the load) on the corresponding value is carried out by pre-tensioned reinforcement (fig. 1, b), which after release tries to return to its initial state. During this the beam gets hogging. When the external load is applied, in structures the pre-compression is compensated firstly and then tensile stresses occur in the concrete (fig. 1, d), they will be gradually increase to the maximum tensile resistance of concrete. Further development of the stress strain stages (SSS) is similar to reinforced concrete structures without pre-stressing. Thus, the pre-stressing makes it possible:

– to increase the crack resistance of concrete in 2 – 3 times compared to structures without pre-stressing (the operation time of structures without cracks in the tensile zone is increased and the width of cracks is limited);

– to increase the stiffness of the elements and its deflection;

– to use high-strength reinforcement which allows up to 50% reduction in steel consumption;

– to increase fatigue limit of structures;

– to increase the overlapping of spans.

Pre-stressing does not affect the strength of structures.

P. Jackson was the first, who in 1886 patented the "Structures of bridges with artificial stone and concrete" with a stressed anchor. In 1910 in Germany Bah performed a series of tests of pre-stressed beams. In 1928 the Frenchman E. Fresnel received a patent for the using of high-strength steels, which previously was stretched to stress higher than 400 MPa.

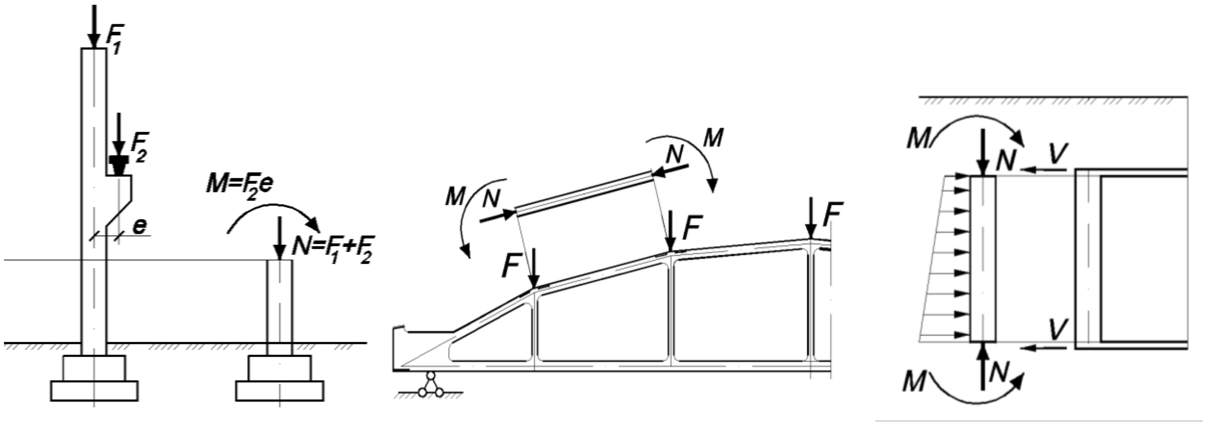
1.5. The classification of reinforced concrete structures according to stress state, by destination and according to their manufacture method

According to stress state, reinforced concrete elements are classified into:

- flexural members – slabs and beams. Slabs are flat members, which thickness is less than length and width. Beams are elongated elements which length is much longer than cross sections size. Slabs and beams are elements of flat reinforced concrete floors and roofs;

- compressed members. Axially compressed members include: middle columns of buildings and structures; top chords of trusses, which are loaded in a panel point; verticals and rising diagonals of truss. Eccentric compressed (fig. 1.3) members include: columns of single-storey industrial buildings which are loaded by resistance of cranes; top chords of diagonals without trusses; arches; walls of rectangular-plan underground tanks that take up lateral pressure of ground or liquid and vertical pressure of roof. In such elements normal force N, bending moment M and shear force V are acting;

- tensile members. Axial tensile members (fig. 1.4) include: ties of arches; bottom chords and lowering diagonals of trusses; walls of water tanks circular in plan. Eccentric tensile members include: walls of rectangular-plan tanks that take up an internal pressure; bottom chords of diagonals without trusses etc.;



a *b* *c*

Fig. 1.3. Structures and their members under compression: a – a column of an industrial building; b – a top chord of a truss; c – a wall of underground tank rectangular in plan

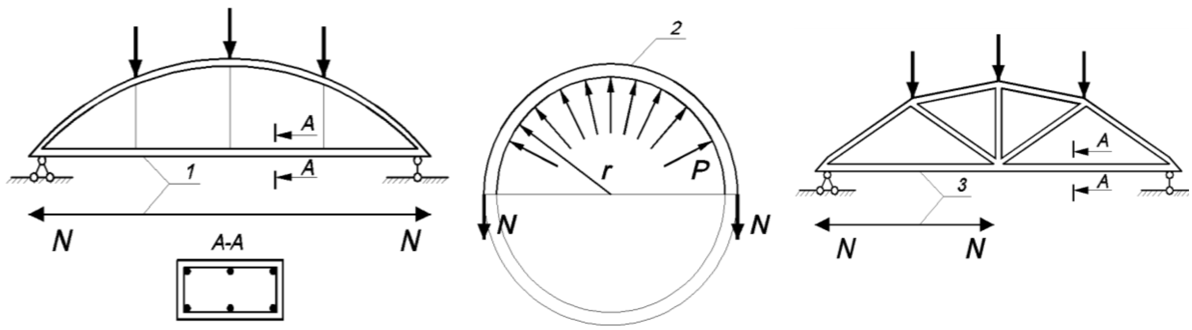


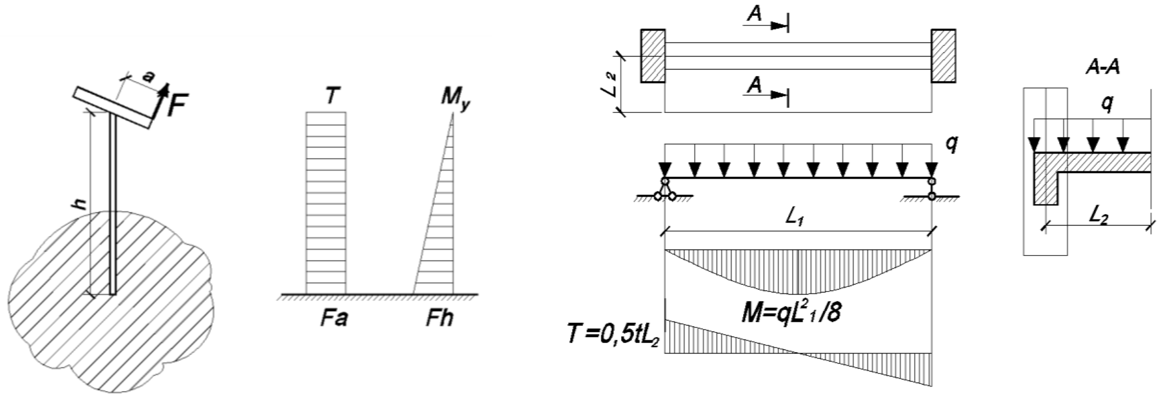
Fig. 1.4. Axial tensile members: 1 – a tie of an arch; 2 – a wall of a tank circular in plan; 3 – a bottom chord of a truss

- flexural-torsional members (fig. 1.5): mast under the action of a horizontal force that is applied with an eccentricity relative to the longitudinal axis; a beam with one-sided corbel slab.

According to destination, reinforced concrete structures can be classified as bearing and enclosing structures.

According to manufacture method, reinforced concrete structures are: precast structures, cast-in-place structures and precast-cast-in-place structures.

Precast structures are manufactured in highly mechanized, automated industrial plants, which specialize in the manufacture of definite range of products and structures. They are widely spread because their usage does not depend on weather conditions and they reduce a labor intensity of construction. At the same time, they have serious disadvantages: labor intensity, high cost, and steel intensity of elements of joints; decrease of stiffness of the structure in total as a result of general space continuity disturbance (redundancy); significant costs of establishing and rebuilding of industrial base and transportation costs. If their number of members is limited and an amount of usage is large the precast structures are effective.



a *b*

Fig. 1.5. Flexural-torsional members: a – a mast under the horizontal force *F* that is applied eccentrically relative to longitudinal axis; b – a beam with one-sided corbel slab

Cast-in-place structures are made of concrete mix, which is placed into formwork on site. Their main advantage is a space continuity, which provides less steel intensity (without embedded item and transport and erection reinforcement). Their effective usage is possible in complex geological conditions, in seismic regions, during reconstruction and application of complex architectural forms. The main disadvantages of cast-in-place structures are: seasonality of works; necessity of formwork; dependence of building time from concrete normal-curing terms; low level of industrialized methods of construction.

Precast-cast-in-place structures are composite structures in which precast and cast-in-place concrete work under the load as a whole. The bond is carried out by selecting the shape and size of elements, hacking of their surfaces, making keys and free lengths of reinforcing bars. Precast-cast-in-place structures combine positive properties of cast-in-place structures and precast structures: relatively lower labor intensity of construction and space continuity. They are mainly used in buildings (structures) under large loads and during reconstruction.

1.6. Materials of masonry and reinforced masonry structures. Buildings and structures parts are made of stone

Masonry is a building material, which consists of different types of stone units joined together by mortar. *Masonry structure* is a masonry that has a certain shape carrying out enclosing and bearing functions. In many cases, steel mesh or longitudinal bars are placed in masonry to reduce its cross-sections or for occurrence of tensile forces in cross-sections. Such structures are called *reinforced masonry structures.*

The use of masonry structures lasts for thousands of years. The Stone Age buildings that were made of large natural stone blocks and slabs are partially survived. The later period includes ramparts made of massive blocks (for example, Genoese fortress in Sudak, Crimea) and tombs of Egyptian pharaohs – the pyramids were built of large cut blocks.

With the development of society and the improvement of manufacture methods, small cut stones or rough-cut stones began to be used instead of large sized blocks. There were bricks on the basis of clay and gypsum binders, from which the buildings in hot climate countries were built more than six thousand years ago. Baked clay bricks began to be produced in Babylon four thousand years ago. Natural stone, adobe bricks and baked bricks were the main materials for construction in ancient Greece, Rome and other countries at that time, and in Western Europe during the Middle Ages. Famous architectural monuments of that time have survived to the modern day.

With the foundation of Kieran Russ, stone buildings and structures were built intensively – firstly by natural stones and baked clay bricks, and then with white ashlar.

First reinforced masonry structures were used in the 11th century in Sveti-Tskhoveli Cathedral in Georgia (forged metal strips were inserted into the horizontal seams of walls). First flat masonry floor structure, reinforced by strip steel, was applied during the construction of Pokrovsky Cathedral in Moscow in the 16th century.

The empirical rules of masonry construction were developed in the 19th century and the empirical formulas for design of structure were developed in the 1930.

In our country natural and manufactured stones and baked bricks are the most important types of the building materials for almost all types of construction.

Masonry and reinforced masonry structures are widely used in various construction fields because of its advantages: adequate strength and high durability; fire-resistance; possibility of using local materials and giving any shape to structures; low operating costs. However, masonry structures have some disadvantages: difficulty in mechanization of construction; insufficient heat resistance; problems of construction in winter.

Using of modern technology makes it possible to produce different highly-efficient stone materials, which provides extensive usage of masonry structures in any field of construction: industrial, civil, hydro technical, energy sector etc. At the present stage of construction, which is characterized by more intensive usage of individual project designs, the use of masonry structures finds new development, especially in enlargement of stone products, creation more efficient masonry, usage of stone blocks.

Various types of natural and manufactured stones (weighing up to 40 kg), stone products and mortars are used for construction of masonry structures. Heavy natural stones (limestone, sandstone, granite) are used mainly for wall covering and foundation; light natural stones (tuff, limestone, coquina) are used for walling in the southern regions.

Manufactured stones are widely used. They include different types of bricks (solid and hollow clay bricks, calcium-silicate bricks, etc.), ceramic stones, heavyweight and lightweight concrete blocks (solid and hollow) and others. A brick is a manufactured stone, which has a regular parallelepiped shape with thickness not exceeding 130 mm and weighing less than 4.3 kg. Clay bricks and blocks are formed by dry-pressing or plastic molding processes using clay and silicic (tripoli, diatomite) sediments and industrial waste (coal production and coal beneficiation, flue ash, slag, etc.) with or without mineral and organic impurities; then they are burned in kilns. Solid clay bricks are used for walling, columns, capacitive structures, wells, tunnels. They have a higher thermal conductivity; therefore, the thickness of external walls of solid masonry is significant and usually defined by thermal requirements. A bearing capacity of such walls is much higher than necessary; and the bricks are not used in full as a structural material. The desire to overcome this problem brought to the creation of multilayer and light weight wall masonry and also more efficient types of bricks. Clay and concrete blocks are used for walling and floor; large heavyweight concrete blocks, in addition, are used for foundations of walls. The manufactured stone maximum weight is 40 kg, so the manufactured stone masonry does not satisfy all building requirements. That’s why stone products – large blocks and wall panels (they are factory-made, and their weight is limited by lifting cantilevering equipment) – are widely used for walling and foundation. Blocks can be made of heavyweight and lightweight concrete (solid and hollow blocks), bricks, ceramic stones, natural stones and other materials. Panels for exterior walls can be single-layer using lightweight concrete, double-layer using bricks or ceramic stones with efficient insulation (vibrated brickwork panels), three-layer (two layers of reinforced concrete and one layer of insulation between them); solid *heavyweight* concrete panels and single-layer vibrated brickwork panels are used for interior walls. When choosing the type of materials for masonry structures, local natural materials should be preferred.

Depending on binding material, there are following mortars: cement mortar, lime mortar and mixed. A mortar should be placed easy to improve brick-mason productivity, masonry quality, its strength and density. Therefore, a mortar includes plasticizers (clay or lime). Cement mortars are used for structures, which are under regular influence of water (for example, foundations that are located below the groundwater level); lime mortars and lime-cement mortars are widely used for above-ground masonry.

Carbon or austenitic stainless reinforcing steel is used for reinforcement of masonry structures. Reinforcing steel can be plain or ribbed. Plate, strip and shaped steel can be used as rigid reinforcement.

1.7. Questions for knowledge control

1. What is reinforced concrete?
2. What is the positive properties of concrete?
3. What is the disadvantages of reinforced concrete?
4. Which of the properties characterizes concrete?
5. For how many times does concrete work better in compression than in tension?
7. What water-cement ratio (W/C) is necessary for chemical compound of water with cement?
8. Which of the properties characterizes reinforcement?
9. Where the first normal crack in reinforced concrete beam is formed?
10. For how many times would the bearing capacity of the concrete beam increase if longitudinal reinforcement will install in its tensile zone?
11. What a reinforced concrete structures floor is consist of?
12. What reinforced concrete members are flexural?
13. What reinforced concrete members are compressed?
14. What reinforced concrete members are tensile?
15. Why reinforcement in compressed reinforced concrete members is installed?
16. How are reinforced concrete products classified according to destination?
17. How are reinforced concrete products classified according to manufacture method?
18. How are reinforced concrete products classified according to stress state?
19. What is the essence of manufacturing pre-stressed structures?
20. What are precast-cast-in-place structures?

2 THE STRENGTH AND DEFORMABILITY OF CONCRETE

2.1. Types of concrete. Concrete structure and its influence on the strength and deformability

Concrete is manufactured complex material in which large and small stone aggregates connected by binder resists the load as a single monolithic body. Although concrete is a heterogeneous material, it has predetermined strength, strain and physical properties. They are required resistance to compression and tension; good adhesion with reinforcement; sufficient density to protect it from corrosion. There are same physical qualities: frost-, water-, corrosion- and fire-resistance.

Depending on the starting materials, the structure and composition there are the following types of concrete: heavy; lightweight; no-fines concrete; cellular concrete; special.

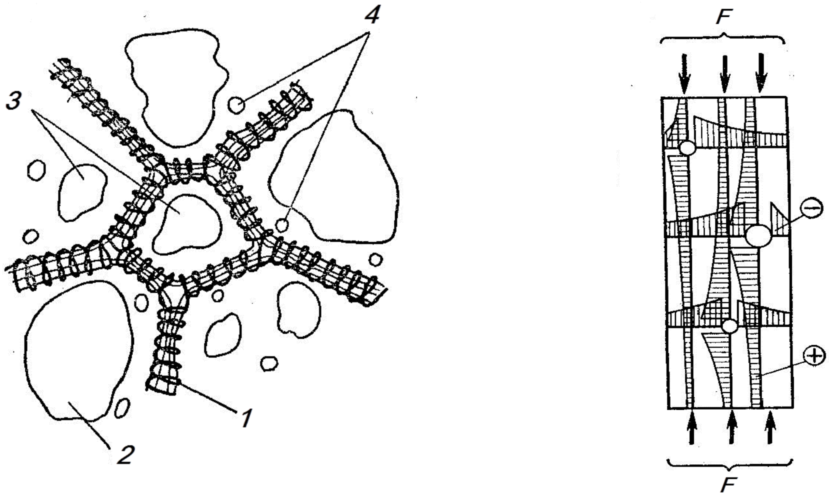
Concretes are classified by:

- function (structural and special);
- structure (heavy, no-fines, cellular, porous);
- average density (especially heavy with γ > 2500 kg/m^3, heavy – γ = 2000 – 2500 kg/m^3, especially light – γ = 1800 – 2000 kg/m^3, light – γ = 800 – 2000 kg/m^3);
- type of binder (cement, polymer, lime, plaster or on mixed binders, special);
- type of aggregates (on dense natural, porous natural, porous manufactured, special aggregates);
- curing conditions (natural, steam curing, autoclave curing).

Concrete structure is formed as the three-dimensional lattice of cement stone. Lattice of cement stone is filled with grains of large and small aggregates and is riddled with many micro pores and capillaries. They contain water, water vapor and air (fig. 2.1, a).

Hardened concrete is complex composite material. It is a material with impaired integrity of the structure and it has all three phases: solid, liquid and gaseous. Cement stone is heterogeneous material and it consists of crystal formation and elastic gel that fills it.

A chemical reaction takes place during mixing aggregate and cement with water, as a result a gel is formed. The gel is porous gelatinous mass that is saturated with water and cement particles that have not reacted and small crystalline formations. During concrete mixing mixture gel connects aggregates, and gradually converts the mass to the monolithic body. The process of crystal formation lasts for years. The phenomenon of crystallization and reducing of the volume of gel during time changes the state of concrete and has a significant influence on its strength and deformability.



a *b*

Fig. 2.1. The structure of concrete (a) and the stress state of the compressed concrete sample (b): 1 – cement stone; 2 – crushed stone; 3 – sand; 4 – pores, which contain air and water

2.2. The strength of concrete

The strength of solid body is its ability to resist the influence of external forces without failure. Since concrete is a heterogeneous body, external load creates combined stress state in it.

In concrete compressed specimen stresses concentrate at hard particles, as a result there are efforts on the surface of their connection that try to break the connection between them. Simultaneously stresses concentration occurs in the place where pores weaken concrete. In this case the tensile stresses act on surfaces which are parallel to compressive force. Since concrete contains a large number of pores, stresses near neighboring pores are superposed. As a result, a longitudinal compressive and transverse tensile stresses take place in concrete specimen under axial compression. This is the field of secondary stresses (fig. 6, b).

Since the resistance of concrete to tension is much lower than compression, firstly tensile stresses reach limit values and micro cracks are formatted. Cracks develop on the boundary between aggregate and cement stone (which is typical for heavy concrete) or on aggregates and cement stone (for lightweight concrete). With increasing the value of load the number of micro cracks increases, they are combined in macro cracks. After that macro cracks are developed to mistral crack and element fails.

Thus, concrete element under the axial compressive forces fails from tensile stresses.

Modern theories of concrete strength do not take into account its structure. Therefore, the relationship between the concrete structure and its properties is not fully studied.

There are two groups of influence factors on the strength of concrete. The first is the concrete composition, grade of cement, its type and quantity, aggregates quality, water-cement ratio (W/C). For chemical compound of water with cement W/C ≈ 0.2 is needed. However, this ratio is taken higher according to technological requirements (0.3 – 0.4 for low-slump concrete and 0.5 – 0.6 for high-slump concrete). The part of chemically free water reacts later with less active particles of cement ore fills the numerous pores and capillaries and then gradually evaporates, releases them. If the W/C decreases, the porosity decreases and the strength of concrete increases.

The second group of factors that influence the strength includes: age of the concrete, conditions of its preparation and curing (humidity, temperature), the size and shape of the samples, the stress state and load types. The strength of concrete when favorable conditions of the natural curing gradually increases during first 10 years or more (fig. 2.2). The concrete acquires strength during the first 28 days in the most intensive way, so the standard test samples is carried out in exactly the same age. If it occurs earlier, the results are leaded to 28-day strength of concrete.

If the temperature and humidity of environment are higher the hardening of concrete significantly accelerates. Therefore, in the enterprises of precast reinforced concrete products they undergo to steam (90°C temperature and humidity to 100%) or autoclave processing under the high pressure of steam and temperature nearby 170 °C.

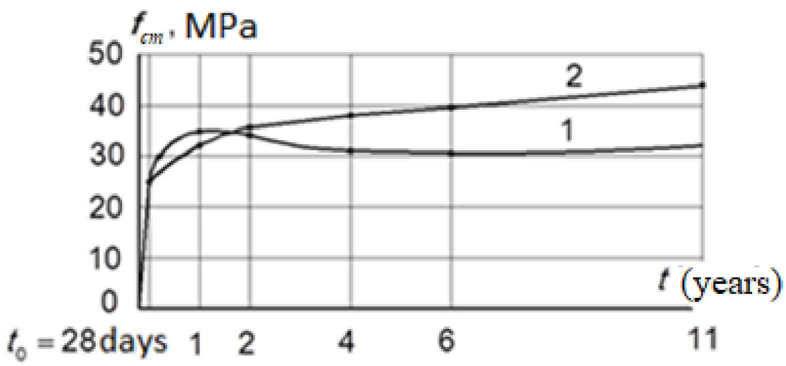


Fig. 2.2. The increase of strength of concrete over time during storage: 1 – dry environment; 2 – wet environment

These methods allow receiving strength equal to 70% of the design strength per day. Concrete hardening significantly slows down at temperature below +5 °C. When the concrete mix temperature is -10°C, the concrete hardening is practically suspended.

For 28 days hardening at temperature -5°C, concrete takes no more than 8% of the strength compared with the strength of concrete hardening in normal conditions. When the temperature is 0°C, concrete takes no more than 40 – 50 %

of design strength. When the temperature is +5°C, concrete takes no more than 70 – 80 % of design strength. After thawing of concrete mix concrete hardening is resumed, but its final strength is lower than the strength of concrete hardening in normal conditions. During concreting at low temperatures conditions (up to -30 °C) the cooled mixture is electrical warmed to +70°C. Using of the quick-hardening cement and heat insulation of structures allows to acquire the strength of concrete up to 70% from the design strength before its freezing. This allows eliminating the influence of freezing concrete on its strength. Anti-frost supplements provide of concrete curing at temperatures down to -10°C.

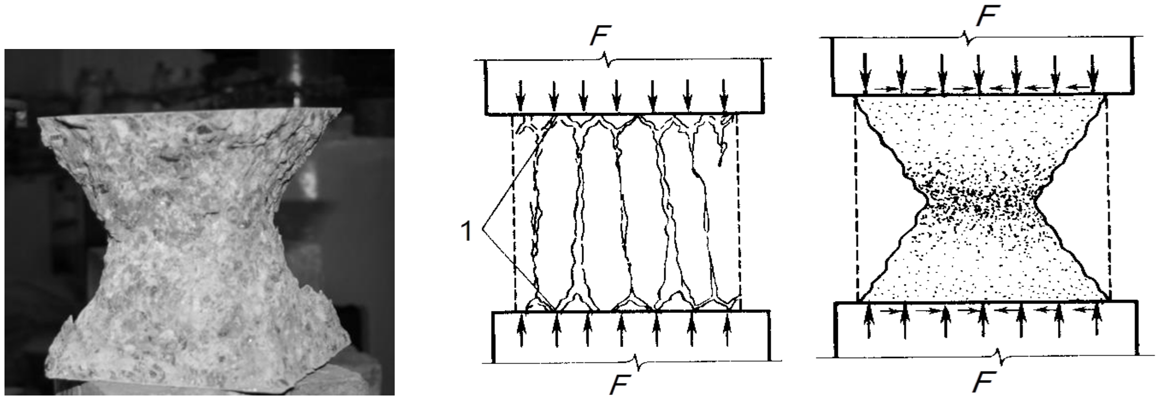
In reinforced concrete structures concrete is usually used to perceive compressive stress. Therefore, its strength under axial compression is a fundamental characteristic of concrete. The simplest and most reliable way of determination of the strength of concrete is crushing of standard cubes with dimensions 15x15x15 cm in the press. The maximum resistance of standard cubes $f_{cm,cube}$ is adopted as *cube strength*.

The shape and size of the samples significantly effect on value of strength: the smaller the cube, the larger the strength is. Thus, the compression resistance of concrete cubes with sides 10 cm is higher up to 10% than the strength of standard cubes. If there is the cube with sides 20 cm its strength is lower up to 7% than the strength of standard cubes and for the cube with sides 30 cm strength is lower up to 11 – 13%. Different resistance to compression of samples of different shapes is due to the influence of friction forces between the surface of the sample and the base plate of press and is due to heterogeneity concrete structure. Near the press base plates friction forces that are directed into the middle of sample create casing and increase compressive strength of samples. Concrete cube after failure takes the form of two truncated pyramids, turned off by peaks to each other (fig. 2.3, a). As the forces of friction by means of lubrication (wax, stearin) decrease then failure takes place by cracks splitting which are parallel to the direction of force (fig. 2.3, b). In this case, the ultimate resistance of concrete is reduced.

The *prism strength* $f_{cm,prizm}$ is an ultimate resistance to axial compression of concrete prism with ratio of the height *h* to the size of the square cross section which is equal to 4 (fig. 9). There is no influence of friction forces on support surfaces and ratio of slenderness of sample to character of failure and the value of ultimate load.

In real structures the stress state of concrete is close to the stress state of prisms, that’s why the prism strength is a main characteristic for calculation of structures. It is approximately equal 0.75 of cube strength for concrete with class C20/25 or higher and equal 0.8 for the other classes of concrete.

Local compressive strength (crushing), according to experiments, significantly is higher than the prism strength. This phenomenon is explained by the influence of unloaded part of concrete element (concrete casing effect).



a *b*

Fig. 2.3. The character of failure of concrete compressed cubes: a – with friction forces on support surfaces; b – without friction; 1 – the lubrication

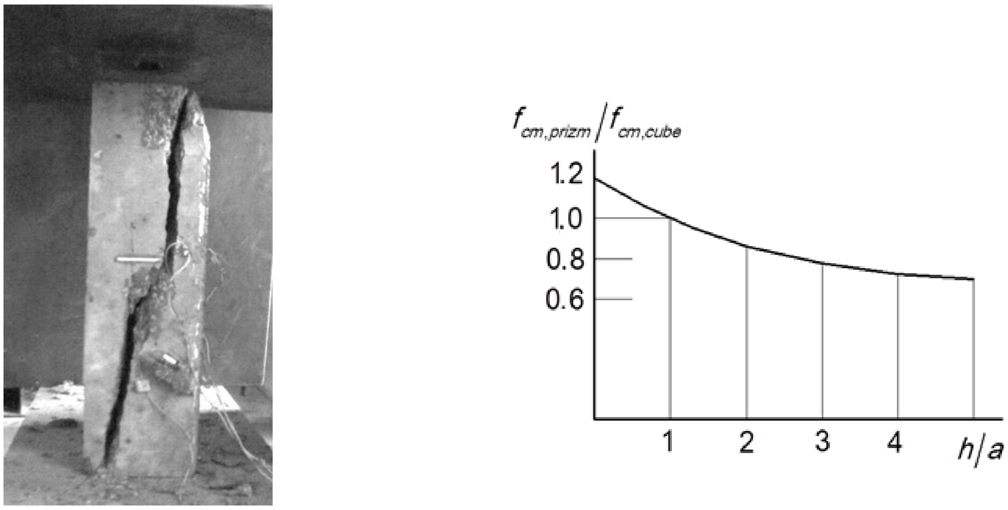


Fig. 2.4. For determination of the prism strength of concrete

Such cases in reinforced concrete structure are quite common, namely: under supports of beams, in joints of precast columns, under anchors in pre-stressed structures. Local compressive resistance of concrete $f_{cm,loc}$ according to the standards is determined by the Bauschinger formula and depends on the prism strength and coefficient of conditional increasing of concrete strength: $\sqrt{A_{c1}/A_{c0}} \le 3$, where A_{c1} is maximum design area of distribution; A_{c0} is area of loading (fig. 2.5).

Because of the complexity of force centering, the *limit axial tensile resistance* f_{ctm} (fig. 11, a) is hard to be got. Therefore, in practice it is often determined by indirect methods, for examples by results of tests of cylindrical samples for splitting and bending experimental beams (fig. 2.6, b, c). The values f_{ctm} are determined by the Fere formula

$$f_{ctm} = 0.232\sqrt[3]{f^2_{cm,prizm}}. \qquad (2.1)$$

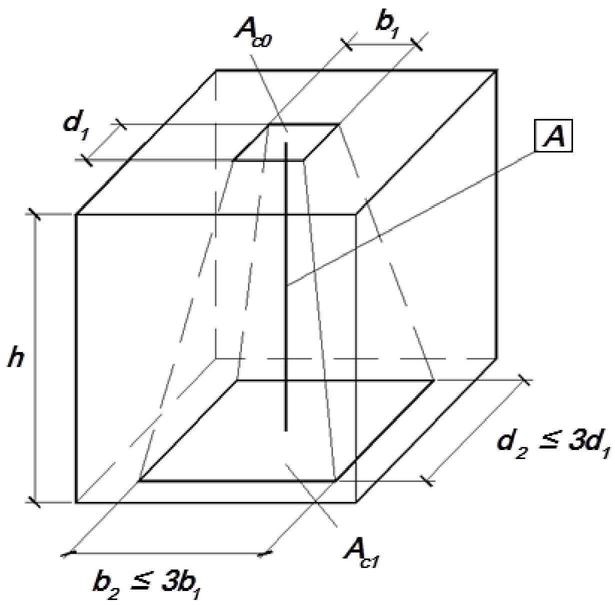
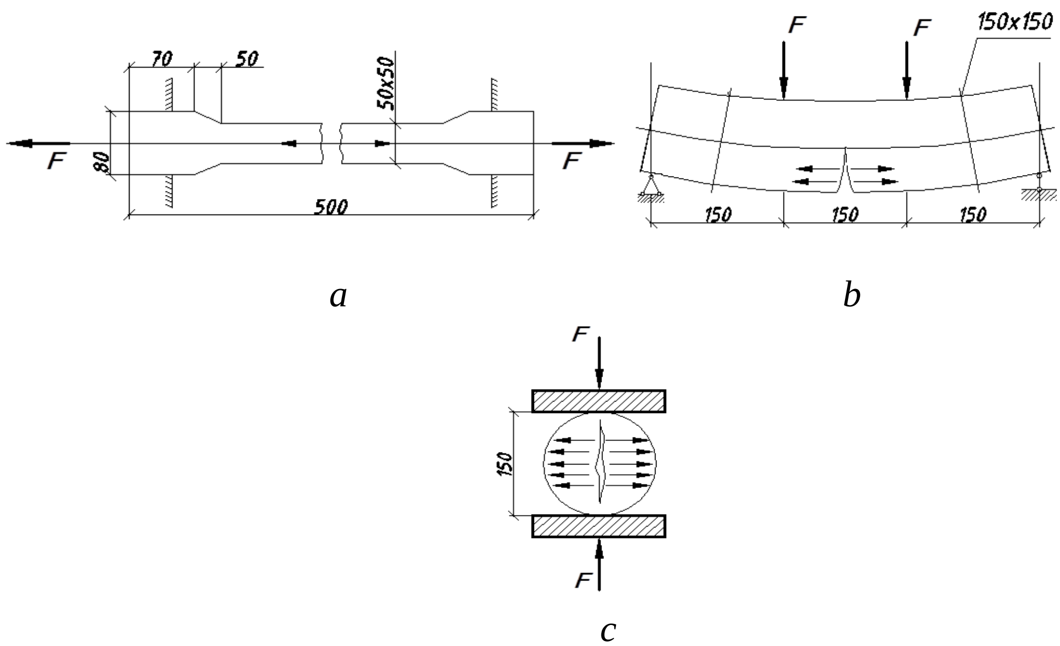


Fig. 2.5. For determination of local compression strength of concrete: *A* is the line of action; $h \geq (b_2 - b_1)$; $h \geq (d_2 - d_1)$

The value of axial tensile strength of concrete is $0.1f_{cm,\ prizm}$ for concrete with class C8/10 and is $0.05f_{cm,\ prizm}$ for concrete with class C40/50. Reasons for the low axial tensile strength are heterogeneity of concrete structure, the presence of internal stress, weak or impaired bond between cement stone and aggregates.



a *b*

c

Fig. 2.6. Scheme of sample test to determine the tensile strength of concrete: a – the tear; b – the bend of concrete beams; c – the splitting cylinder

The *maximum shear resistance of concrete* f_{cshm} is not normalized and is accepted equal to *2* f_{ctm}.

The greatest statically constant stress that concrete can withstand during long time without failure is the *limit long-term resistance of concrete*. The limit long-term resistance of concrete depends on the loading mode, initial strength and age of samples. Long-term resistance $f_{cm,l}$ is up to 90% of the short-term resistance f_{cm} (fig. 2.7).

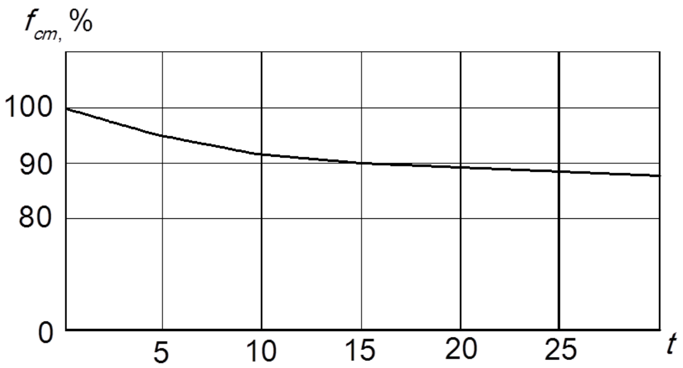


Fig. 2.7. The dependence of the strength of concrete on the loading time

The strength of concrete at repeated load $f_{cm,fat}$ (*the fatigue strength*) is the stress at which the number of cycles to failure the sample is at least 10^6. The fatigue strength $f_{cm,fat}$ is below then the prism strength and depends on the asymmetry of the cycle (the ratio of the largest to smallest stresses of the concrete) and is equal (0.5 – 0.95) f_{cm}.

The short-term dynamic load of high intensity is the increasing of strength of concrete. It is a dynamic strengthening. It is increasing when the time of loading the sample is decreasing. It is a result of energy absorbing capacity of concrete, which is elastic under short-term dynamic loads. *Dynamic resistance* is $f_{cm,d} = \gamma_d f_{cm}$, when the time of load is equal 0.1 s, the coefficient of dynamic concrete strength is $\gamma_d = 1.2$.

As a *class of concrete compressive strength C* (MPa) is understood the limit compression resistance of concrete cubes with dimensions 15x15x15 cm, which are tested according to the standard after 28 days storage at 20 ± 2°C taking into account the statistical variability of strength (the limit strength with probability of 0.95).

$$f_{ck,cube} = f_{cm,cube}(1\text{-}1.64\text{x}0.135) = 0.78\, f_{cm,cube}\,. \quad (2.2)$$

The concrete with class of concrete compressive strength, which is lower C12/15, is not recommended to use for reinforced concrete structures.

The main physical properties of concrete are evaluated by grades. There are grade of concrete: by frost resistance F (F50, F75, F100, F150, F200); and by concrete water resistance: W (W2, W4, W6).

2.3. Non-force strain of the concrete

Deformability of solid bodies is their ability to change the size and shape under the influence of force and non-force factors. There are force and non-force (three dimensional) strains. The force strains are developed along the line of force and are divided into elastic and plastic. They are classified according to the time of loading. There are strains under the action of single short-term load, long-term load, repeated load. Shrinkage, expansion and temperature strains (shortening or lengthening) are non-force strains.

Shrinkage is a decrease of the volume of concrete during hardening it on the open air. The quantity of cement, its type, W/C ratio, temperature and humidity in which concrete is hardened, type of aggregates are factories of influence. The relative shrinkage strain ε_{cs} is (30 – 70)x10^{-5}. The aggregates prevent free shrinkage of cement in the initial period of hardening concrete. They are internal connections and cause initial tensile stresses in cement stone. During hardening of gel, crystal formations also act as connection. Uneven drying of concrete volume cause to uneven shrinkage, and the formation of the initial shrinkage stresses. There are tensile stresses on the open parts of outside elements, which dry quickly, and compressive stresses in inside volume of concrete which is more humid. The result of the initial tensile stresses is formation of shrinkage cracks in concrete, especially on the surface of element (fig. 2.8). Reduction of shrinkage, shrinkage stresses and the development of cracks can be achieved by both technological and design methods. For example: selection of grain size composition and type of aggregate to reduce their surface and voids volume and reducing quantity of cement and W/C ratio, increasing the density of concrete and moisturizing of its outside surface. There are such constructive methods: the reinforcing of structures and creation shrinkage joints in its.

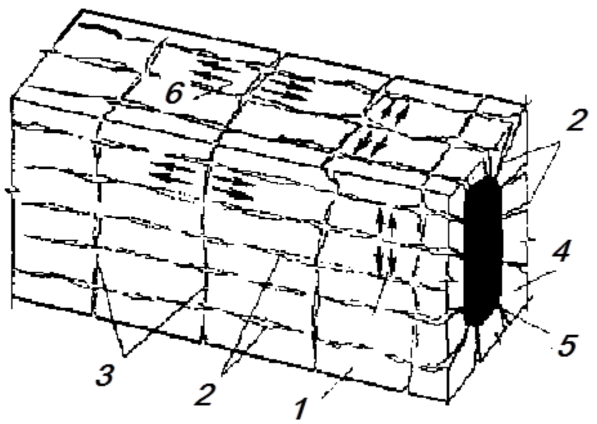


Fig. 2.8. Concrete shrinkage: 1 – part of concrete beam; 2, 3 – longitudinal and transverse shrinkage cracks; 4 – the outer surface; 5 – the inner part; 6 – the tensile stresses

Total shrinkage strain ε_{cs} consists of two components:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}, \tag{2.3}$$

where ε_{cd} is drying shrinkage strain,

ε_{ca} is autogenous shrinkage strain.

The first component develops slowly, because it depends on the migration of water in concrete at its hardening, the large part of the second component is observed in the first hours after concreting and linearly depends on its strength.

Development of the drying shrinkage strain is described by the function

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) k_n \varepsilon_{cd,0}, \tag{2.4}$$

where k_n is coefficient that depends on the notional size of element cross-section $h_0 = 2A_c / u$;

A_c is area of concrete section;

u is perimeter of the part, which is exposed to drying (h_0 ranges from 1.0 to 0.7);

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}}, \tag{2.5}$$

where t is the age of the concrete at the moment which is considered;

t_s is the age of the concrete at the beginning of drying shrinkage (normally this is at the end of curing).

The total value of drying shrinkage strain is $\varepsilon_{cd,\infty} = k_n \varepsilon_{cd,0}$. Its basic value $\varepsilon_{cd,0}$ can be accepted at standards depending on the class of concrete and relative humidity.

Autogenous shrinkage strain is defined as

$$\varepsilon_{ca}(t) = \beta_{as}(t) \varepsilon_{ca}(\infty), \tag{2.6}$$

where $\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \times 10^{-6}$, but $\beta_{as}(t) = 1 - \exp(-0.2t^{0.5})$,

here t is taken in days.

Shrinkage of concrete is considered in some calculations of RCS, for example, at determination of losses of pre-stressing of reinforcement in pre-stressed structures and calculation of resistance to cracking of RCS.

There is the *expansion of the concrete* during curing in water – increase of its volume at strong moisture. This strain is much smaller than the shrinkage strain, and it does not consider in the calculations of structures.

With increasing temperature, the concrete expands, with decreasing temperature the concrete reduces. This strain is characterized by the coefficient of linear thermal deformations (of relative elongation (or shortening) of the sample at heating (or cooling) by 1°C (when the temperature changes within -40 to +50° C).

2.4. Strain of the concrete under the load action

Diagram of mechanical state of the concrete specimen (fig. 2.9) shows the relationship between stress σ_c and strain ε_c. It looks like a curve. Its curvature changes with increasing level of stress. At an early stage when $\sigma_c \leq (30 - 40)$ % f_c the total strain is close to the line of elastic strain. When tension of concrete is observed there is a similar situation. There are two components of total strain of concrete under stresses $\varepsilon_c = \varepsilon_{el} + \varepsilon_{pl}$, were ε_{el} is elastic strain that is matched to instant download of sample; ε_{pl} is inelastic (plastic) strain that depend on the speed of loading and on the level of stress in concrete (fig.2.9, fig. 2.10).

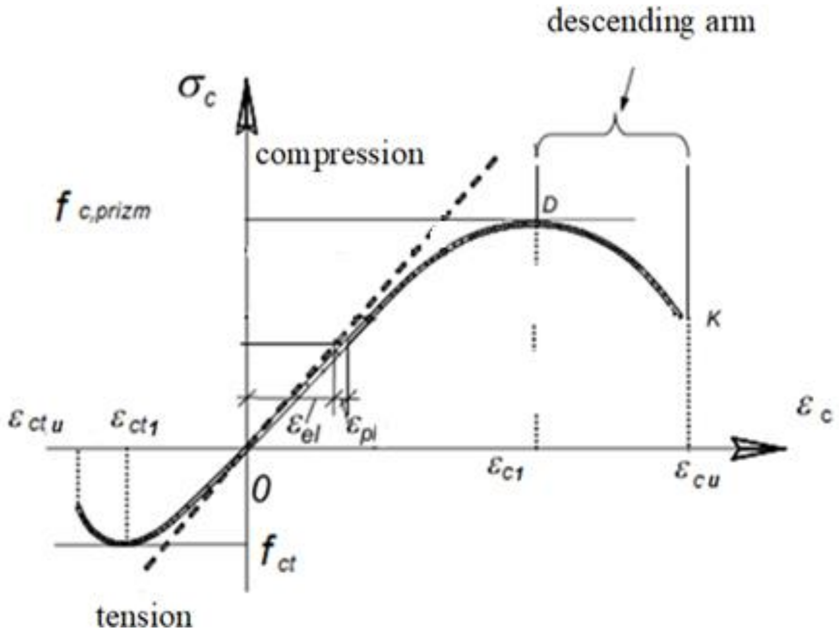


Fig. 2.9. The diagram of the mechanical state of concrete: ε_{c1} – total compressive strain; ε_{cu} – nominal ultimate compressive strain; ε_{ct1} – total tensile strain; ε_{ctu} – nominal ultimate tensile strain

During long-term load the plastic strain of concrete continues to grow for a long time (for years). They increase in the first 3 – 4 months the most intensively. Part 0 – 1 (fig. 2.11) describes the strain of concrete during loading; part 1 – 2 describes the growth of plastic strain at constant stress.

The property of concrete, which is characterized by increasing plastic strain under long-term constant load, is called *creep*. Nature of creep is explained by change in time of structure of cement stone that hardens. The creep is determined by the redistribution of stresses from gel to crystalline formation and aggregate grain. As a result, more elastic components of concrete are loaded and additional strain is created.

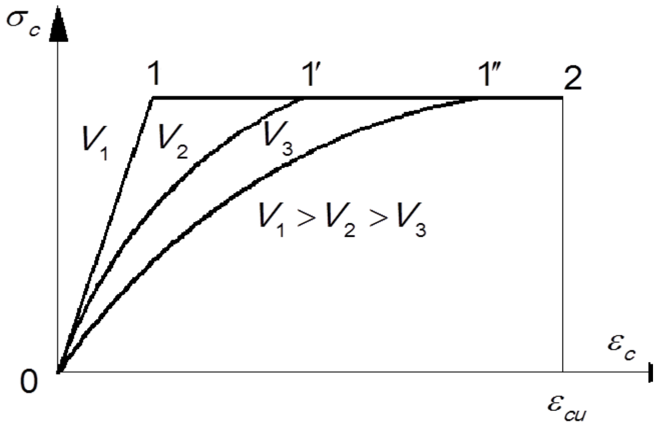


Fig. 2.10. The dependence of the strain on speed of loading *v*

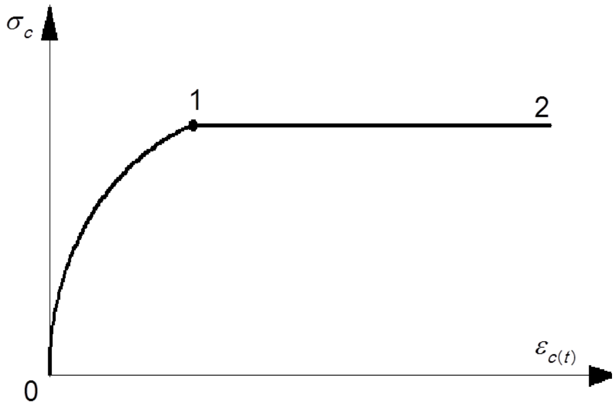


Fig. 2.11. The diagram σ_c –ε_c concrete sample under long-term compression test

Creep of concrete increases with: increasing the level of stress and under long-term load; with increasing the W/C ratio and humidity of the environment; reducing the size and age of the samples which are tested; the use of stone aggregates with high strength and modulus of elasticity.

The strain of creep of concrete $\varepsilon_{cc}(\infty,t_0)$ for t=∞ at constant compressive stress σ_c, which is applied to the concrete at the age t_0, can be described by the formula

$$\varepsilon_{cc}(\infty,t_0) = \varphi(\infty,t_0)(\sigma_c / E_c)\,, \tag{2.7}$$

where $\varphi(\infty,t_0)$ is the total creep coefficient, which is determined by standards and depends on the class of concrete and relative humidity of the environment;

tangential module E_c can be taken as $1.1E_{cm}$.

If concrete compressive stress at the age of t_0 is more then $0.45 f_{ck}(t_0)$, then creep should be considered as non-linear. In such cases, the nonlinear creep coefficient can be defined as

$$\varphi_k(\infty,t_0) = \varphi(\infty,t_0)\exp\left[1.5(k_\sigma - 0.45)\right], \tag{2.8}$$

where k_σ is the ratio of «stress – strength" $\sigma_c \,/\, f_{cm}(t_0)$;

here σ_c is compressive stress;

$f_{cm}(t_0)$ is mean compressive strength of concrete at the time of loading.

The relaxation is closely connected with creep. The *relaxation* is the quality of the concrete that is the reduction of the stress in the course of time at a constant initial strain. Multiple repetition cycles of loading and unloading of concrete sample leads to a gradual accumulation of plastic strain. After a large number of loading cycles when plastic strain reaches to its limit, the concrete begins to work elastically. This behavior of concrete occurs if stresses do not exceed the fatigue strength. If $\sigma_c > f_{c,fat}$, after several cycles of load the diagram becomes reverse curvature, the plastic strain increases without limit, and the sample is destroyed (fig. 2.12).

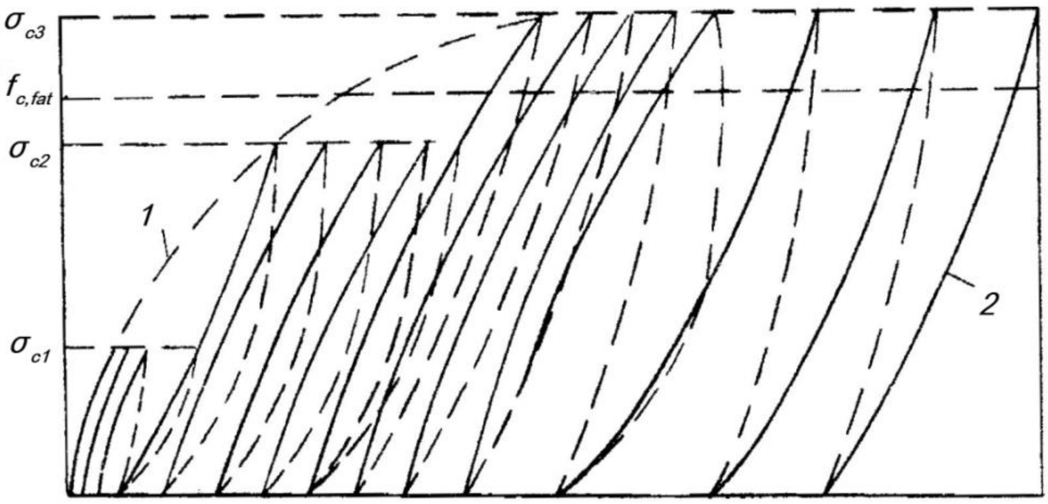


Fig. 2.12. The deformability of concrete at repetitive loading: 1 – the diagram of the strain in the primary load; 2 – the same as in multicycle load to stress that exceeds the fatigue strength

Deformation properties of materials are characterized by module of strains. *The initial module of elasticity of concrete E_{cm}* depends on the class and type of concrete. It is defined as $E_{cm} = \sigma_c / \varepsilon_{el} = tg\alpha_0$.

Total strain of concrete is characterized by the *module of strain* that for each level of stress is equal to *tgα* (*α* is the angle of inclination of the tangent line to the curve $\sigma_c - \varepsilon_c$ (fig. 2.13). $E_{cm,pl} = \sigma_c / \varepsilon_c = \sigma_c / (\varepsilon_{el} + \varepsilon_{pl}) = tg\alpha$. Angle α is a variable. It depends on the level of stress and speed of loading. *Mean value of strain module* is equal to $tg\alpha_1$ of secant to the curve $\sigma_c - \varepsilon_c$. The angle α_1 is angle to horizontal axis.

Module of strain is always less than the initial module of elasticity. Since the development of inelastic strain depends on many factors and time, *module of strain is variable value*.

At long-term load the value of module of strain of concrete is determined by the dependence

$$E_c(t,t_0) = E_{cd} / (1+\varphi(\infty,t_0)) . \qquad (2.9)$$

Total strain of concrete ε_{c1} is corresponded to the maximum stress f_c. It characterizes the failure of sample during loading it by increasing force. *Nominal ultimate compressive strain* ε_{cu} and *tensile strain* ε_{ctu} of concrete in the test diagram $\sigma_c - \varepsilon_c$ (fig. 2.14) depend on strength of concrete, it composition and speed of loading.

According to experiments, strain, which characterizes the failure of axial compressed concrete samples, is ranged from 0.001 to 0.003; strain, which characterizes the failure of axial tensile concrete samples, is ranged from 0.00015 to 0.00030. There is wide variation of experimental data, even for a concrete with same composition and strength, because they are obtained during loading by increasing force. Since reaching the ultimate resistance of concrete the strain becomes extremely fast (failure of the sample is completed in a fraction of a second), and reading from apparatus depends only on reaction of the experimenter.

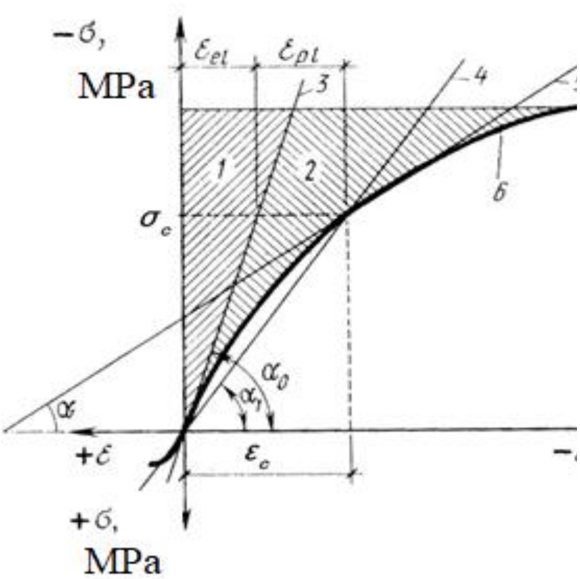


Fig. 2.13 Relationship between strain and stresses: 1 – the part of elastic strain; 2 – the part of plastic strain; 3 – the boundary of elastic strain; 4 – the secant line; 5 – the tangent line; 6 – the curve of total strain

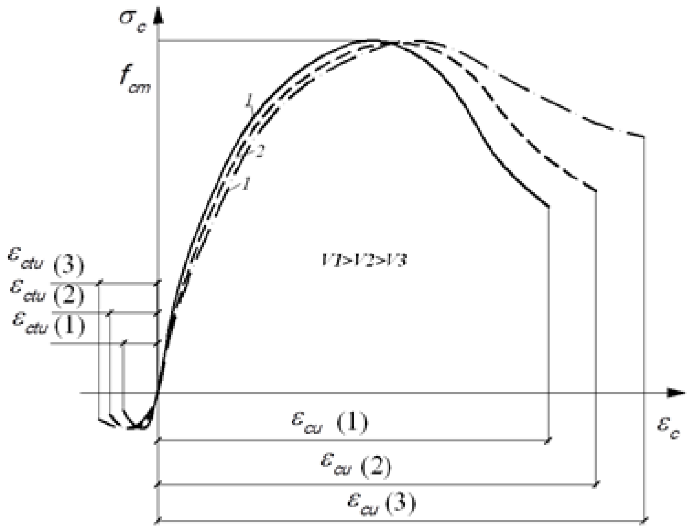


Fig. 2.14. Diagram $\sigma_c - \varepsilon_c$ of concrete specimen under loading at different speeds

Under the action of short-term load, concrete undergoes not only longitudinal but also transverse strain. They are characterized by *Poisson ratio* v. Experimental study of concrete strain shows that at low stress, not exceeding $0.5f_c$, v can be taken with a certain approximation constant for all concretes such that equal 0.2 for concrete without cracks and equal 0 for cracked concrete.

2.5. Questions for knowledge control

1. What is concrete?
2. What is the positive properties of concrete?
3. What is $f_{cm,cube}$?
4. What is $f_{cm,prizm}$?
5. The axial tensile strength of concrete f_{ctm} is …
6. What parameters are classified concrete?
7. What is a class of concrete compressive strength?
8. What the minimum class of concrete for reinforced concrete structures is used?
9. What physical properties of concrete are evaluated by grades?
10. Which of the non-force deformations are most dangerous for reinforced concrete structures?
11. What is shrinkage?
12. What is creep?
13. The concrete strain $E_{cm,pl}$ modulus is a constant value?
14. What are the negative effects of shrinkage of reinforced concrete elements?
15. Which are the components of total strain of concrete under load?

3 REINFORCEMENT
FOR REINFORCED CONCRETE STRUCTURES

3.1. Types of reinforcement by its function

The *reinforcement* is a flexible or hard steel bars, placed in concrete in accordance with the diagrams of bending moments, longitudinal and shear forces. The main function of reinforcement is to perceive tensile stress (in bending, in eccentric compression, in tension) and also shrinkage and temperature stress in the elements of structures. Considerably rarer it is applied for strengthening of compressed area of concrete.

By its function, reinforcement (fig. 3.1) is divided into principal and constructive (mounting, distribution) bars.

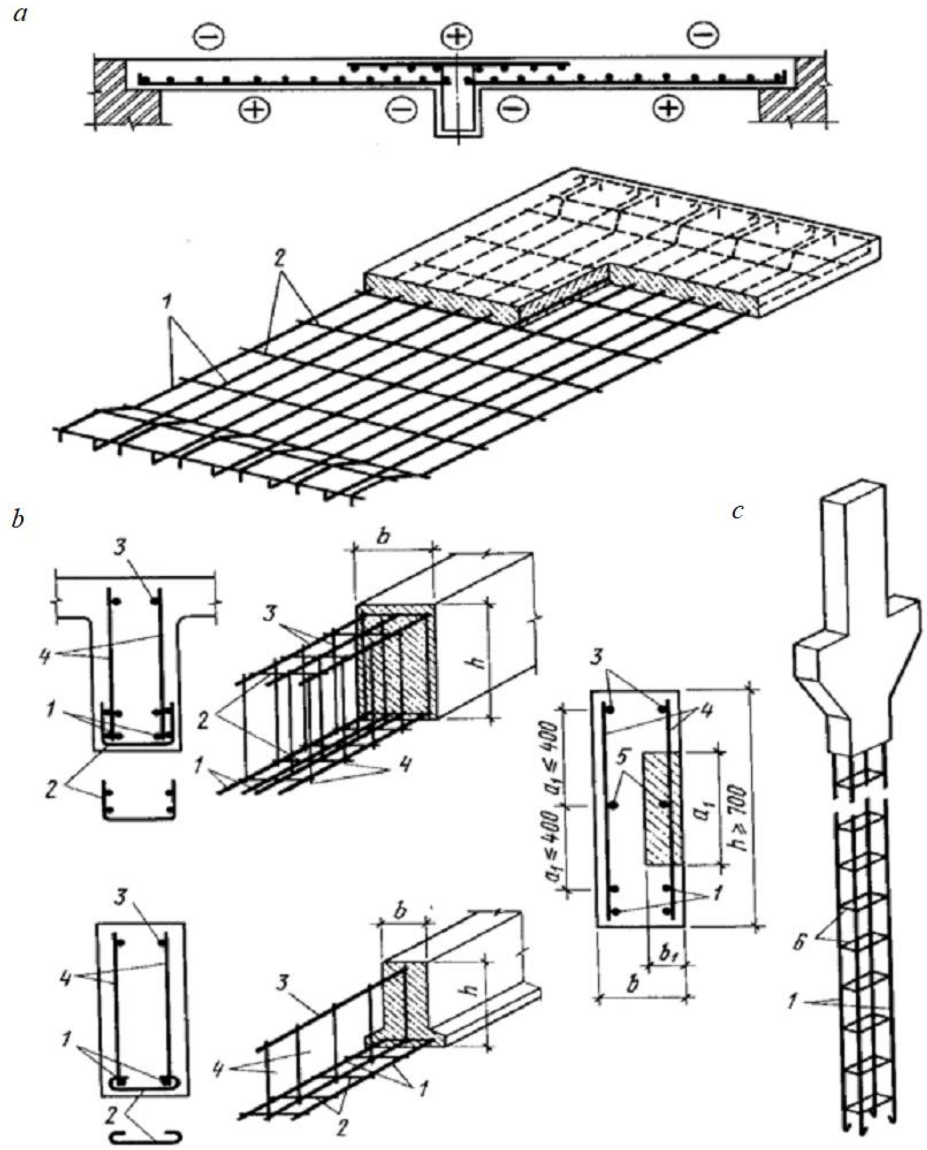


Fig. 3.1. Reinforcement of RCS: a – slabs; b – beams; 1 – the longitudinal reinforcement; 2 – the constructive reinforcement; 3 – the mounting reinforcement; 4 – the shear reinforcement; 6 – lateral ties of column cage

Area of the principal reinforcement (A_s) is determined by calculation on the action of external load. Depending on forces it takes, the principal reinforcement is divided into longitudinal and transverse (shear). The longitudinal reinforcement takes longitudinal forces; it is parallel to the longitudinal axis of element. The shear reinforcement is oriented perpendicularly or at an angle to the longitudinal axis and takes shear forces.

The amount of the longitudinal reinforcement in the reinforced concrete members is determined by the ratio of the longitudinal reinforcement area (A_s) to effective cross-sectional area of a concrete section (A_c). It is called *reinforcement ratio* for longitudinal reinforcement ($\rho_f = A_s / A_c$).

The area of the constructive reinforcement is not calculated. It is accepted by constructive or technological conditions. It is intended for more uniformly distribution of the concentrated effort between the separate bars of the longitudinal reinforcement or maintaining the design position of the longitudinal reinforcement during concrete casting (distribution bars). The constructive reinforcement is used also for perception of stress which is caused by shrinkage and creep of concrete, temperature stress, local stress, random stress that arise during the manufacture and storage of structures, as well as the impact on them the mounting and transport effort (assembling bars).

The diameter of the constructive reinforcement in beam is accepted not less than 10 – 12 mm and not less than diameter of the shear reinforcement.

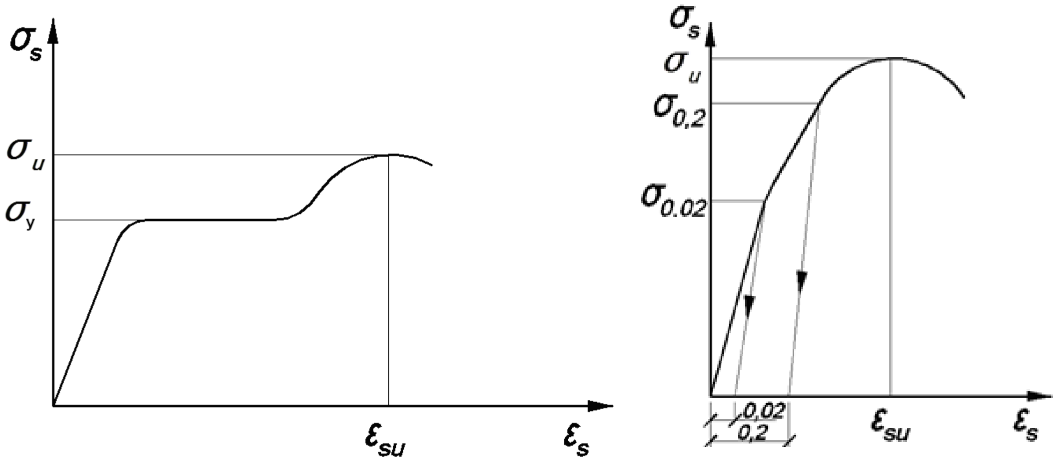
The constructive reinforcement is used as principal in precast reinforced concrete members (RCM) at transporting and assembling.

3.2. Physical and mechanical properties of reinforcing steel

Basic characteristics of reinforcement are strength and deformability, which depend on composition and technology of reinforcement making.

The characteristics of strength are *physical* σ_y and *proof at 0.2 percent set* $\sigma_{0.2}$ *yield stress of reinforcement, ultimate strength* σ_u. They are set according to the diagram σ_s – ε_s, which was obtained during tensile testing the samples (fig. 3.2). For hot rolled steels of classes A240C and A400C the presence of a linear dependence between stress and strain is characterized by in this diagram (elastic steel behavior) and a line of yield is clearly defined (fig. 3.2, a). The stress on yield line is called physical yield stress (σ_y). The plastic strains on yield line increase without increasing external load. Steels with a pronounced yield line are called “mild”. After the yield line, they can again increase resistance with increasing strain, there comes the so-called stage of self-strengthening of steel, the relative elongation with it increases to 15 – 25%, depending on the steel class. The highest point of the diagram corresponds to the ultimate strength of the steel. At this point there is a narrowing of the sample (there is a so-called neck) and a break.

The cold-deformed reinforcement of class B500 conventionally is attributed to the reinforcement which has the physical yield stress for providing necessary reliability of structures.



a *b*

Fig. 3.2. Stress-strain diagrams $\sigma_s - \varepsilon_s$ for reinforcing steels with physical (a) and proof at 0.2 percent set (b) yield stress

The basic strength index of mild steels is the physical yield stress σ_y for RCS. When the stress in the reinforcement reaches σ_y in the tensile concrete zone, the crack width reaches an unacceptable value, and then, due to significant deflection of the structure, the concrete of the compressed zone is destroyed. The ultimate strength of steel σ_u exceeds its yield strength by more than 1.5 times, is usually not achieved and not used in the bearing capacity of the structures.

For reinforcement of high strength bar and high-strength wire of so-called solid steels, there is no clear boundary of elasticity (plasticity) in the diagram $\sigma_s - \varepsilon_s$ (fig. 3.2, b). Therefore, the concepts of the offset limit of elasticity and the proof at 0.2 percent set $\sigma_{0.2}$ yield stress are used here. The *offset limit of elasticity* $\sigma_{0.02}$ is the stress at which the residual relative strain appears, which is equal 0.02% of the ultimate residual strain. The proof at 0.2 percent set yield stress $\sigma_{0.2}$ is the stress corresponding to the residual strain of 0.2%.

Under the action of repeated loading, the value of the yield strength of the steel decreases, and the failure becomes fragile. The *fatigue strength* is the strength, for the achievement of which there is no fragile failure of steel with the number of cycles 10^6.

Characteristics of steel ductility, which are determined by the shape of the diagram $\sigma_s - \varepsilon_s$, the bend angle or the number of intersections in the cold state, the creep of steel (rheological properties) is steel deformability. The value of the total relative elongation at the maximum load of the reference sample and the ratio σ_t/σ_y characterize the failure of the structure (fragile or plastic), the possibility of processing for reinforcing steels depends on the angle of bend and the number of intersections.

The increasing of strength and decreasing of the relative lengthening at a break is observed with introduction to composition of reinforcement steel carbon and alloying additions (the manganese, silicon, and chrome).

The *creep of reinforcement* is the growth of strain in time under load, it increases with increasing of stresses and temperature.

Relaxation is decreasing of pretesting in time at the hard fixing of reinforcement ends (which restrains free strain of reinforcement). Relaxation of stress depends on strength, chemical composition of steel, technology of making, temperature, geometry of surface, value of stress and terms of application. This quality negatively affects to behavior of pre-stressed structures. It predetermines the considerable pre-stress loss, which reduce crack resistance and rigidness of structures. The pre-stressed reinforcement is classified in obedience to norms by character of relaxation as follows: class 1 is a wire or rope with ordinary relaxation; class 2 is a wire or rope with low relaxation; class 3 is hot-rolled or treated bars.

In addition to the above, the behavior of the reinforcing steel is determined by parameters such as bond properties (f_R), shear strength, flexibility, weld ability.

3.3. The classification of reinforcement

Reinforcement is classified by: the its functionality (see p. 3.1); the method of application (pre-stressed and without pre-stressing); depending on the method of manufacture (bar diameter of 5.5 – 40 mm and wire diameter of 3 – 12 mm); the method of further strengthening (thermomechanical-strengthened and strengthened in the cold state); surface form (smooth and ribbed profile).

The *type of reinforcement* is determined by the method of manufacturing, surface form and further strengthening: *hot rolled smooth* (Ø 5.5 – 40 mm) and *ribbed profile* (Ø 6 – 40 mm); *cold deformed wire ribbed section*; *thermo-mechanical strengthened ribbed profile*; *reinforcing rope diameters from 6 mm to 15 mm*.

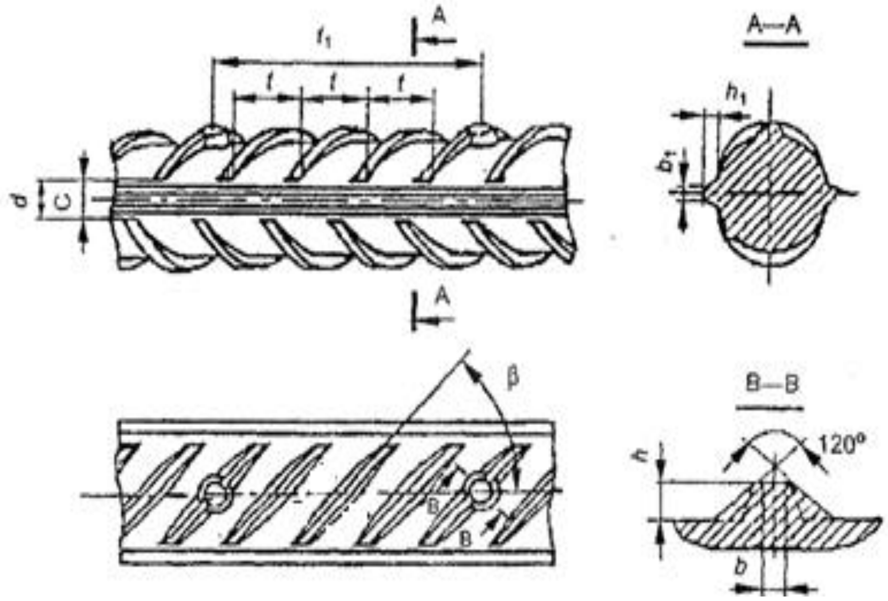
Depending on the yield strength reinforcement is divided into classes. There are such classes of hot-rolled rebar: A240C (smooth); A400C (ribbed profile (fig. 3.3) with the surface "herring bone", reinforcing bars have crescent-shaped transverse projections, which are not connected with longitudinal ridges, the latter are optional). Classes of ribbed profile reinforcement A400, A500C, A600, A600C, A600CK, A800, A800K, A800CK, A1000 are subject to thermomechanical strengthening.

Depending on the properties reinforcing bars are divided into:

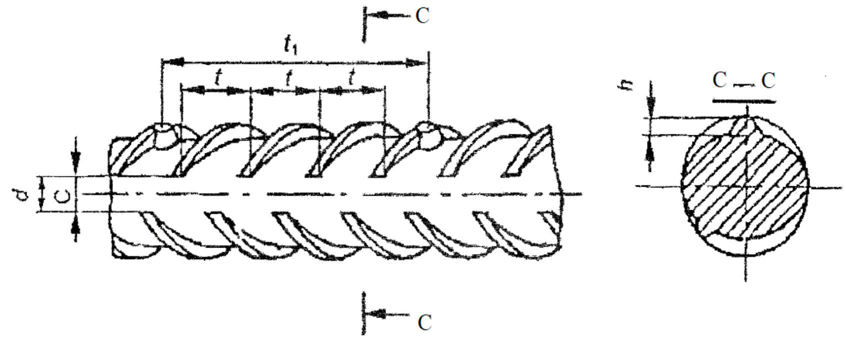
- welded (labeling index has C);
- not welded (no index C);
- resistant to stress corrosion cracking (index K);

- unstable to stress corrosion cracking (no index K);
- welding and resistant to corrosion cracking (CK index).

Weldability of steels is characterized by carbon equivalent, expressed as a mass fraction of carbon and aggregate to it alloying elements in steel.



a



b

Fig. 3.3. Bar reinforcement with ribbed profile with longitudinal projections (a) and without (b): t – the distance between adjacent projections; t_1 – the distance between the marks, the court indicates class reinforcement; h (h_1), b (b_1) – height and width of projections

Resistance to corrosion cracking is the ability of the material not to fail in time under the combined action of tensile stress and corrosive environment (provided by its chemical composition, level of mechanical properties, manufacturing technology). Reinforcing bar is considered to be resistant to corrosion cracking if during the test of samples in certain nitrate solution at a temperature of 98 – 100˚C and stresses of $0.9\sigma_{0.2}$ time to the failure from corrosion cracking is not less than 100 hours.

Steel wire reinforcement is cold deformed reinforcement. It is denoted the letter B, and is made of ribbed profile classes B500, Br1200 – 1500.

Smooth A240C and A400C ribbed profile, A500C, and wire B500 are mostly used as non-tensioned principal reinforcement. Hot rolled reinforcing steel A240C is used as principal shear and mounting reinforcement and as mounting loops for precast concrete elements and concrete structures. However, its usage is limited to operation at temperatures not lower than -30 ˚C.

3.4. Reinforcing products

To accelerate the manufacture of reinforced concrete structures, they are reinforced with a reinforcing meshes, plane cages and spatial cages, which include principal (non-tensioned), constructive reinforcement, embedded items, mounting loops. Welded mesh (fig. 3.4) is used mainly with perpendicular arrangement of principal wire and distribution wire of class B500 with a diameter of 3 – 5 mm and bar class A400C with a diameter 6 – 10 mm.

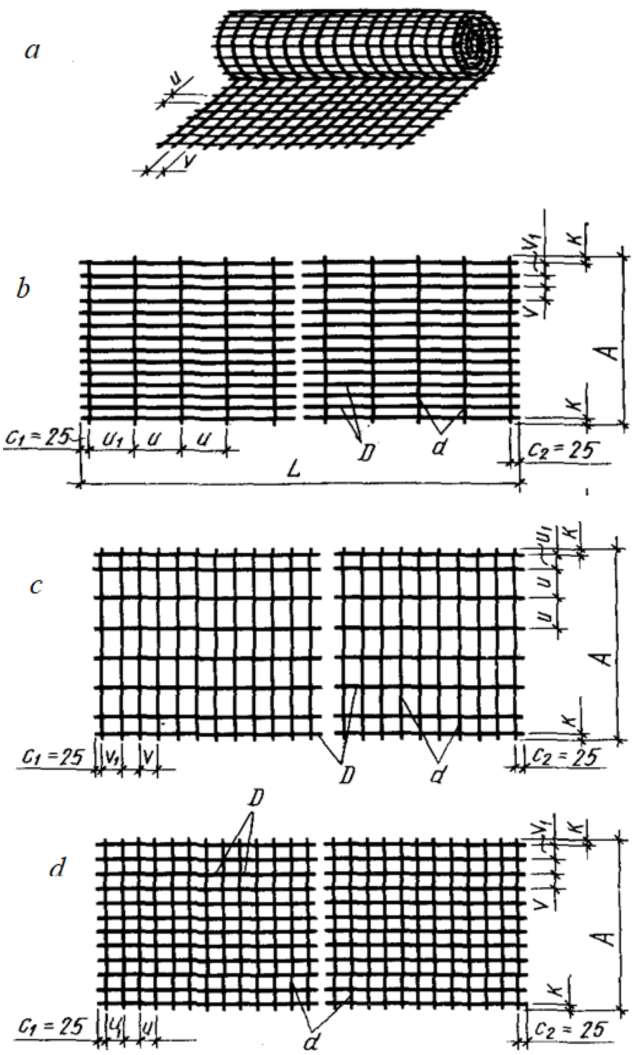


Fig. 3.4. Welded reinforcing mesh as: a – rolled; b, c, d – flat with the principal reinforcement respectively longitudinal, transverse and in both directions

Meshes are made rolled and flat. In rolled mesh, the largest diameter of the longitudinal bars is 8 mm, the width is within 1040 – 3630 mm, the length is depending of the mesh mass 900 – 1300 kg. Welded meshes, as a rule, are made up to 3800 mm wide.

Welded cages (fig. 3.5) are made plane or combined in spatial cages by mounting bars. The longitudinal principal bars are placed in one or two levels on one or both sides of the shear bars. Unilateral placement of longitudinal and shear reinforcement is considered the best for contact spot welding and to ensure adhesion to concrete.

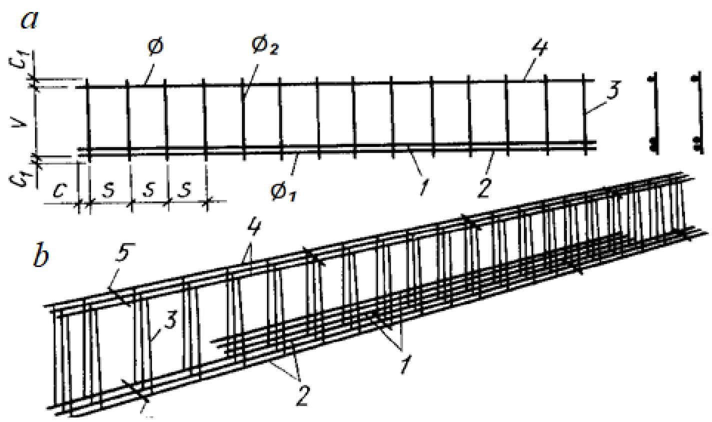


Fig. 3.5. Types of reinforcement cages: a – plane; b – spatial; 1 – a second level of principal reinforcement; 2 – the lower level of principal reinforcement; 3 – the shear reinforcement; 4 – the assembly bars; 5 – the constructive bars, which combine plane cages in spatial cage

The minimum size of the free lengths of longitudinal and shear bars in the meshes and cages should be $c \geq 0.5Ø_1 + Ø_2$ or $c \geq 0.5Ø_2 + Ø_1$ and not less than 15 mm.

It is also allowed to use the tied reinforcement (especially in cast-in-place RCS), which consists of individual longitudinal and shear bars connected at the intersections of tying wire. This method of reinforcement requires a high manual labor costs, but excludes stress concentration in the areas of pin-point welding and eliminates the risk of burning of shear bars typical for welded products.

The use of high-strength wire as pre-stressed reinforcement is the most economical because it has a lower unit cost compared to bar reinforcement. However, these advantages are significantly reduced due to the increase of a concrete croc-sectional sizes for accommodate the large number wires, limiting the maximum size of a coarse aggregate of concrete, the need for more plastic concrete, increasing of metal intensity of anchor and gripping devices, increasing costs for corrosion protection. Therefore, high-strength wires are used as ropes, bundle, packages (fig. 3.6).

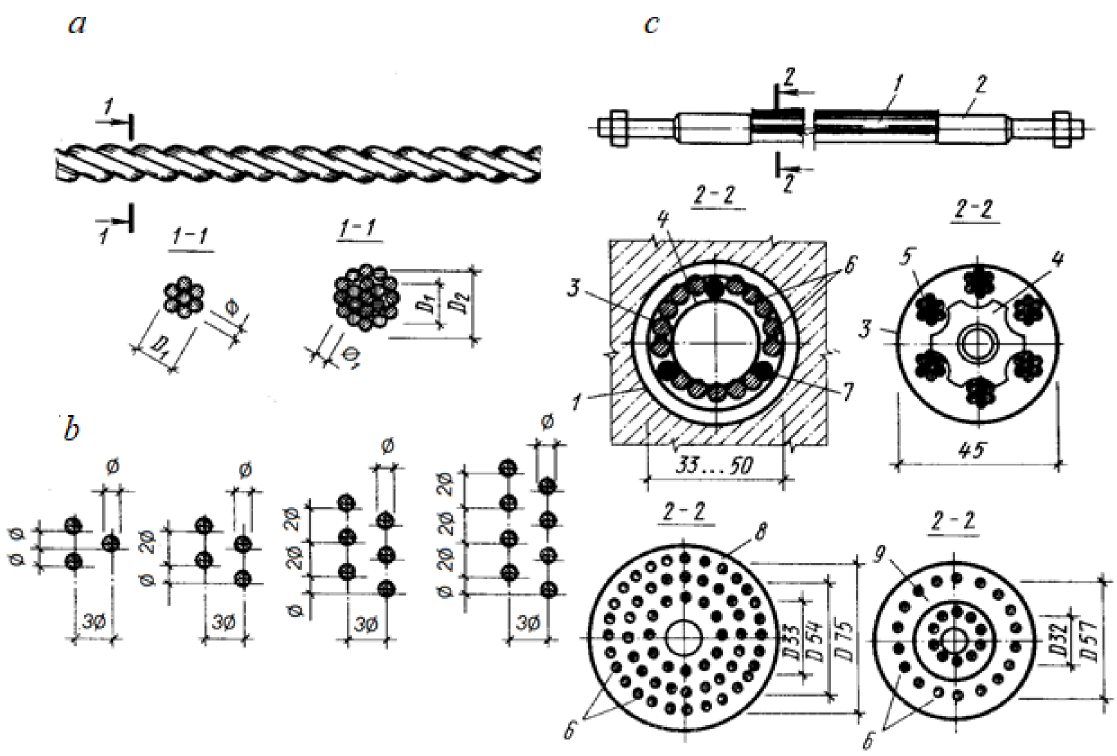


Fig. 3.6. The reinforced wire products: a – wire ropes K1400 and K1500; b – packages with wire of class Br1200 and with diameter of 5 mm; c – single row (with 18 individual wires and with 6 seven-wires ropes) and multi rows (60 and 28 wires) bundles; 1 – the tube of roofing steel; 2 – the anchor; 3 – the clamps of soft wire diameter 3 mm; 4 – the pieces of spiral wire with a diameter of 2 mm (the star distribution in bundles of ropes); 5 – seven-wires ropes; 6 – separately deposited wires; 7 – units with diameter 18 mm, length 100 mm, with step 1000 mm increments for comfortable filling of bundle cavity by mortar; 8 – multi-row bundles

3.5. Joints and intersections of reinforcement

The B500 reinforcing wire rod and bar reinforcement of all classes with a diameter of 8 mm (inclusive) are produced in bobbins and are called rod steel. The length of such reinforcement is sufficient for reinforcing structures of large sizes. The bar reinforcement with diameter of 10 mm and above are produced like rebar with length of 6.5 – 14 m, which is needed to join together. It is also necessary to join the reinforcement meshes and cages of precast elements.

According to the method of manufacturing, the joints of bar reinforcement are divided into welded and laps without welding and according to the place of manufacture they are divided into assembly and factory.

As the most economical, the welded joints have become widespread in construction. Depending on the type of reinforcement and manufacturing conditions various types of welded joints are used (tab. 3.1 and fig. 3.7).

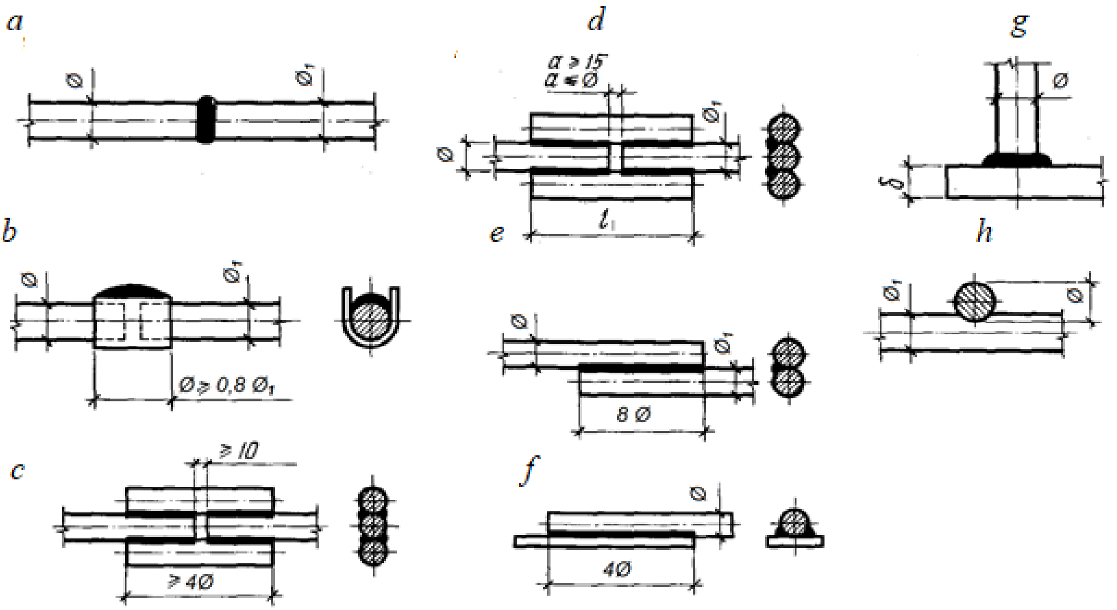


Fig. 3.7. Welded joints of non-tension reinforcement: a – contact; b – in the inventory form; c – a two-way seam with the straps; d – a one-way seam with the straps; e – in lap by connecting two bars; f, g – the same one when bar is connected to plate; h – contact-point joint by connecting bars, meshes and cages that intersect

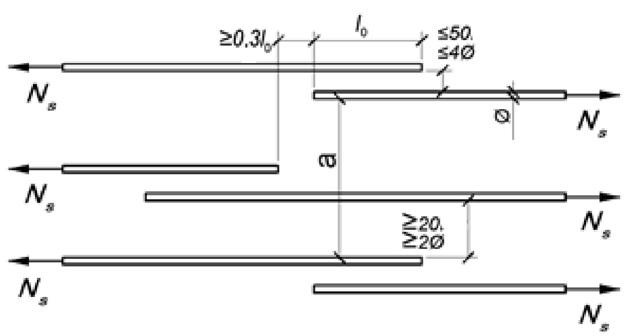
Table 3.1 – Recommended methods of welding

Loading case	Welding method	Bars in tension [1]	Bars in compression [2]
Predominantly static	flash-welding	butt joint	
	manual metal arc welding and metal arc welding with filling electrode	butt joint with Ø ≥ 20 mm, splice, lap, cruciform joints[3], joint with other steel members	
	metal arc active welding[2]	splice, lap, cruciform[3] joints & joint with other steel members	
	resistance spot welding	lap joint[4] , cruciform joint[2, 4]	
Not predominantly static	flash-welding	butt joint	
	manual metal arc welding	–	butt joint with Ø ≥ 14 mm
	metal arc active welding	–	butt joint with Ø ≤ 14 mm
	resistance spot welding	lap joint[4], cruciform joint[2,4]	

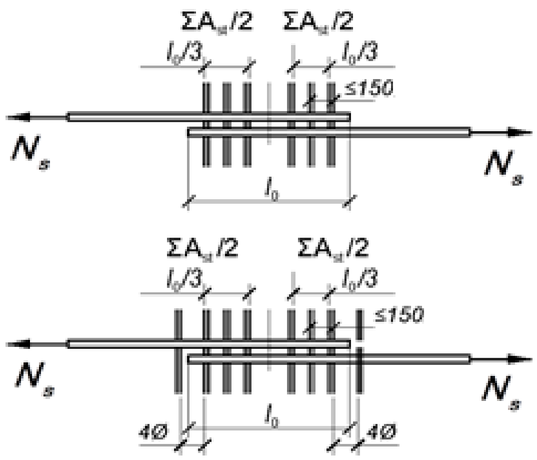
Note:
1. Only bars with approximately the same nominal diameter may be welded together.
2. Permitted ratio of mixed diameter bars is ≥ 0.57.
3. For bearing joints Ø ≤ 16 mm.
4. For non-bearing joints Ø ≤ 28 mm.

The transfer of force from one bar to another can also be done with the lap of reinforcement with hooks or without it (fig. 3.8, a). This connection should not be placed in areas of maximum bending moments (forces), the lap should be placed symmetrically. The distance between the bars, which are connected, shouldn't be more than 4Ø (50 mm), otherwise the length of the lap is increased by value of this distance; longitudinal distance between two adjacent places of overlap should be at least 0.3 of the length l_0 lap, in case of allied laps clear distance between adjacent bars should not be less than 20 mm or 2Ø. When all of the above conditions are fulfilled, if all lap bars are placed in one layer, 100% reinforcement is allowed to connect, in several layers 50%.

a



b



c

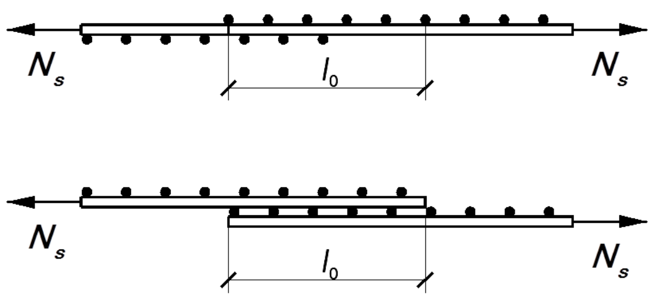


Fig. 3.8. Rules of joining the individual bars (a), transverse reinforcement in the joining area (b), the overlap of welded meshes (c)

Compressed and distribution reinforcement may be connected in the same section.

The design lap length is defined by the expression

$$l_0 = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_{b,rqd} \geq l_{0,min}. \quad (3.1)$$

The basic required anchorage length for straight bars assuming constant bond stress that is equal to f_{bd} and is calculated by the formula

$$l_{b,rqd} = (\varnothing / 4)(\sigma_{sd} / f_{bd}), \quad (3.2)$$

were σ_{sd} – the design stress of the bar at the place from which the length anchorage is determined.

The design value of the ultimate bond stress for ribbed bars is

$$f_{bd} = 2,25\eta_1\eta_2 f_{ctd}, \quad (3.3)$$

were η_1 – a coefficient related to the quality of the bond condition and the position of the bar during concreting;

η_2 – a coefficient that depends on the diameter of the bar (is equal to 1 at Ø ≤ 32 mm or (132 - Ø) / 100 – for Ø> 32 mm);

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ – coefficients that are determined by the form of the bars assuming adequate cover, concrete minimum cover, confinement by transverse reinforcement, effect of welded transverse bars along the design anchorage length, effect of the transverse pressure to the plane of splitting along the design anchorage (the product of $\alpha_1\alpha_3\alpha_5 \geq 0.7$).

The minimum anchoring length should be:

- for anchorages in tension $l_{b,\min} \geq \max\{0.3l_{b,rqd}; 10\varnothing; 100\}$ mm;
- for anchorages in compression $l_{b,\min} \geq \max\{0.6l_{b,rqd}; 10\varnothing; 100\}$ mm.

In the lap zone transverse reinforcement is installed (fig. 3.8, b) to resist transverse tension forces. If the diameter of the lapped bars is less than 20 mm, or the percentage of lapped bars in any section is less than 25%, then any transverse reinforcement or links necessary for other reasons may be assumed sufficient for the transverse tensile forces without further justification. If the diameter of the lapped bars is greater than or equal to 20 mm, the transverse reinforcement should have a total area of not less than the area of one lapped bar.

If more than 50% of the reinforcement is lapped at one point and the distance *a* between adjacent laps at a section is ≤ 10Ø, transverse reinforcement should be united. Transverse reinforcement should be placed along the outer sections of the lap. Transverse reinforcement for compressed bars should be set according to the aforementioned rules, and also a transverse bar should be placed outside each end of the lap length end and within 4Ø of the ends of the lap length.

Lap connection is also used for welded and tied cages and mesh of wires with periodic structure (fig. 3.8, c). Overlap in these cases may be made either by intermeshing or by layering of the meshes. If fatigue loads occur, intermeshing is required. For layered meshes, the laps of the main reinforcement usually must be placed in zones where the calculated stress in the reinforcement at ultimate state is not more than 80% of the design strength. For intermeshed reinforcement lap bars percentage can range from 25% to 50%. For layered meshes, the permissible percentage of the principal reinforcement, that may be spliced by lapping, in any section is determined by performance of conditions: $A_s / s \leq 1200$ mm^2/m – 100%; $A_s / s > 1200$ mm^2/m – 60 %. The joints of the multiple layers should be staggered by at least through $1.3l_0$.

If it is necessary, the joints of pre-stressed reinforcement are carried out through the cage with the help of appropriate equipment. This joint is the most economical. In some cases, joint is done with nuts, bushings with threaded plugs (at the end of a bar movable sleeve is put, and the moving threaded plug is put at the other end of a bar, anchor head is planted and the joint is tightened) (fig. 3.9).

a

b

c

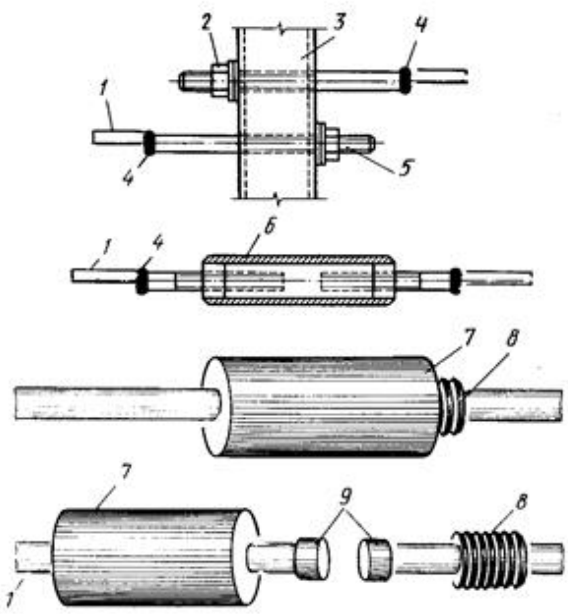


Fig. 3.9. Joints of pre-stressed reinforcement: a – using nuts; b, c – by sleeve; 1 – reinforcement; 2 – tensioning nut; 3 – steel channel stock; 4 – electric contact welding; 5 – threaded end; 6 – tension sleeve; 7 – beads; 8 – threaded plug; 9 – anchor

The ropes are joined by pressing the coupling sleeve at their ends, inventory lamps, etc.

3.6. Questions for knowledge control

1. What efforts take a longitudinal principal reinforcement?
2. What class of reinforcement has a smooth (plane) surface?
3. What formula determines the reinforcement ratio ρ_f?
4. What reinforces products are used in the manufacture of reinforced concrete structures?
5. What is the difference between principal reinforcement and constructive reinforcement?
6. How the joints of the reinforcement by the manufacturing method are divided?
7. Reinforcement of what classes is used as principal in reinforced concrete structures without pre-stressing?
8. What reinforces products are used in the manufacture of reinforced concrete structures?
9. What parameters are classified reinforcement?
10. What are the reinforced wire products?

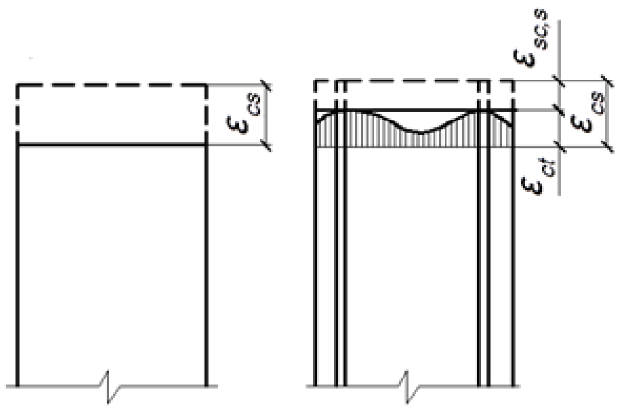
4 THE PHYSICAL AND MECHANICAL PROPERTIES OF REINFORCED CONCRETE

4.1. Shrinkage and creep of reinforced concrete

The most important properties of concrete that affect its work are creep and shrinkage. The shrinkage of reinforced concrete elements is much less than of concrete members. Steel reinforcement due to its bond with concrete, is an internal connection that prevents free shrinkage of concrete. As a result, self-balanced state of stress is implemented in RCS (caused without external forces): a tension occurs in concrete; a compression occurs in reinforcement.

In the symmetrically reinforced structure, the tensile strain of concrete is equal to the difference between the free shrinkage strain of concrete and reinforced concrete elements $\varepsilon_{ct} = \varepsilon_{cs} - \varepsilon_{cs,s}$ (fig. 4.1). In this case stresses in concrete and reinforcement are equal to $\sigma_{ct} = \varepsilon_{ct} E_{ct}'$, $\sigma_s = \varepsilon_{cs,s} E_s$ and $\sigma_s A_s = \sigma_{ct} A_c$. From here $\sigma_s = \sigma_{ct} A_c / A_s = \sigma_{ct} / \rho$. Substituting in equation initial strain, expressed in terms of stress, we have $\sigma_{ct} / E_{ct}' = \varepsilon_{cs} - \sigma_s / E_s$. After completing the transformation $\sigma_{ct} / E_{ct}' = \varepsilon_{cs} - \sigma_{ct} / \rho E_s$, $\sigma_{ct}(1/E_{ct}' + 1/E_s \rho) = \varepsilon_{cs}$, we get

$$\sigma_{ct} = \varepsilon_{cs} / (1/E_{ct}' + 1/\rho E_s). \qquad (4.1)$$



a *b*

Fig. 4.1. Strain of shrinkage for specimens: a – concrete; b – reinforced concrete

So, the dangerous for concrete tensile stress depends on the value of free shrinkage, the amount of the reinforcement, reinforcement and concrete class. With a large amount of reinforcement, it can grow so much that concrete shrinkage cracks formed. This stress also contributes to an earlier formation of cracks in tensile zones of reinforced concrete element under the load. However, with the formation of cracks during the operation, effect of shrinkage decreases, in failure stage shrinkage does not affect the bearing capacity of statically indeterminate reinforced concrete structures. In statically indeterminate structures additional connections prevent shrinkage and cause additional internal efforts.

The reinforcement prevents free creep of concrete similarly to shrinkage.

Under the influence of creep, the redistribution of forces occurs between concrete and reinforcement (stress in concrete decreases, stress in reinforcement increases leading to the full use of its bearing capacity).

The creep affects differently on the behavior of RCS:

- in short compressed elements – provides total use of the strength of concrete and reinforcement;

- in flexural compressed elements – increases the initial eccentricities, which can reduce the bearing capacity of structures;

- during bending – causes an increase in deflections;

- in pre-stressed RCS – causes the loss of pre-stressing of reinforcement.

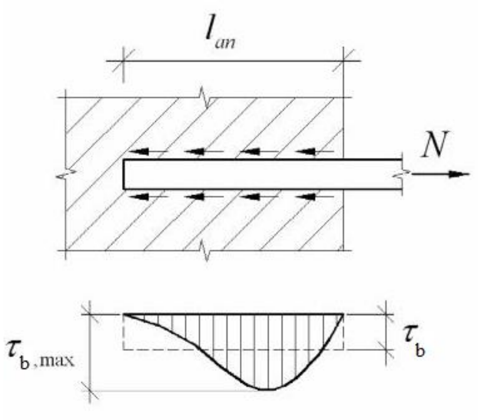
Under the influence of temperature in RCS the internal self-balanced forces occur caused by the difference of the coefficients of linear expansion of cement, aggregates and steel reinforcement. During the action of $t \leq 50°C$ they are small and do not affect the strength of concrete, at t = 60 – 200°C slight decrease of mechanical strength of concrete should be considered (by 30 %). At prolonged heating to t = 500 – 600°C with subsequent cooling the element is destroyed due to an increase in the volume of free lime released during dehydration of cement minerals and is extinguished by moist air. The bond strength of reinforcement with concrete decreases at t = 500° C at 30% (for deformed reinforcement bar) and at 250°C sharply (for reinforcement with smooth section). In statically indeterminate RCS under the influence of seasonal changes in temperature additional stress occurs, which in large size structure can be significant. To reduce it the building is divided into separate units by temperature seams, which usually coincide with shrinkage seams.

4.2. The bond of reinforcement with concrete

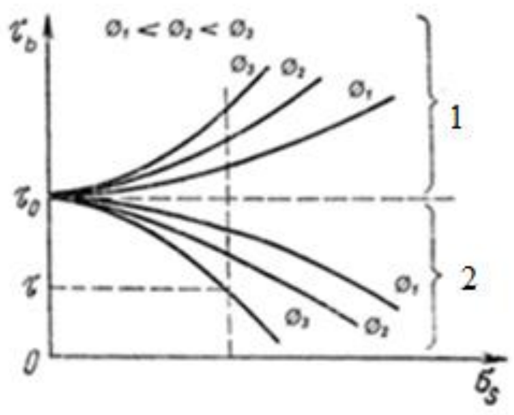
The basis of effective collaboration of reinforcement and concrete in one monolithic body – reinforced concrete – is the *bond – a set of physical and mechanical phenomena on the contact surfaces of reinforcement and concrete, which ensure their communication and shear resistance of reinforcement in concrete.* The bond strength depends from:

- class and properties of concrete: with increased grade of cement and its quantity, age of concrete, increased density, reduced W/C ratio the bond strength increases;

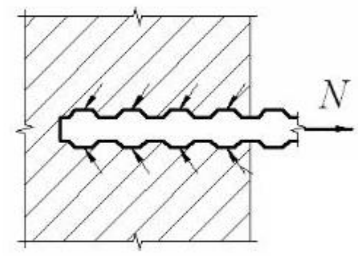
- quality of reinforcement: the surface and cross section forms; the bar diameter. With the increasing the diameter of bar and the stress in it, the bond stress increases in compression and decreases in tension (fig. 4.2, c).



a



c



b

Fig. 4.2 The bond of reinforcement with concrete: a – smooth bar; b – ribbed profile bar; c – relationship of bond stress on the diameter of the bar and the stress in it; 1 – compression; 2 – tension

During the designing of reinforced concrete elements tensile bars diameter should be limited for better bond of the reinforcement and concrete;

- manufacturing technology.

Experiments have shown that bond of the reinforcement and concrete is possible due to:

- resistance of concrete for crushing and shearing due to irregularities and projections on the surface of the reinforcement (70 – 75%). The smooth profile bars have the bond 3 times less than the deformed bars;
- gluing the reinforcement with concrete (10%).

To test the bond various methods are used: (fig. 4.2, b)

$$\tau_b = \frac{N}{ul_{bd}}; \qquad N = \sigma_s A_s; \qquad u = \pi\varnothing; \qquad A_s = \frac{\pi\varnothing^2}{4}; \qquad \tau_b = \frac{\sigma_s\varnothing}{4l_{an}}.$$

The norms provide determining the estimated values of the bond stress f_{bd} for deformed bar by the formula (3.3).

4.3. The anchoring of reinforcement

The bond usually provides joint work of reinforcement and concrete. However, sometimes these forces are not enough. Then additional means of ensuring the collaboration of concrete and reinforcement are used – the anchoring (fig. 4.3).

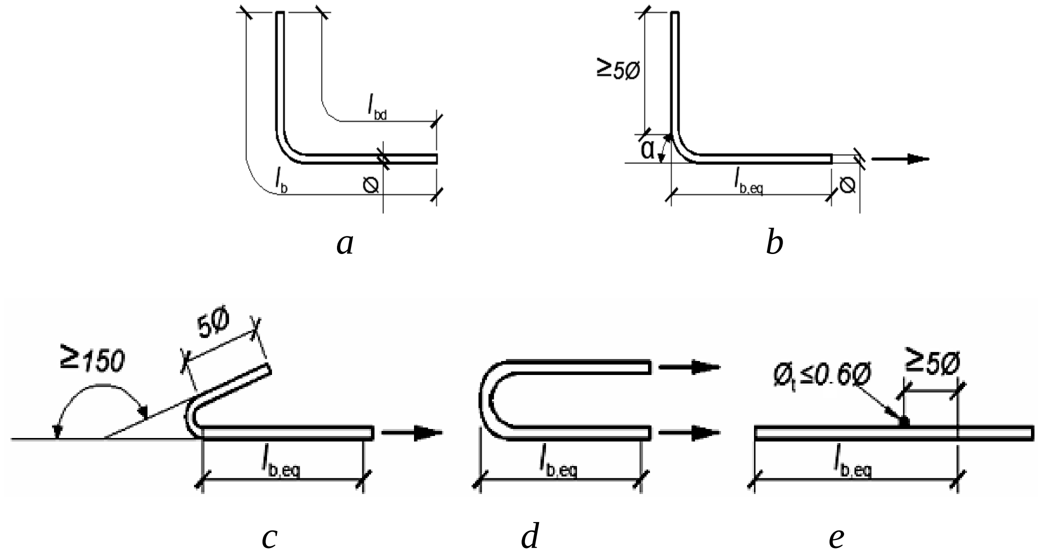


Fig. 4.3. Methods for indirect anchoring bars: a – anchoring base length l_b for any form along the axis; b – the anchoring equivalent length for standard bend bar; c – the anchoring equivalent length for standard hook; d – the anchoring equivalent length for standard loop; e – the anchoring equivalent length for welded bar

The length of the anchoring zone, at which the bond is ensured, must be as higher as higher the strength of reinforcement and diameter of bar are. To reduce the length of the anchoring zone (for saving steel) the tensile reinforcement diameter is limited, class of concrete is increased and deformed reinforcement bars are used. During determining the design anchorage length, it is required to take into account the bond characteristics.

The minimum length of anchoring is adopted by norms. Anchoring at tension can be achieved through an equivalent length of anchoring $l_{b,eq}$, which is taken $\alpha_1 l_{b,rqd}$ (fig. 4.3, b – d) and $\alpha_4 l_{b,rqd}$ (fig. 4.3, e).

Pre-stressing reinforcement – deformed bars or wire ropes – with pre-tensioning and sufficient strength of concrete is used in the structures without special anchors; with post-tensioning or pre-tensioning in low bond to concrete special anchors are used. The disposable technological anchor for bar reinforcement is taken in the form shown in fig. 4.5.

Factory sleeve anchorage of tendons (fig. 4.4) consists of a bar with chasing that wound up in a tendon, and sleeve of mild steel, which is superimposed on top of the tendon. When pulling out through a special metal ring the sleeve metal flows (yields) and presses the wires of tendon. Fixing of this anchor after post-tensioning of tendon by jack is made by nut of end bar that is tightened to the better end to the end surface of element.

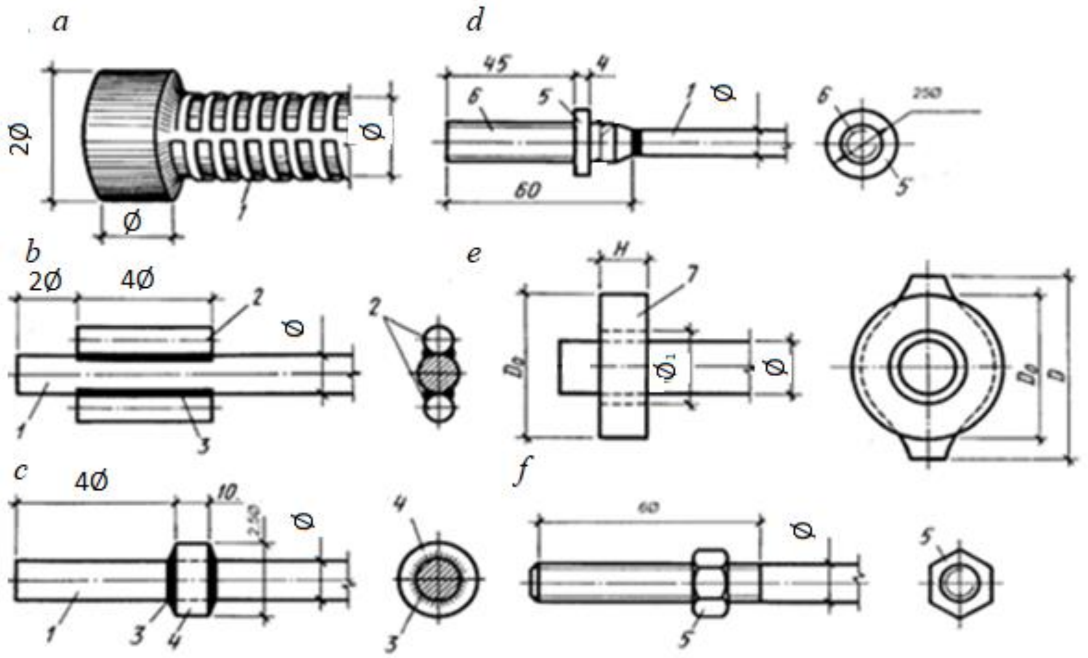
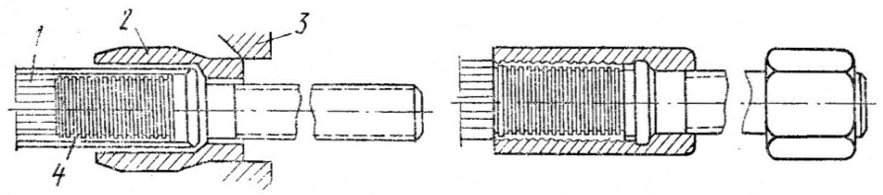


Fig. 4.3. Technological anchor of bars of pre-stressed reinforcement: a – the blown head; b – the welding of short bars; c – welding ring; d – wrap nut; e – compressed gasket; f – wrap nut; 1 – reinforcing bars; 2 – short bars; 3 – welding; 4 – ring; 5 – nut; 6 – stamped steel tip with chasing, which is welded to the reinforcement; 7 – compressed gasket



a *b*

Fig. 4.4. Sleeve anchorages: a – before pressing tendon; b – after pressing; 1 – the tendon; 2 – the anchorage; 3 – the special ring; 4 – the bar with chasing

Anchor in which reinforcing steel tendon is fixed by conical stopper (fig. 4.5) during tensioning by jack of double action, is created as follows: using the support of jack to the end surface of element the reinforcement tendon is stretched to a given stress; then the wires tendon is blocked up by conical stopper in a steel ribbon.

Barrel-type anchor (fig. 4.6) is used to consolidate a strong anchor tendon with multiple rows of concentrically arranged wires. A jack grabs and pulls anchor with the better end to concrete; in the gap, which is formed between the end surface and the anchor of element, gaskets with slots are installed, thus reinforcing tendon is held in a stress state.

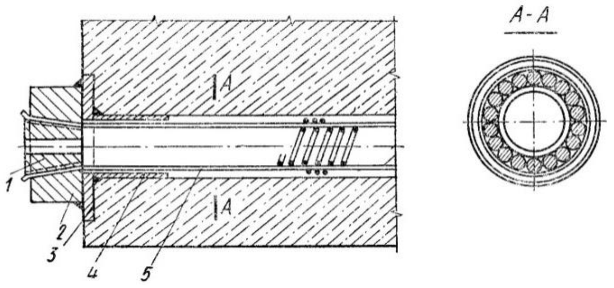


Fig. 4.5. Anchor with conical stopper: 1 – conical stopper; 2 – ribbon; 3 – steel plate; 4 – adapter sleeve; 5 – steel tendon

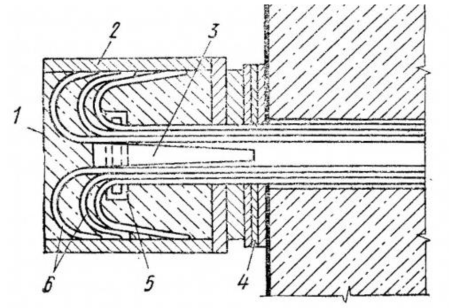


Fig. 4.6 Anchor barrel type: 1 – concrete, which provides a pressing of tendon in anchor; 2 – steel welded barrel with welded on bottom; 3 – conical steel bar; 4 – steel gasket; 5 – ring; 6 – hooks at the ends of wires

4.4. The concrete cover

Concrete cover is prescribed to protect against corrosion, prevent rapid heating at high temperatures and better bond reinforcement with concrete. The thickness of the concrete cover is prescribed depending on the type and diameter of reinforcement, cross-sectional dimensions of the elements, the type and class of concrete, application conditions of structures. The nominal cover should be specified on the drawings. It is determined by a minimum cover (tab. 4.1) and allowance in design for deviations

$$C_{nom} = C_{b,\min} + \Delta c_{dev} \ . \tag{4.2}$$

The greater value satisfying the requirements for both bond and environmental conditions should be used. In order to transmit bond forces safely and to ensure adequate compaction of the concrete, the minimum cover should not be less than $C_{b,\min}$.

Table 4.1 – Minimum cover requirements to ensure the bond

Arrangement of bars	Minimum cover $C_{b,\min}$
Separated	The diameter of the bar
Bundled	The equivalent diameter $Ø_p$
Note. If the nominal maximum aggregate size is greater than 32 mm, $C_{b,\min}$ should be increased by 5 mm; $Ø_p$ – equivalent diameter is determined according to relevant norms	

From the design experience, the following minimum values of concrete cover for structures without pre-stressing are recommended:

- in slabs and walls with thickness of 10 cm, which are made of heavy concrete – at least 10 mm, for lightweight concrete – 15 mm;

- in slabs and walls with thickness of more than 10 cm, as well as beams and ribs with height of less than 25 cm – 15 mm;

- in beams and slabs with height of 25 cm or more, and in the columns – 20 mm;

- in foundation beams and foundations in the presence of foundation mattress – not less than 35 mm, without foundation mattress – not less than 70 mm.

The thickness of the concrete cover for shear bars in the beams and columns should not be less than 15 mm.

For pre-tensioning reinforcement recommended values are: 1.5Ø for smooth rope wire or rope; 2.5Ø for ribbed profile bar.

With post-tensioning reinforcement the thickness of the concrete cover should exceed: with round cross-section of the channel – Ø; with a rectangular – must be more than two values: the smaller side or half more.

When the thickness of the concrete cover exceeds 45 mm, it is necessary to provide constructive reinforcement.

4.5. Corrosion of reinforced concrete

From the large number of different types of concrete corrosion, the most typical should be identified. The first type includes processes that occur during action of water with low rigidity on reinforced concrete. Products of dissolution are carried up by water on the surface of concrete, which are white flakes or smudges.

The second type of corrosion occurs under the influence of gas or liquid aggressive environment: acid gases, combined with high humidity, acids and other solutions. Concrete fails under interaction of acid with calcium oxide hydrate. Products of chemical aggressive environment interaction and concrete during crystallization gradually fill the pores and channels in concrete, leading to rupture of its walls. Salts of some acids, especially sulfuric, have the most negative influence to a concrete.

Corrosion of reinforcement is the result of chemical or electrolytic action of the environment. Products of steel corrosion have a larger volume than the reinforcement; as a result, the considerable radial pressure on the layer of the concrete is generated. In this case, along the reinforcing bars cracks with partial exposure of the reinforcement appear.

For corrosion protection the filtering ability of concrete should be reduced by introduction of special additives, the thickness of concrete cover should be increased, mastic paint and coatings should be applied, Portland cement in special types should be replaced, acid-concrete, polymer-concrete should be used.

4.6. Questions for knowledge control

1. What factors do not affect the bond of reinforcement with concrete?

2. What is the main factor that ensures the bond of reinforcement with concrete?

3. Assign the minimum value of concrete caver for the slab with thickness of 10 cm which is made from heavy concrete?

4. What are the negative effects of shrinkage of reinforced concrete elements?

5. How creep effects on the operation of reinforced concrete elements?

6. What factors affect the size of the protective layer in the reinforced concrete?

7. What measures do you know for anchoring the reinforcement?

8. Describe the types of corrosion of concrete?

9. What measures prevent corrosion of reinforced concrete?

10. What factors affect the size of the concrete cover in the reinforced concrete?

5 MASONRY

5.1. Strength characteristics of the masonry

The main characteristic of masonry units is their strength, which is characterized by grades and classes. The grade of the masonry unit is set according to the value of the limit of the strength in compression, and for the brick also according to the value of the limit of the strength under bending. If a masonry unit has a different structure in different directions, then its grade defines a limit of the strength in the direction in which it works in the masonry. Limit of the strength of hollow stones is calculated gross.

For bricks from the experimental batch, the samples are taken (5 pieces for compression test and 5 pieces for bending). At the same time, the brick, which will be tested for bending, should not have through-the-thickness cracks on the stretcher face for the entire thickness of more than 40 mm in length. Compression-tested bricks are sprayed on two halves, which are applied by bed one by one with the faces of the cut in the opposite sides and connected by a cement paste. The top and bottom of the samples are aligned with the same paste to ensure parallelism. The thickness of the joint between the bricks should be no more than 5 mm, and the leveling joint – no more than 3 mm. Thus prepared samples are kept before the test for 3 – 4 days in a closed room at an air temperature of 20 ± 2 ºC. Testing of samples takes place in the press.

When tested for bending bricks are invested according to the scheme of the beam with simply supported ends in the form of rollers with the diameter of 20 – 30 mm at a distance of 200 mm. In the places of supporting and applying the load strips of cement paste are applied on the bricks with a width of 20 – 30 mm and a thickness of no more than 3 mm, after which they are kept indoors for 3 – 4 days. For the bend test, a press is used; the load is applied in the middle through the roller.

The strength of the mortar is characterized by its grade, which is determined by the compressive strength. The choice of the grade of the mortar is carried out according to the standards depending on the durability of the buildings and the conditions of operation of the structures. As experiments show, a masonry unit and a mortar in masonry are in a complex stress state, even with uniform distribution of the load across the entire section of the compressed member. They simultaneously resist to eccentric and local compression, bending, shear, tension (fig. 5.1). This is explained by the fact that the density and stiffness of the mortar along the length and width of the joint due to different factors (irregularity of water return and shrinkage, uneven placing of the mortar, the presence of vertical joints and voids) are heterogeneous. That is why the quality of the masonry (the completeness and uniformity of filling the joints, keeping their rational thickness (10 – 12 mm)) is essential. Increasing the flow-ability of the mortar contributes to uniform filling of joints and, as a consequence, leads to increased strength of the masonry. It is affected by: strength, size and shape of masonry units, as well as cavities in them; strength and deformability of the mortar, its flow-ability; method of binding of masonry units; bond of the mortar with the units; masonry quality.

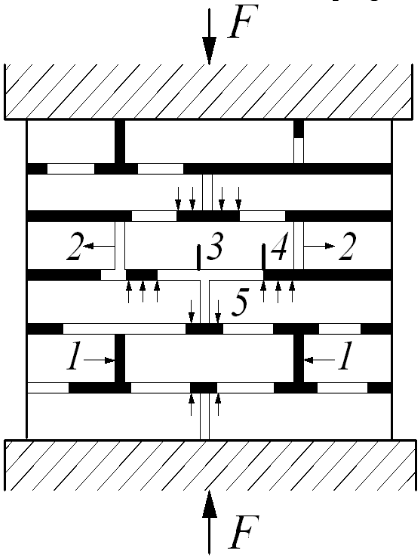
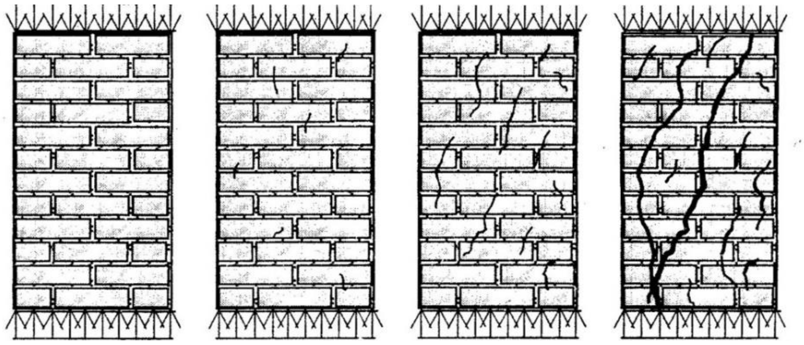


Fig. 5.1. Stress strain state of units in a masonry at central compression: 1 – compression; 2 – tension; 3 – bend; 4 – shear; 5 – local compression

In the behavior of masonry on a compression four stages are distinguished (fig. 5.2). The first one corresponds to the normal operation of the masonry, when the effort that it generates under load does not cause visible damage *($N \leq N_{crc}$)*.



a *b* *c* *d*

Fig. 5.2. Stages of masonry behavior in compression: a, b, c, d – first, second, third, fourth respectively

The transition to the second stage of behavior is characterized by the appearance of small cracks in individual bricks. Here the masonry accepts loads in the range of 60 – 80% of the ultimate, and the further development of cracks at its constant value is not observed (($N=N_{crc}$). As the load increases, new cracks arise and develop, which interconnect with each other and cross a significant part of the masonry in a vertical direction. This is the third stage ($N_{crc} < N < N_u$). Within it, under long-term action of the load, which corresponds to this stage, even without its increase, gradual development (due to the development of plastic strain) will result in the further development of cracks, which divides the masonry into thin flexible columns. Then the third stage passes into the fourth stage of destruction. Since the destruction of the compressed masonry occurs due to the loss of stability of the flexural columns formed after cracking, the strength of the masonry, even in a very strong mortar, is always less than the strength of the stone compression. The theoretical maximum masonry strength on a mortar with infinity strength is known as the *structural strength of the masonry*. Actual characteristic of its strength is much less than structural strength. In addition to the grade of masonry units its grade is influenced by the grade of the mortar f_m and the type of masonry. The actual strength of the masonry can be obtained according to the empirical Onishchyk's formula

$$f_k = Af_b(1 - \frac{a}{b + f_m / 2f_b})\eta , \qquad (5.1)$$

where A, a, b, η − coefficients that depend on the type of masonry and always are less than 1.

From the formula (5.1) it can be seen that an increase in the strength of the masonry attenuates with an increase in the strength of the mortar. Therefore, the application for ordinary masonry of high-grade mortars (more than 75) is not feasible. The design strength of the masonry is obtained by dividing its characteristic strength by the safety factor k (when compressed, 2 is taken).

The characteristic strength of the masonry on compression can also be determined by testing masonry samples, in addition to the norms (corresponding to (5.1)) or by the formula

$$f_k = Kf_b^{\alpha} f_m^{\beta} , \qquad (5.2)$$

where α, β – constants;

K – coefficient, which depends on the type of elements of masonry, masonry groups, type of construction mortar (of general purpose, thin-layer with thickness of horizontal joins 0.5 – 3 mm, facilitated by different density).

The following types of masonry units are distinguished: common brick; sand-lime brick; concrete products (with heavy and light aggregates); blocks of autoclaved lightweight concrete; concrete stones and artificial stone blocks; building elements made of natural stone.

All elements of the masonry are divided into 4 groups. Units of autoclaved lightweight concrete, building elements of artificial and processed natural stone are in the 1st group. Geometric requirements (volume of cavities, declared values of the thickness of the internal and external partitions for the brick and their total thickness, vertical and horizontal cavities) are given in norms for the determination of the common, sand-lime brick and concrete blocks.

For masonry using general purpose construction mortar and light construction mortar, dependence is recommended

$$f_k = K f_b^{0.7} f_m^{0.3}, \tag{5.3}$$

for masonry using in horizontal joints a thin-layer construction mortar with the thickness of 0.5 – 3 mm and common brick of the 1st and 4th groups, sand-lime brick, concrete blocks of autoclaved lightweight concrete

$$f_k = K f_b^{0.85}, \tag{5.4}$$

and for masonry using in horizontal joints a thin-layer construction solution with a thickness of 0.5 – 3 mm and common brick of groups 2 and 3

$$f_k = K f_b^{0.7}. \tag{5.5}$$

The strength of the masonry depends on the duration of the load. *The boundary of the long-term masonry resistance* is considered to be the maximum stress, which the masonry can withstand indefinitely without destruction. For masonry, it is smaller than the strength of a short-term load and is (0.6 – 0.88) f_k depending on the mortar.

The destruction of the tensile masonry (fig. 5.3, a, b) and the masonry when bending (fig. 5.3, c) can occur on the wall-bound cross-section (in case if the tensile strength of the mortar is less than the bond between the stone and the mortar, there is mortar destruction, otherwise – by a cross section that passes through a stone and a mortar) or on an unbounded cross-section (the masonry collapses in the plane of connection of a stone and a mortar in horizontal seams).

The central tension of the masonry on the wall-bound section is found in round tanks, silos and other structures, and tension on the unbounded sections – in eccentrically compressed pillars and walls. The strength of the unbounded cross section f_{xk1} is always lower than that of the wall-bound f_{xk2}.The characteristic strength of the masonry in the shear f_{vk} (with the use of a general purpose construction mortar, thin layer mortar in bedding seams with a thickness of 0.5 – 3 mm, as well as a lightweight construction mortar) with fully filled seams is determined by the equation

$$f_{vk} = f_{vko} + 0.4\sigma_d, \tag{5.6}$$

but not more than 0.065 f_b or f_{vlt},

where f_{vk0} – the characteristic initial strength of the masonry for zero compression (determined from the results of the masonry test or by [6], depending on the type of masonry elements, type and grade of the mortar);

f_{vlt} – ultimate value f_{vk};

σ_d – the designed value of the compression perpendicular to the direction of shear action in the structural element;

f_b – the characteristic strength of the compression of the elements of the masonry in the direction of the load on the test piece perpendicular to its bed surface.

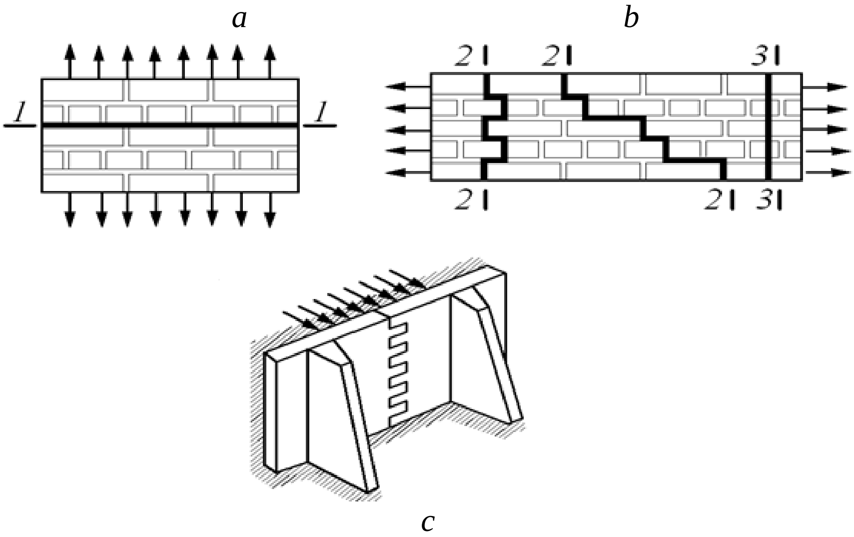


Fig. 5.3. Destruction of masonry from tension: a – for unbounded section; b – for wall-bound section; c – during bending; 1-1, 2-2 – cross sections passing through the mortar; 3-3 – section that passes through the stone

For the description of the first two types of mortar, or a light construction mortar with incomplete perpendicular seams and masonry elements embedded in such a way that their faces are tightly adjacent to one another, the formula can be used

$$f_{vk} = 0.5 f_{vko} + 0.4\sigma_d, \qquad (5.7)$$

but not more than 0.045 f_b or f_{vlt}.

5.2. Deformability of the masonry

Masonry is elastic-plastic material. Under the load action, it undergoes elastic ε_{el} and plastic ε_{pl} strains. The relationship between stress and total relative strain – curvilinear, but with the purpose of designing it can be considered as linear, parabolic, parabolic-rectangular (fig. 5.4).

Module of strain is the variable value, is equal to the tangent of angle of inclination tangent line to the curve of the mechanical state of the masonry at the point which meets certain stress (fig. 5.5)

$$E = \partial\sigma / \partial\varepsilon = E_0 \left[1 - \sigma / (1.1 f_k\right], \qquad (5.8)$$

where E_0 is the module of masonry elasticity (determined by the results of the test or can be taken from a database).

For simplicity of calculation a value of secant module of strain $E = \sigma/\varepsilon = tg\varphi_1$ is taken. According to the norms the value of strain module of masonry E in the calculation of structures on strength is determined by the formula $E = 0.5E_0$.

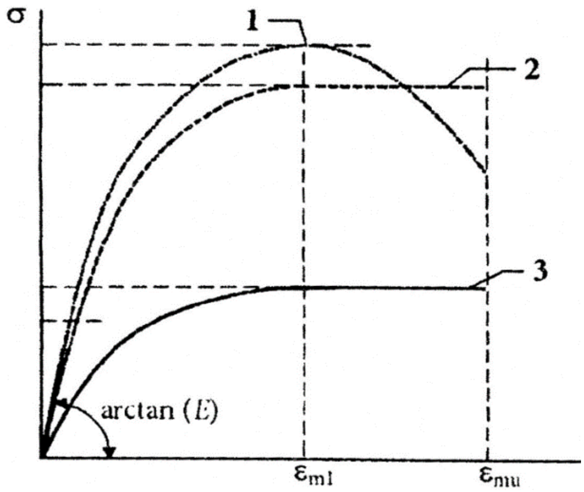


Fig. 5.4. The dependence of stress-strain for masonry in compression: 1 – typical dependence; 2 – perfect dependence (parabolic-rectangular); 3 – calculated dependence

Module of shear G is equal 40 % of the value of elasticity module.

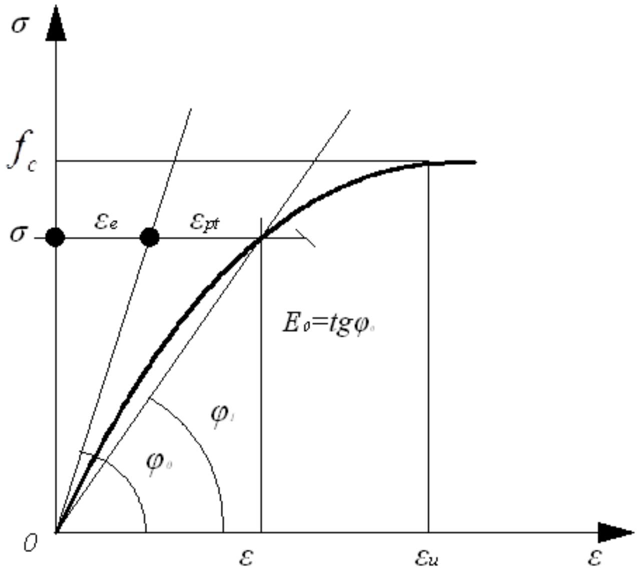


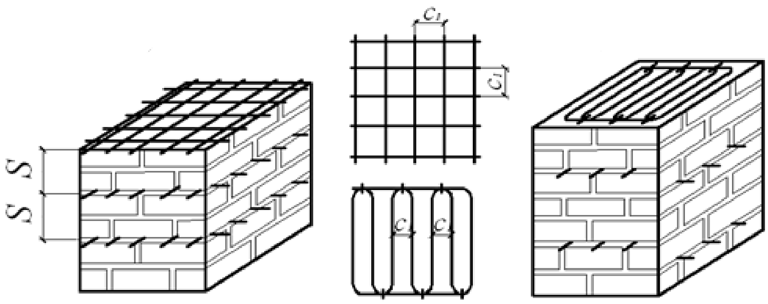
Fig. 5.5. The stress-strain diagram of masonry at compression

5.3. Reinforcing the masonry

Masonry structures are reinforced to increase the bearing capacity by: transverse reinforcement in the form of meshes, which are placed in horizontal joints of masonry; longitudinal reinforcement, which is placed in the middle of the masonry or on the outside in a layer of mortar and connected by stirrups.

Mesh reinforcing is the most common because of the simplicity of the production work. It is effectively used in brick pillars and small spaces with flexibility $l/h \leq 15$ and with small eccentricity $e_0 \leq 0{,}17h$. Strengthening of stone compressed elements by transverse reinforcement is carried out due to the tensile bars of reinforcement that prevent strain of masonry in the transverse direction, increasing its bearing capacity. Experiments show that the central compressed masonry meshed reinforcing is much more effective than longitudinal reinforcement, taken in the same amount.

The transverse reinforcement is used in the form of rectangular meshes and meshes such as "zigzag" with steel classes A240C and B500 (fig. 5.6). The use of rectangular meshes requires a significant increase in the thickness of the seam and the limitation of the wire diameter up to 3 – 6 mm. Meshes "zigzag" are laid in two adjacent horizontal seams so that the direction of the bars in them was mutually perpendicular. Two such meshes are equivalent to one rectangular, the largest diameter wires in them – 8 mm.



a *b*

Fig. 5.6. Indirect reinforcing of masonry: a – rectangular meshes; b – meshes "zigzag"

The distance between the bars (c_1, c_2) should not be more than 120 mm and not less than 30 mm. With increasing distance between meshes *S* effectiveness of their work decreases, that’s why meshes are placed at least after 5 rows of masonry for the common brick or 400 mm for other types of stones. Saturation masonry by mesh reinforcement is characterized by volume percent of reinforcement of masonry ρ

$$\rho = \frac{2A_{st}}{cs}100 \quad \%. \tag{5.9}$$

The minimum percent of reinforcing is ρ =0.1 %, maximum ρ = 1.0 %. Grade of mortar for reinforced stone structures should be not less than 50.

6 GENERAL PROVISIONS ABOUT THE CALCULATION OF BILDING STRUCTURES

6.1. Types, steps and task of analysis

Structural analysis is a static (or dynamic) analysis, an individual member section analysis, and detailing.

Static analysis includes: making of analytic models, which the most closely correspond to the real structural behavior; calculation of external loads, which act on structures during operation; analysis of forces (bending moments *M*, shear forces *V* and normal forces *N*) in specific sections of structures, which are designed. Loads and actions on the structure are determined in accordance with National Building Code B.1.2-2: 2006 "Loads and actions" []. Efforts are determined by the methods of structural mechanics or by the ultimate equilibrium method on each external load separately, and then they are totted to get the most unfavorable combination.

Member section analysis of reinforced concrete structures is a determination of the rational shape and size of the normal sections, the optimum concrete class, the class and area of principal reinforcement and its configuration in plan with allowance for requisite crack resistance and elements stiffness; or a verification of strength, crack resistance and deformability of elements. Section analysis is performed by methods of the reinforced concrete theory.

Detailing is the selection of structural solutions in general, rational scheme of principal and constructive reinforcement arrangement, development of formwork and reinforcement engineering design shop drawings, drawings of structural joins and members. *Detailing* of elements and structures in general is based on a section analysis data with allowance for building codes, which guarantee bearing capacity, crack resistance and stiffness of elements and structures that are subjected not only to the calculated forces at all construction and operational stages but also to the forces and effect which are ignored by the analysis (temperature, shrinkage). Rationality of designed structures is measured by the degree of conformance to their technological and operational requirements and by economic indicators.

6.2. The concept of stress-strain state stages of reinforced concrete elements

In 1890 Lviv Polytechnic University Professor M. Toullie introduced the concept of stages – the *stress-strain state* (SSS) *stages* of reinforced concrete flexural members, separating them in such a way as it is nowadays.

Numerous experiments show that there are three typical successive SSS stages in reinforced concrete members during increasing of the external load (fig. 6.1).

Stage I – the initial period of member's work till the cracking in the concrete tensile zone; when the load is small (15 – 25% of the ultimate), concrete strain is mainly of elastic character, and stress diagrams in compressive and tensile zones of elements can be taken as triangular, the strain in the concrete of the tensile zone is less than the ultimate values (this is the stage of work of the flexural member without cracks in the tensile zone).

With increasing of the load there is an intense development of plastic strain in the concrete tensile zone, the stress diagram here becomes curvilinear and the maximum strain in the concrete tensile zone achieves the value of nominal ultimate tensile strain ε_{ctu}; at the same time the concrete compressive zone is still mainly subjected to elastic strain, the stress diagram here is similar to triangle. Such condition characterizes the end of stage I or the end of *element work without cracks*. It is called stage *Ia* or *the moment of the formation of the first normal crack*.

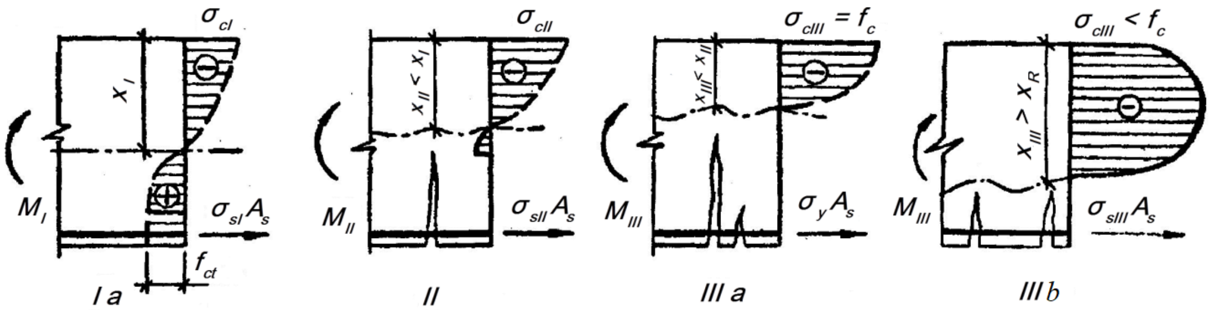


Fig. 6.1. SSS stages of flexural reinforced concrete members without pre-stressing

Stage II occurs with further increase in the load immediately after the cracking in the concrete tensile zone, tensile forces at the crack areas are perceived by the reinforcement and concrete area above the crack, tensile forces between the cracks are perceived by the reinforcement and concrete jointly.

Strain in the concrete compressive zone has plastic-elastic character with a gradual dominance of plastic strain during increasing of the load (in flexural members the stress diagram is curvilinear). The maximum stress and strain in the concrete compressive zone and in the tensile reinforcement is significant but doesn't achieve ultimate values. Most of reinforced concrete structures during operation are in stage II (*the operation loads stage* or *stage of work of reinforced concrete structures with cracks*).

Stage III. With an increase in the load, the II stage passes to stage III – the failure stage. It is characterized by a decrease in neutral axis depth and an increase in stress and strain in compressed zone. The strains in the reinforcement reach values corresponding to the beginning of the yield point, the crack opening width increases rapidly, the neutral axis depth decreases sharply, and the strains in concrete ε_c reach their ultimate values (at this moment, crushing of the compressed concrete occurs). The failure of an element in a normal section is plastic (case I-III *a*, fig. 6.1).

When reinforcing elements in the tensile zone with a high-strength wire with a small elongation at failure, along with wire break the crushing of concrete compressed zone occurs, and it is also classified as case I.

In *over reinforced* elements (with unreasonably large amount of reinforcement in the tensile zone) the failure occurs at the concrete compressive zone. Stage II changes into stage III immediately. The failure has a fragile character. It is called case II (fig. 6.1, III *b*).

Along the length of flexural members, sections can be subjected to different stages of SSS, depending on value of *M*: from stage I (for sections with a small bending moment) to stage III (for sections with maximum *M*).

6.3. Main conditions of the permissible stress method

The main point of the *permissible stress method* is that reinforced concrete was understood to be an elastic material that made it possible to use formulas of strength of materials with account of the peculiarities of its properties. The calculation is based on stage II of SSS with account of the following conditions (fig. 6.2):

- concrete of the tensile zone does not work; tensile forces are perceived by reinforcement;
- concrete of the compressive zone is elastic, the dependence σ_c - ε_c is linear, the stress diagram is triangular;
- there is a valid plane-sections hypothesis;
- using of the notion of the *reduced concrete section* – the reinforcement is reduced to the concrete according to ratio of their modules.

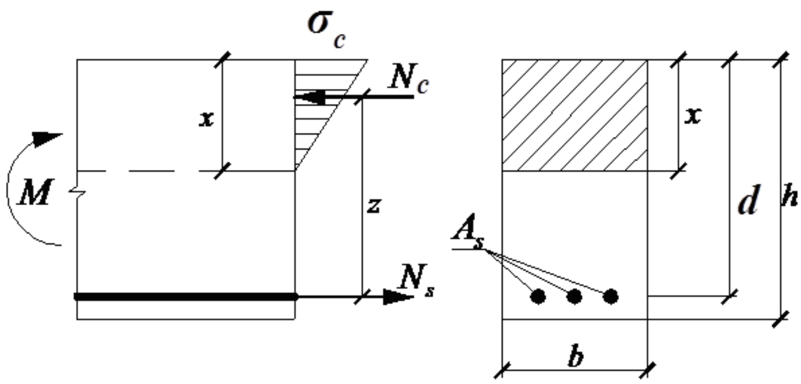


Fig. 6.2. Analytic model of the flexural member, according to calculations by the permissible stress method

Basing on these conditions, the reinforcement service stress σ_s and the concrete service stress σ_c are defined, and then they are compared with the corresponding permissible stress. Permissible stress is meant as a certain part of the yield point of reinforcement and the concrete resistance limit

$$\sigma_s \le [\sigma_s] = 0.5\sigma_y, \ \ \sigma_c \le [\sigma_c] = 0.45 f_{c,cube}. \tag{6.1}$$

According to equality of strain at the collaboration of steel and concrete, basing on Hooke's law, it can be written: $\varepsilon_s = \varepsilon_c = \dfrac{\sigma_c}{E_c} = \dfrac{\sigma_s}{E_s}$, then $\sigma_s = \dfrac{\sigma_c E_s}{E_c} = \alpha\sigma_c$, where $\alpha = \dfrac{E_s}{E_c}$. Therefore, reinforcement stress is α times more than concrete stress. That is why in reduced section the reinforcement is changed to the equivalent section area of concrete αA_s.

The fiber stress in compressed concrete and in tensile reinforcement in flexural members are defined for reduced section by strength of materials formulas

$$\sigma_c = \frac{Mx}{I_{red}}, \ \sigma_s = \frac{\alpha M(d-x)}{I_{red}}. \tag{6.2}$$

The moment of inertia of the reduced section about the neutral axis is calculated as

$$I_{red} = \frac{bx^3}{3} + \alpha A_s (d-x)^2. \tag{6.3}$$

The neutral axis depth x is defined from the condition, that the first moment of reduced section area about the neutral axis is equal to 0,

$$S_{red} = bx^2/2 - \alpha A_s (d-x). \tag{6.4}$$

While designing reinforced concrete structures according to the permissible stress method the obtained results are often different from the experimental values. It is caused by the fact that concrete is an elastic-plastic material, and stress diagram in the concrete compressive zone is curved. The modular ratio of steel and concrete is a time-variable value, because with the development of inelastic strain the concrete modulus decreases, that isn’t covered by design. As a result of this, the reinforcement stress that is calculated by (6.2) is larger than working stress. It results to material overconsumption. Besides, the safety factor is prescribed separately for concrete and reinforcement, and the overall safety factor continues to be unascertained.

6.4. Main conditions of the ultimate-strength method

In 1932 A. Loleyt proposed a theory of the *ultimate-strength method*, which was introduced to the building code of USSR in 1938. It is based on stage III of SSS (stage of failure) and on conditions (fig. 6.3):

- the concrete work of the tensile zone is not taken into account;
- using of notions of the ultimate stress of concrete and steel (there is no need to use α coefficient);
- the stress diagram in the concrete compressive zone is rectangular;
- using of ultimate factor of safety, which is equal to the ratio between the ultimate force and force during operation (for flexural member $k = \frac{M_u}{M}$). The safety coefficient depends on the building (construction) criticality rating, the type of structure and combination of external loads (for example, for cast in place slabs and beams at the ratio between live and dead loads *v/g* $\leq 2 - \kappa = 1{,}8$; *v/g* $>2 - \kappa = 2$; for precast concrete units the admitted coefficient is less than 0.2).

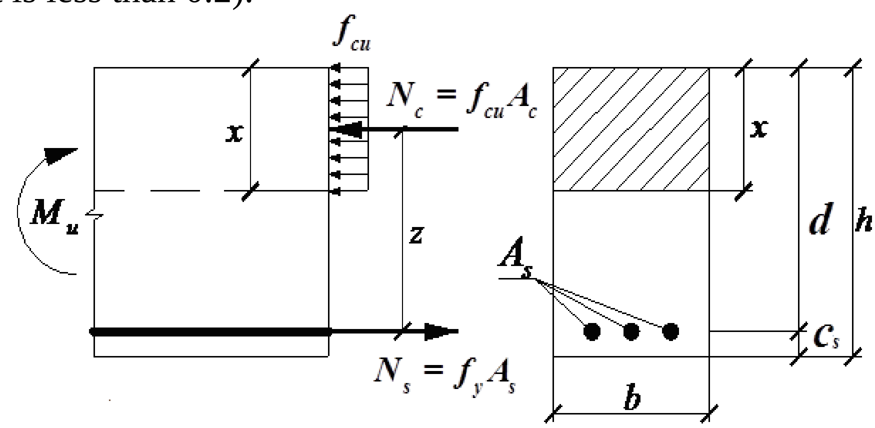


Fig. 6.3. Scheme of efforts in the normal section of the flexural element, according to calculations by the ultimate-strength method

Strength of the member at the normal section is determined using the equation of equilibrium

$$\sum X = 0: f_{cu}bx = f_yA_s, \tag{6.5}$$

$$\sum M = 0: f_{cu}bx(d - x/2) = M_u, \tag{6.6}$$

where $f_{cu} = 1.25 f_{c,prizm}$ – concrete compressive strength under bending.

A big advantage of this method is that it is based on the consideration of the failure stage. Within the limits of design, the overall ultimate factor of safety, which is related to the real, is determined as a whole instead of its individual components. The main disadvantage of the ultimate-strength method is the fact that it’s not possible to take adequately into account the effect of a large number of factors on structural bearing capacity by the only ultimate factor of safety. Such factors are: the variability of the strength property of concrete and reinforcement; the difference of actual loads from assumed loads; the influence of operating peculiarities of materials and structures.

6.5. The main provisions of limit states design method

6.5.1. Essence of the method

Limit states design method was proposed by A. Gvozdev in 1955. To the present time it is normative. The main difference is that it establishes clear boundaries of structures limit states and introduces a system of design factors that prevents the occurrence of these states in the structures under the most adverse combination of loads and minimum strength of materials.

Limit state is a state of structure in which it ceases to meet the requirements that are made to it, i.e. loses its ability to resist to external influence or obtains abnormal deformations or local injuries.

According to the causes that may lead to the loss of the required functional properties, there are two groups of limit states:

- the first – *ultimate limit state* – for unsuitability for further operation;
- the second – *serviceability limit state* – for unsuitability to normal operation.

The ultimate limit state includes the following limit states:

- fragile, ductile and other failure (strength calculation);
- loss of shape stability (calculation of stability of thin-walled structures) or position (calculation at overturning and sliding retaining walls, the calculation of pivoting);
- fatigue failure (endurance calculation of structures that are under the influence of repetitive load – crane beams, railway sleepers, frame foundations and floors in some equipment, etc.);
- the failure from the joint influence of loads and adverse environmental effects (intermittent or constant exposure of aggressive environment, the impact of alternate freezing and thawing, fire).

The serviceability limit state includes:

- abnormal movement (deflections, angles of inclination and rotation, oscillation);
- cracking, as well as short-term or long-term crack growth (if the operating conditions are acceptable).

All structures are calculated according to the first group, and the second one – calculation performed only when the following conditions may occur.

The main factors that determine the design achievement of a limit state are:

- loads that act on the structure;
- strength characteristics of materials of which they are made;
- the conditions under which structures work.

All these factors have a certain volatility and may differ from the intended standards. In the calculation method of limit states, it is taken into account by the introduction of *a system of partial safety factors*.

6.5.2. Characteristic and design values of loads. The combination of loads

The loads are divided into dead and variable depending on the variability in time (long, short and episodic) [1].

Dead loads include the weight of bearing and enclosing structures of buildings and structures, weight and pressure of the soil, the pre-stressing of structures.

Long-term variable loads include the weight of temporary partitions, grouting and base concrete for equipment; stationary equipment; pressure of gases, liquids, bulk; weight of specific load in warehouses, refrigerators, archives, libraries; quasi-dead design values of the load from weight of people, animals, equipment on buildings floors, the vertical load from the bridge and underhung cranes, snow load; temperature climatic impacts.

Short-term variable loads include the weight of people, parts and materials in the areas of maintenance and repair of equipment; loads resulting from the manufacture, transportation and installation; snow and wind, crane and ice load with limit design or service values.

Episodic loads include seismic and explosive impact or loads caused by sharp disturbances of the technical process, base deformation.

The basis for the assignment of loads is their *characteristic values*. The design values of loads are determined by multiplying the characteristic values by the *load safety factor* γ_f. Depending on the character of load and purpose of calculating the load there are four types of design loads: the limit; service; cyclical; quasi-dead. To check the limit states of the first group the limit design values of loads should be used, the second group – depending on the operating conditions of the structure.

Characteristic values of weight of precast structures should be defined by the standards, working drawings or passport data of suppliers, and other building structures and soils – the project sizes and density of materials. Characteristic values of equipment weight are determined by standards or directories, passport data or working drawings; characteristic value of snow load equals to the ground snow load and may be exceeded on average once every 50 years (depending on snow determining region); for wind pressure – equals to the average (static) pressure component of the wind at a height of 10 m above the ground, which can be exceeded on average once every 50 years (determined in accordance with the wind area).

Depending on the composition of the load taken into account, there are:

- *basic combinations* that include dead, cyclic or quasi-dead values of variable loads;

- e*mergency combinations* that consist of dead, variable and one episodic influences.

Low probability of simultaneous implementation of design values of multiple loads is taken into account by multiplying their design values by a *combination factor* $\psi \leq 1$.

For basic combinations, including dead and at least two variable loads the last one is accepted with a combination factor ψ_2 = 0.9 for both long-term and short-term loads.

For emergency combinations, including dead and not less than two variable loads, the last one are taken with a combination factor ψ_1 = 0.95 for long-term and ψ_2 = 0.9 – for short-term loads. Emergency load is taken with a combination factor ψ_1 = 1.

While taking into account a work of one crane its load is taken without reduction, for two cranes combination factor ψ = 0.85 is used (for operating modes 1 – 6 K) and ψ = 0.95 (7 – 8 K), while taking into account four cranes 0.7 and 0.8, respectively.

6.5.3. Characteristic and design values of strength of materials

Different calculations require different strength security. As a result, there are two concepts that are used for concrete and reinforcement: the characteristic and design values of strength of materials. There are the following *characteristic values*:

– concrete resistance to axial compression $f_{ck,prizm}$;

– concrete resistance to axial tension $f_{ctk,0.05}$ (if it is necessary, for emergency calculated situations concrete resistance values to axial tension $f_{ctk,0.95}$ can be used). These values are taken depending on the class of concrete to compression C in standards.

Design value of concrete compressive strength

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c \ , \tag{6.7}$$

where γ_c = 1.3 – the *partial safety factor for concrete in compression* is assigned based on the coefficient of variation of concrete compressive strength of 13.5% (can be taken and γ_c = 1.22 if it is proven that the system of quality control in manufacturing provides design coefficient of variation of concrete strength up to 10%);

α_{cc} – the coefficient that takes into account the long-term effects on the compressive strength and unfavorable effects resulting from the way the load is applied (the value α_{cc} can vary from 0.8 to 1.0, is appointed by designer and agreed with the customer, the recommended value is α_{cc} = 1).

The value of the design tensile strength is defined as

$$f_{ctd} = \alpha_{ct} f_{ctk,0.05} / \gamma_{ct}, \tag{6.8}$$

where γ_{ct} = 1.5 – *partial safety factor for concrete in tension*, is assigned based on the variation coefficient of tensile strength of concrete at 15%;

α_{ct} – a coefficient that takes into account the long-term effects on the tensile strength and unfavorable effects, resulting from the way the load is applied (the recommended value is α_{ct} = 1).

Design values of concrete compressive and tensile strength are taken depending on the class of concrete in compression C in standards.

Design values of concrete strength decreases and sometimes increases by *appropriate coefficients of concrete work conditions* γ_{ci}, that take into account the peculiarities of its work in structure:

γ_{c1} – duration of static load (γ_{c1}= 1 – with short-term action load γ_{c1} = 0.9 – with its long action);

γ_{c2} – failure character of concrete structures (γ_{c2} = 0.9);

γ_{c3} – introduced for concrete and reinforced concrete structures that are concreted in a vertical position, at an altitude of more than 1.5 m γ_{c3} = 0.85).

The main characteristic of reinforcement strength is its characteristic value close to physical f_{yk} *or proof at 0.2 percent set* $f_{0.2k}$ *yield strength,* which is usually given in the relevant regulations for the reinforcement.

Design yield strength of reinforcement f_{yd} is given by

$$f_{yd} = f_{yk} / \gamma_s, \tag{6.9}$$

γ_s – *partial safety factor for reinforcement* (depends on the type and class of reinforcement and is assigned based on the coefficient of variation of strength for steel from 7 to 10 5%: A240C – 1.05; A400C – 1.10; A600 – A1000 – 1.2; B500 – 1.2; Br1200 – 1500 – 1.25; K1400 – 1500 – 1.2).

Design values of compressive strength of reinforcement are equal to the *design yield strength of reinforcement* f_{yd}, but no more than those comply with the limits of concrete compressive strain, where the reinforcement is placed, with short-term or long-term load action.

Under certain conditions, the *design yield strength of reinforcement* in the calculations by the first group of limit states is multiplied by *a factor of reinforcement conditions* γ_{si},, which takes into account character of loading, reinforcement purpose, welding seams presence etc. Ultimate values of compression resistance of reinforcement with class B500 are adopted with a factor of safety – 0.9. *Design yield strength of shear reinforcement* f_{ywd} is decreased compared to f_{yd} by multiplying by a factor of working conditions 0.8

and is taken no more than 300 MPa.

6.5.4. Safety factor for responsibility

Depending on the material damage and (or) social losses associated with the cessation of operation or the loss of integrity of the object the following *classes of responsibility of buildings and structures* are determined (tab. 6.1): *CC*3 (significant consequences), *CC*2 (secondary consequences), *CC*1 (minor consequences). A tentative list of objects by classes of effects (liability) is given in standards. Depending on the effects that can be caused by the rejection, there are *three categories of accountability of structures and their elements*:

Table 6.1 – Safety factor for responsibility γ_n

<table>
<tr><td rowspan="3">Class of consequ-ences (liability)</td><td rowspan="3">Category of responsibility of structure</td><td colspan="5">The value of γ_n, which is used in the design situations</td></tr>
<tr><td colspan="2">steady*</td><td colspan="2">transitional **</td><td>emergency ***</td></tr>
<tr><td>ultimate limit state</td><td>service-ability limit state</td><td>ultimate limit state</td><td>service-ability limit state</td><td>ultimate limit state</td></tr>
<tr><td rowspan="3">CC3</td><td>A</td><td>1.250</td><td rowspan="3">1.000</td><td>1.050</td><td rowspan="3">0.975</td><td rowspan="3">1.050</td></tr>
<tr><td>B</td><td>1.200</td><td>1.000</td></tr>
<tr><td>C</td><td>1.150</td><td>0.950</td></tr>
<tr><td rowspan="3">CC2</td><td>A</td><td>1.100</td><td rowspan="3">0.975</td><td>0.975</td><td rowspan="3">0.950</td><td rowspan="3">0.975</td></tr>
<tr><td>B</td><td>1.050</td><td>0.950</td></tr>
<tr><td>C</td><td>1.000</td><td>0.925</td></tr>
<tr><td rowspan="3">CC1</td><td>A</td><td>1.000</td><td rowspan="3">0.950</td><td>0.950</td><td rowspan="3">0.925</td><td rowspan="3">0.950</td></tr>
<tr><td>B</td><td>0.975</td><td>0.925</td></tr>
<tr><td>C</td><td>0.950</td><td>0.900</td></tr>
</table>

* s*teady* – such design situations which have a duration of implementation T_{sit} the same manner as an established service life of building project T_{ef} (for example, operating cycle between major repairs or changes in technical process);

** *transition*, which have a small duration of implementation T_{si} compared with established service life T_{ef} (for example, during the construction of the object, major repair, reconstruction);

*** *emergency*, which have a low probability of occurrence P_{sit} and usually short duration of implementation $T_{sit} << T_{ef}$, but which are important in terms of the consequences of possible failures (such as situations that arise during the explosions, fires, equipment accidents, vehicle collisions, and immediately after the failure of any element of structure).

– A – structures and elements rejection of which can result in complete unsuitability to the operation of the building (structures) the whole or a substantial part thereof;

– B – structures and elements rejection of which may lead to complication of the normal operation of the building (structures) or failure of other structures that do not belong to the category A;

– C – structures which failure do not lead to the disruption of operation of other structures or elements.

Categories of responsibility are established by designers and must be given in the design plans and specification.

Safety factor for responsibility γ_n [2] is determined depending on the class of object effects (liability), category of structure liability and type of *design situations* as conditions complex that is taken into account in the calculations and determines the design requirements for the structure. *Design situation* is characterized by the design model of structure, types of loads, the values of the coefficients of working conditions and safety factor, the list of limit conditions that should be considered in this situation. Ultimate values of bearing capacity, the calculated values of resistance, and ultimate values of deformations or crack opening width are divided by this coefficient or the calculated values of loads and efforts are multiplied by it.

6.5.5. The essence of calculation for various limit states

Condition that ensure the bearing capacity of structures, i.e. impossibility of realization of ultimate limit states is written as

$$\Phi_{\max}(g_k, v_k, \gamma_f, \gamma_n, c) \le \Phi\left(f_{ck}, \gamma_c, \gamma_{ci,} f_{ctk}, \gamma_{ct}, f_{yk}, \gamma_s, \gamma_{si}, s\right), \qquad (6.10)$$

where Φ_{max} – design efforts;

Φ – design bearing capacity;

c – design scheme of structure (member);

s – shape and sizes of the section.

The same equation can be written in reduced form as

$$\Phi_{\max}(g, v, \gamma_n, c) \le \Phi\ (f_{cd}, f_{ct}, \gamma_{ci}, f_{yd,} \gamma_{si}, s). \qquad (6.11)$$

Conditions for calculating of crack resistance in general terms can be written depending on whether cracking in the structure is permitted:

– in the case when the cracks are not allowed, the condition should be fulfilled in the calculations of their formation (for flexural members)

$$M \le M_{crc}, \qquad (6.12)$$

where M – moment of maximum external load;

M_{crc} – cracking moment (the moment at which cracks in structures may cause);

- in the case when the width limited cracks are admitted, i.e. for the calculations for crack opening

$$w_k \le w_{k,u}, \tag{6.13}$$

where w_k – the largest crack opening width calculated theoretically; $w_{k,u}$ – ultimate value of width of the crack opening imposed by standards.

Condition for calculation by deformations is in form

$$f \le f_u, \tag{6.14}$$

where f – deformation calculated theoretically;

f_u – ultimate value of deformation imposed by standards.

The main advantage of the method of calculation by the limit states is that the introduction of designed coefficients system allows a more differentiated assess of the impact of all determinants on structures operation, and thus more accurately reflect its actual stress state.

6.6. Questions for knowledge control

1. What is the essence of static calculation of building structures?
2. What is the limit state?
3. How many groups of limit states are there?
4. What factors determine the structure to reach its limit state?
5. What are the dead loads?
6. What short-term loads do you know?
7. What long-term loads do you know?
8. How many stages of SSS of reinforced concrete structures are there?
9. What is the first stage of SSS?
10. What is the second stage of SSS?
11. What is the third stage of SSS
12. What stage of SSS is the basis for calculating the bearing capacity of reinforced concrete structures?
13. How a condition that ensure the bearing capacity of structures is written?
14. How a condition that ensure the crack resistance of structures is written?
15. How a condition that ensure the deformations of structures is written?
16. What is γ_{c2} ?
17. By what formula is determined the design yield strength of reinforcement?
18. By what formula is determined the design value of concrete compressive strength?
19. How the design load value is determined?
20. What is γ_n?

7 DESIGN AND DETALING OF REINFORCED CONCRETE STRUCTURES

7.1. Reinforced concrete flexural members

7.1.1. Design features

Flexural members include slabs (according to DBN: elements in which the minimum side size is at least 5 times more of the total thickness) and beams (elements in which the span is at least 3 times more of the total section height). Slabs and beams (fig. 7.1) can be members that work independently or can be as a part of the floors (roofs) in combination with each other. Reinforced concrete slabs in cast-in-place structures are made with a thickness of 50 – 100 mm, precast – 25 – 40 mm, reinforced mainly by welded flat or rolled meshes. Principal bars of reinforcing mesh are placed along the span of the slab to perceive the tensile forces that arise when bending under load, according to the diagram of moments. In single-span slabs, meshes are placed only from below, in multi-span – from below in spans and above intermediate supports (fig. 46). In this case, the distance between the principal bars is usually taken in the range from 100 to 200 mm with a slab thickness of up to 150 mm (the maximum possible step of the bars according to the norms should not exceed 2*h* and 400 mm for principal reinforcement, 3*h* and 450 mm for constructive reinforcement).

a

b

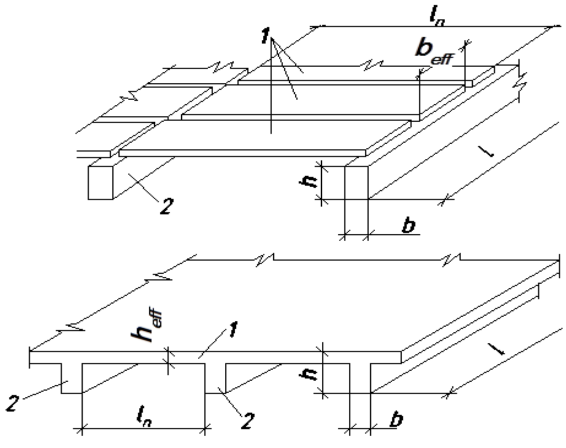
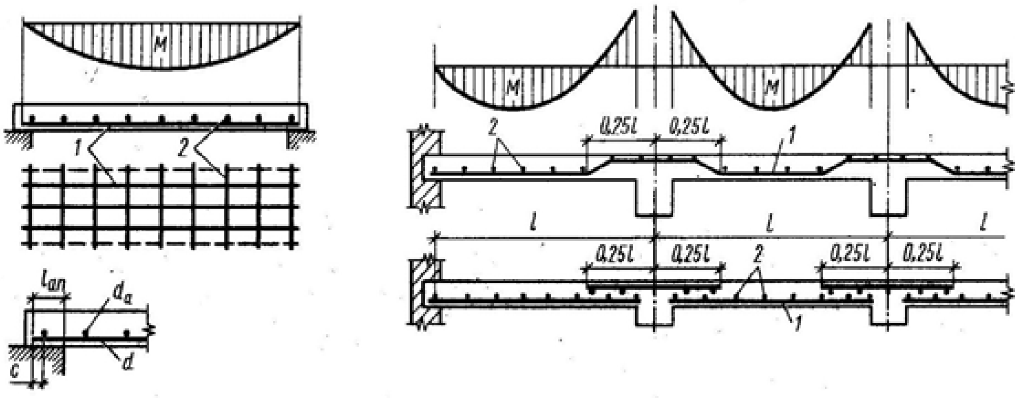


Fig. 7.1. Reinforced concrete floor structures: *a* – precast; *b* – cast in place; 1 – a slab; 2 – a beam

When reinforcing continuous slabs with welded roll meshes (fig. 7.2), near the intermediate supports, the lower meshes can be completely bent to the upper zone. To save reinforcement, part of the bars can be cut off without reaching the support. The cross-sectional area of the bars that are brought to the support should be at least 1/2 of the area of the principal bars, which is calculated by the maximum bending moment, and the distance between them should not exceed 400 mm. The standards recommend that, when using welded mesh, at least one transverse bar go beyond the edge of the support, welded to all principal bars that bring to the support.

a *b*



d *c*

Fig. 7.2. Reinforcement of slabs: *a* – single-span; *b, c* – multi-span; *d* – on a support; 1 – principal and 2 – distribution reinforcement

In cast-in-place structures, the sections above the supports (even under the assumption of hinge support) should be calculated for the bending moment that arises from partial restraint and is equal to at least 0.15 of the maximum bending moment in the span.

Precast and cast-in-place slabs are often designed as supported on two sides, cast-in-place often supported on three or four sides. In the latter case, the slabs can be bent in two directions, and they are reinforced with meshes with principal reinforcement in both directions.

Welded meshes are made only by welding from steel of classes Br-I with a diameter of 3 – 5 mm and A400C – 6 – 14 mm. The concrete cover for principal reinforcement under normal operating conditions is taken at least 10 mm, and in slabs more than 100 mm thick – 15 mm.

Reinforced concrete beams can be rectangular, T-profile, I-profile (fig. 7.3), trapezoidal cross-section.

The height of the beams h is 1/10 – 1/20 of the span, depending on the load and type of structure. For the purpose of unification, the height of the beams is assigned a multiple of 50 mm if it does not exceed 600 mm, and a multiple of 100 mm at a higher height. The width of the rectangular cross sections b is taken to be (1/4 – 1/2)h. The optimal ratio b/h = 0.5 – 0.66.

Longitudinal principal reinforcement in beams, as well as in slabs, for the perception of tensile forces is placed in the tensile zones in accordance with the diagram of bending moments. As a rule, reinforcing bars with a diameter of 12 – 32 mm are used as longitudinal bars in beams. In beams with a width of 150 mm or more, at least two longitudinal bars are installed. With a width of less than 150 mm, the installation of one reinforcing cage is allowed. The distance between the individual bars must be at least its maximum diameter, $k_1 + (d_g + k_r)$ (k_1 – 1 mm, d_g – the maximum size of the aggregate, k_r – 5 mm) or 20 mm. When placing the bars in different horizontal rows, they should be located one above the other.

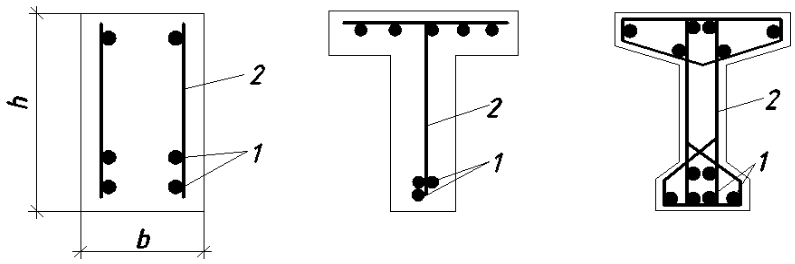


Fig. 7.3. Cross-sectional shapes of beams and schemes for their reinforcement: 1 – longitudinal bars; 2 – shear bars

Longitudinal principal reinforcement, which is placed only in the tensile zones, is called single. In the case of insufficient concrete strength of the compressed zone, and also when the moments of two signs act in the cross section, principal reinforcement is also installed in the upper (compressed) zone and beams have double reinforcement. Longitudinal and shear reinforcement is set by calculation. The area of longitudinal tensile reinforcement should be taken not less than, $A_{s,\min} = 0.26\dfrac{f_{cm}}{f_{yk}} b_t d$ but not less than $0.0013 b_t d$ (b_t – the average width of the tensile zone of the element). The cross-sectional area of the tensile and compressed reinforcement should not exceed $A_{s,\max} = 0.04 b_t d$.

In the absence of compressed principal bars, mounting longitudinal bars with a diameter of 10 – 12 mm are installed. In beams with a height of more than 700 mm, additional longitudinal bars are placed at the side faces at a distance (in height) of no more than 400 mm, its total area should be at least 0.1% of the cross-sectional area of the beam. These bars together with the shear reinforcement restrict the opening of cracks on the side surfaces of the beam.

Beams are reinforced with welded and tied cages. In the first case, individual plain welded cages are combined into a spatial one using horizontal connecting bars. As a rule, longitudinal principal reinforcement without pre-stressing is a periodic profile of class A400C, shear – of class A240C, Br-I and A400C with a diameter of 3 – 10 mm.

Pre-stressed bars should not come into contact with conventional reinforcement.

7.1.2. Design of the bearing capacity of reinforced concrete members on the normal section

There are two possible failure types of the reinforced concrete simply supported one-span beam which is loaded with concentrated forces (or uniformly distributed load):

– in a section that is normal to the longitudinal axis of member, where maximum bending moment acts;

– in a section that is inclined to the longitudinal axis of member, where bending moment M and shear force V act.

In accordance with this, strength design of flexural members is performed separately for sections that are normal and inclined to the longitudinal axis of member.

7.1.2.1 Calculation of the bearing capacity of reinforced concrete members of a rectangular profile with a single reinforcement with using a rectangular diagram of stress in a compressed zone of concrete

The design preconditions. III stage of stress-strain state of flexural members is the strength design basis. Thus there are two possible failure cases: *plastic* and *brittle* which should be avoided.

Differentiation of possible failure cases is made by observance of condition

$$\xi = \frac{x}{d} \le \xi_R = \frac{x_R}{d}, \tag{7.1}$$

where ξ – relative and x – neutral axis depth;

ξ_R – ultimate relative and x_R – ultimate neutral axis depth;

d – effective depth of cross-section (distance from center of gravity of tensile reinforcement to most compressed concrete fiber) $d = h - c_{nom} - \varnothing / 2$;

c – concrete cover;

Ø – diameter of longitudinal principal reinforcement.

According to the ξ and ξ_R values failure case is determined. If $\xi \le \xi_R$ the section is not over reinforced and by suitable load failure may occur according to case 1. If $\xi > \xi_R$ the section is over reinforced and failure occurs according to case 2.

Flexural member's strength in normal section is determined taking into account consideration of stressed state of sections that fail by 1st case scheme. Thus:

– the work of concrete of tensile zone isn't taken into consideration;

– stress in tensile reinforcement reaches value of *design yield strength* f_{yd}, in compressed – *design yield strength of reinforcement* f_{yd};

– concrete resistance to compression is taken as uniformly distributed along the compressed zone (rectangular diagram) and equals to *design value of concrete prizm compressive strength* f_{cd};

– designed formula are brought out from the section balance conditions, where bending moment from external design load M_{Ed} and internal forces in concrete compressed zone and longitudinal reinforcement M_{Rd} act.

The design of rectangular section members with single reinforcement. In flexural members, reinforcement should be placed in tensile zone. Reinforcing of compressed zone is carried out only in case of need of its strengthening, which is placed according to the design. At first it is foreseen that reinforcement in compressed zone isn't necessary (in most cases it's really so) and the design is carried out based on *single reinforcement*. Condition that sum of projections of all normal forces on the longitudinal axis of element is equal to zero is following (fig. 7.4)

$$f_{yd}A_s - f_{cd}bx = 0\,. \tag{7.2}$$

Strength of elements in normal section is provided if bending moment from external design forces M_{Ed} doesn't exceed design bearing capacity the same section by bending moment M_{Rd} (design moment, which can be carried by section by its attaining of ultimate state), i. e.

$$M_{Ed} \le M_{Rd}\,. \tag{7.3}$$

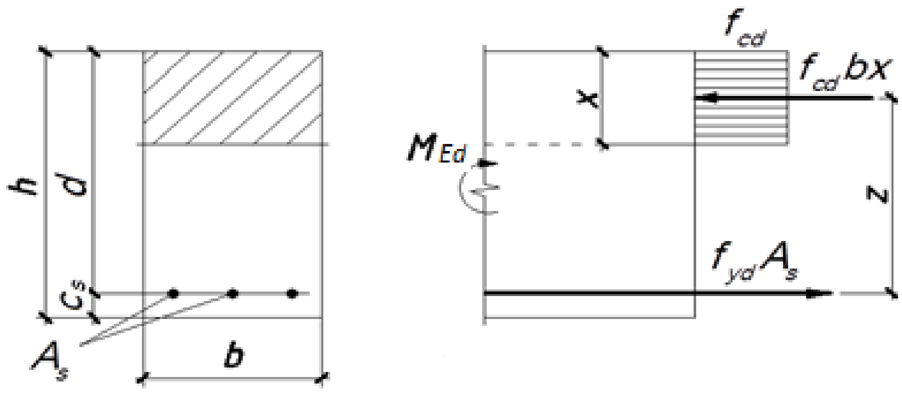


Fig. 7.4. Design diagram of normal section in the strength design of rectangular members with single reinforcement

The value of bending moment is determined about to axis that is normal to bending plane and passes through a centroid of tensile reinforcement (conditionally center of gravity of tensile reinforcement)

$$f_{cd}bx(d - x/2) - M_{Ed} = 0\,, \tag{7.4}$$

or in compressed concrete (conditionally center of gravity of concrete compressed zone)

$$f_{yd}A_s(d - x/2) - M_{Ed} = 0\,. \tag{7.5}$$

Location of neutral axis (value x) is found from the equation (7.2). For an easing of design and possibility of tabulation of values formula (7.2), (7.4) and (7.5) can be transformed using value of relative neutral axis depth $\xi = \frac{x}{d}$ and indicating from $\zeta = 1 - 0,5\xi$, $\alpha_m = \xi(1 - 0,5\xi)$

$$f_{cd}b\frac{x}{d} - \frac{f_{yd}A_s}{d} = 0,\ \ \xi = \frac{f_{yd}A_s}{f_{cd}bd},\ \ \rho = \frac{A_s}{bd},$$
$$\xi = \rho\frac{f_{yd}}{f_{cd}}\ \ or\ \ \rho = \xi\frac{f_{cd}}{f_{yd}}. \tag{7.6}$$

$$f_{cd}b\frac{x}{d}(\frac{d}{d} - \frac{x}{2d}) - \frac{M_{Ed}}{d^2} = 0,\ \ f_{cd}b\xi(1 - 0,5\xi) - \frac{M_{Ed}}{d^2} = 0,\ \ f_{cd}b\alpha_m = \frac{M_{Ed}}{d^2},$$
$$f_{cd}b\alpha_m d^2 = M_{Ed}. \tag{7.7}$$

$$f_{yd}A_s(\frac{d}{d} - \frac{x}{2d}) - \frac{M_{Ed}}{d},\ \ f_{yd}A_s(1 - 0,5\xi) - \frac{M_{Ed}}{d} = 0,$$
$$f_{yd}A_s\zeta d = M_{Ed}. \tag{7.8}$$

According to the minimum cost of the structure optimum reinforcement percentage and correspondent to them values of relative neutral axis depth are taken: for beams ρ % = 1 – 2 % (ξ = 0.3 – 0.4), for slabs – 0.5 – 0.7 % (ξ = 0.1 – 0.15).

While solving strength problems different versions are possible.

Strength control of elements by normal sections belongs to the problem of first type: the dimensions of cross-section, reinforcement, strength characteristics of materials and bending moment from external load are given.

Given: b, h, f_{cd}, A_s, f_{yd}, M_{Ed}.

Strength control lies in comparison of M_{Ed} and M_{Rd}.

Design is carried out in following sequence:

–neutral axis depth $x = \frac{f_{yd}A_s}{f_{cd}b}$ or $\xi = \rho\frac{f_{yd}}{f_{cd}}$ is found;

– condition $\xi \le \xi_R$ is checked;

– M_{Rd} is determined

$$M_{Rd} = f_{yd} bx(d - x/2),$$
$$M_{Rd} = f_{yd} A_s (d - x/2).$$

A bending moment can be determined, previously found in table $\xi \to \zeta$, α_m

$$M_{Rd} = f_{cd} b \alpha_m d^2,$$
$$M_{Rd} = f_{yd} A_s \zeta d;$$

–condition of $M_{Ed} \le M_{Rd}$ is checked. By its performance, strength can be considered as guaranteed.

Problem of second type is to select the section of longitudinal principal reinforcement.

Given: b, h, f_{cd}, f_{yd}, M_{Ed}.

Find A_s.

Design is made in following sequence:

– find $\alpha_m = \dfrac{M_{Ed}}{f_{cd} b d^2}$;

– find $\alpha_m \to \xi, \zeta$;

– check $\xi \le \xi_R$;

– determine area of reinforcement $A_s = \dfrac{M_{Ed}}{f_{yd} \zeta d}$;

– made section construction.

7.1.2.2. Calculation of the bearing capacity of reinforced concrete members of a rectangular profile with a single reinforcement according to the norms

The above methodology for calculating the bearing capacity has certain disadvantages, primarily due to the fact that the design schemes of reinforced concrete members in the failure stage differ from the real ones, namely, on the neutral axis stress in concrete is not equal to zero. That is a consequence of the applied premise about them uniform distribution within the compressed zone (fig. 48); strength calculation does not take into account the property of concrete to deform after strains reach a value ε_{c1} (fig. 7.5); the technique from the point of view of the mechanics of a solid deformable body is incomplete, since it models only the stress state of the normal section without taking into account the strain state, as a result of which it cannot solve all calculation problems without involving additional empirical dependencies.

According to the applicable standards of DBN B.2.6-98: 2009 [3], it is recommended that the calculation of reinforced concrete structures by bearing capacity under the influence of a bending moment can be performed on the basis of a calculation model of the normal section using the deformation method. *The criterion for the appearance of the limiting state is considered, while the achievement by strains of compressed concrete or tensile reinforcement in the cross section of the ultimate values of the relative strains ε_{cu} and ε_{ud} from the corresponding diagrams of their mechanical state is accepted.*

The essence of the deformation method is that it takes into account the increase in the cross section not of effort, but of strain.

The criterion for the exhaustion of the bearing capacity of the section is accepted:

- loss of balance between internal and external efforts (reaching a maximum in the diagrams "moment - curvature (deflection)" is an extreme criterion;

- destruction of compressed concrete when fiber strains reach the ultimate value ε_{cul} or rupture of all reinforcing bars due to the achievement of ultimate strains in them.

The dependence "stress – strain" for concrete is recommended in the form of a diagram shown in fig. 7.5, which can be described by a fractional rational function

$$\frac{\sigma_c}{f_{cd}} = \frac{K\eta - \eta^2}{1 + (K-1)\eta}, \tag{7.9}$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$ is the level of strains, ε_{c1} is the value of the relative compressive strain of concrete at maximum load (when calculating the *ultimate limit state* it is accepted $\varepsilon_{c1,cd}$), $K = 1.05\, E_{cd}\varepsilon_{c1,cd} / f_{cd}$.

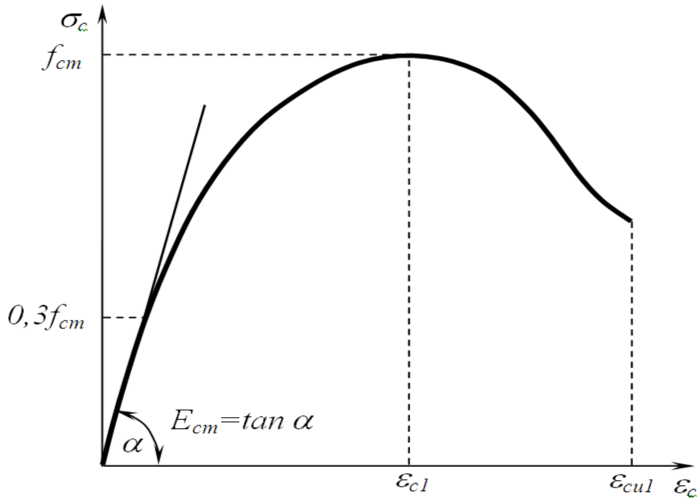


Fig. 7.5. Diagram of the mechanical condition of concrete

Expression (7.9) is valid under the condition $0 \le |\varepsilon_c| \le |\varepsilon_{cu1}|$, where ε_{cu1} is the value of the relative strain of the most compressed concrete fiber at the moment when the reinforced concrete member provides maximum resistance to external load.

The stress in the reinforcement σ_s is determined depending on the relative strain ε_s according to the formulas:

at $0 \le \varepsilon_s < \varepsilon_{s0}$ $\sigma_s = E_s \varepsilon_s$; (7.10)

at $\varepsilon_{s0} \le \varepsilon_s < \varepsilon_{ud}$ $\sigma_s = f_{yd}$, (7.11)

where $\varepsilon_{s0} = f_{yd} / E_s$ is the value of the relative elongation of the reinforcement on the verge of the inclined section of the diagram (fig. 7.6);

ε_{ud} is the value of the relative ultimate elongation of the reinforcement (accepted according to the norms).

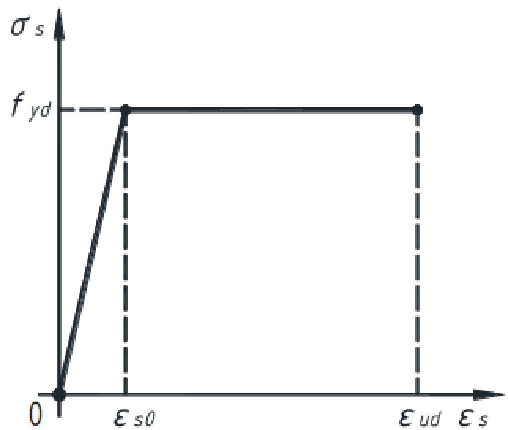


Fig. 7.6. Diagram of the mechanical condition of reinforcement with a physical yield strength

The solution of the strength problem for a beam of rectangular cross-section reinforced with a single reinforcement at the moment when concrete strain of the most compressed fiber reaches the value when the bearing capacity of the member will be maximum is possible in two ways:

- $\varepsilon_{cu,cd}$ in the calculations is taken according to experimental data studies of standard samples of concrete prisms for compression, depending only on the type and class of concrete (recommendation of norms $\varepsilon_{cu1,cd}$)
- $\varepsilon_{cu,cd}$ is determined directly from the calculation based on the concept of extreme strength criterion.

Fig. 7.7 presents a design diagram of a flexural reinforced concrete member over a normal section with a curvilinear stress diagram in the compressed zone of concrete, which is the basis for the use of a nonlinear deformation model.

A fairly simple algorithm for calculating the bearing capacity of reinforced concrete structures during bending, in which the features of the compressed zone stress distribution are taken into account using the corresponding coefficients [4], has been developed at the department of building structures of National University «Yuri Kondratyuk Poltava Polytechnic».

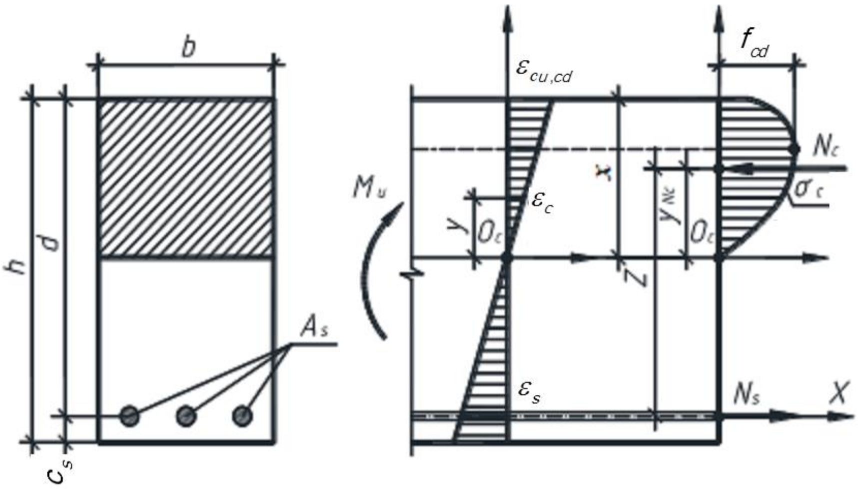


Fig. 7.7. The design diagram of the normal section of flexural RCS of rectangular profile with a single reinforcement [4]

The algorithm for solving the strength problem by the method [4] is as follows:

- calculate the value of the coefficient k and the value of the relative

neutral axis depth $\xi = \dfrac{f_{yd} A_s}{f_{cd} bd}$ at rectangular distribution;

- according to received k and ξ in table A.5 find the values $\bar{\alpha}_m, \bar{\xi}, \bar{\zeta}$;
- determine by the table A.4 $\bar{\xi}_R$;
- verify the fulfillment of the condition $\bar{\xi} \le \bar{\xi}_R$, which ensures that the stress in the reinforcement is equal to yield strength;
- determine M_{Rd} according to the formulas

$$M_{Rd} = f_{cd} b \bar{\alpha}_m d^2,$$

$$M_{Rd} = f_{yd} A_s \bar{\zeta} d;$$

- check the condition $M_{Ed} \le M_{Rd}$, for the fulfillment of which the bearing capacity is provided.

If the stress in the reinforcement reaches the yield strength, the following algorithm can be used to determine A_s according to [4]:

- calculate the value of the coefficient K;
- determine $\bar{\alpha}_m = \dfrac{M_{Ed}}{f_{cd} bd^2}$,
- according to the table A.4 take value $\bar{\alpha}_R$;
- check the condition $\bar{\alpha}_m \le \bar{\alpha}_R$;

- in case of its implementation in tab. A.5 by known k and $\bar{\alpha}_m$ determine $\bar{\zeta}$;

- calculate the required area of the principle longitudinal reinforcement $A_s = \dfrac{M_{Ed}}{f_{yd}\bar{\zeta}d}$;

- carry out the detailing the section.

7.1.2.3. Task №. 1. Determination of the bearing capacity of a reinforced concrete beam with a single reinforcement

Given: a beam on two supports is loaded with a uniformly distributed load $g+v$ with a design span of l_{eff} = 6 m; it has a cross-sectional dimension of bxh = 200x400 mm and reinforced with 3∅18 A400C (A_s = 763 mm^2). The beam is made of heavy concrete of class C16/20.

Let's determine the bearing capacity of the beam.

Solve the problem of the first type. Determine the design characteristics of concrete and reinforcement: f_{cd} = 10.5 MPa (at γ_{c2}= 0.9), E_{cd} = 20x10^3 MPa, $\varepsilon_{c1,cd}$ = 1.62‰=162x10^{-5} (tab. A.1 [4]), f_{yd} = 364 MPa (tab. A.2 [4]).

Calculate the effective depth of the section (take the concrete cover c = 20 mm) $d=h-c-\varnothing/2 = 400-20-18/2 = 371$ mm.

Find $K = 1.05\,E_{cd}\varepsilon_{c1,cd} / f_{cd} = 1.05\times 20\times 10^3\times 162\times 10^{-5}/10.5 = 3.24$,

and value of relative neutral axis depth $\xi = \dfrac{f_{yd}A_s}{f_{cd}bd} = \dfrac{364\times 763}{10.5\times 200\times 371} = 0.356$

According to the values of K and ξ in tab. A.5 [4] find the values $\bar{\xi}$ = 0.437.

Determine the value of relative ultimate neutral axis depth $\bar{\xi}_R$= 0.553 (tab. A.4 [4]).

Check the condition $\bar{\xi} = 0.437 \le \bar{\xi}_R = 0.553$.

According to the values of K and ξ in tab. A.5 [4] find the values $\bar{\alpha}_m = 0.289$.

Determine bearing capacity M_{Rd} by the formula

$M_{Rd} = f_{cd}b\bar{\alpha}_m d^2 = 10.5\times 200\times 0.289\text{x}371^2$ = 83.53 kNm.

The load on the beam, at which its bearing capacity will be preserved, is: $g + v = 8M / l_0^2 = 8\times 83.53/ 6^2$ = 18.56 kN /m (fig. 7.8).

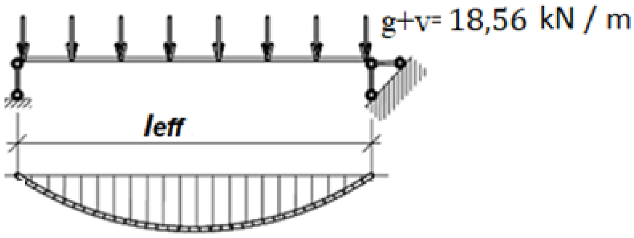


Fig. 7.8. The design diagram of the beam

7.1.2.4. Task №. 2 Determination of the area of the longitudinal principal reinforcement of a reinforced concrete beam of a rectangular profile with single reinforcement

Given: a beam with a span of l_{eff} = 6 m freely lies on two supports, the maximum bending moment that occurs in the beam from the action of an external load is M_{Ed} = 125 kNm. A beam with a section of $b \times h$ =200×550 mm is made of heavy concrete of class C16/20, γ_{c2}= 0.9, reinforcement A240C.

Determine the area of longitudinal principal reinforcement A_s.

Determine the design characteristics of concrete and reinforcement: f_{cd} = 10.35 MPa (at γ_{c2} = 0.9), E_{cd} = 20x10^3 MPa, $\varepsilon_{c1,cd}$ = 1.62‰ = 162×10^{-5} (tab. A.1 [4]), f_{yd} = 229 MPa (tab. A.2 [4]).

Find the effective depth of the section, preliminarily asking the diameter of the longitudinal principal reinforcement 20 mm:

$d = h - c - \varnothing / 2 = 550 - 20 - 20/2 = 520$ mm.

Counting $k = 1.05\, E_{cd}\varepsilon_{c1,cd} \;/\; f_{cd} = 1.05 \times 20 \times 10^3 \times 162 \times 10^{-5} / 10.5 = 3.24$,

$$\bar{\alpha}_m = \frac{M_{Ed}}{f_{cd} b d^2} = \frac{125 \times 10^6}{10.5 \times 200 \times 520^2} = 0.22 .$$

Determine the value $\bar{\alpha}_R = 0.384$ (tab. A.4 [4]) and check the condition $\bar{\alpha}_m = 0.22 \le \bar{\alpha}_R = 0.384$, since it is satisfied, we find it by the known K and $\bar{\alpha}_m$ the $\bar{\zeta} = 0.867$.

Calculate the required area of the principal longitudinal reinforcement $A_s = \frac{M_{Ed}}{f_{yd} \bar{\zeta} d} = \frac{125 \times 10^6}{229 \times 0.867 \times 520} = 1210.74$ mm^2. Select according to the tab. A.6 [4] 4∅20 (1257 mm^2) (fig, 7.9).

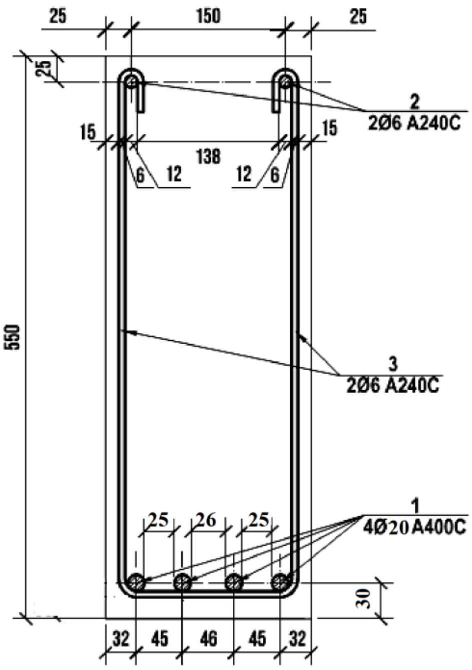


Fig. 7.9. Beam cross-section reinforcement

7.1.2.5. Calculation of the bearing capacity of reinforced concrete members of a rectangular profile with double reinforcement

Members with double reinforcement are called members in which, in addition to the principal tensile reinforcement A_s, the design reinforcement is installed in the compressed zone A_s' (fig. 7.10). Such sections are characterized by an increased cost of steel, so their use should be economically justified.

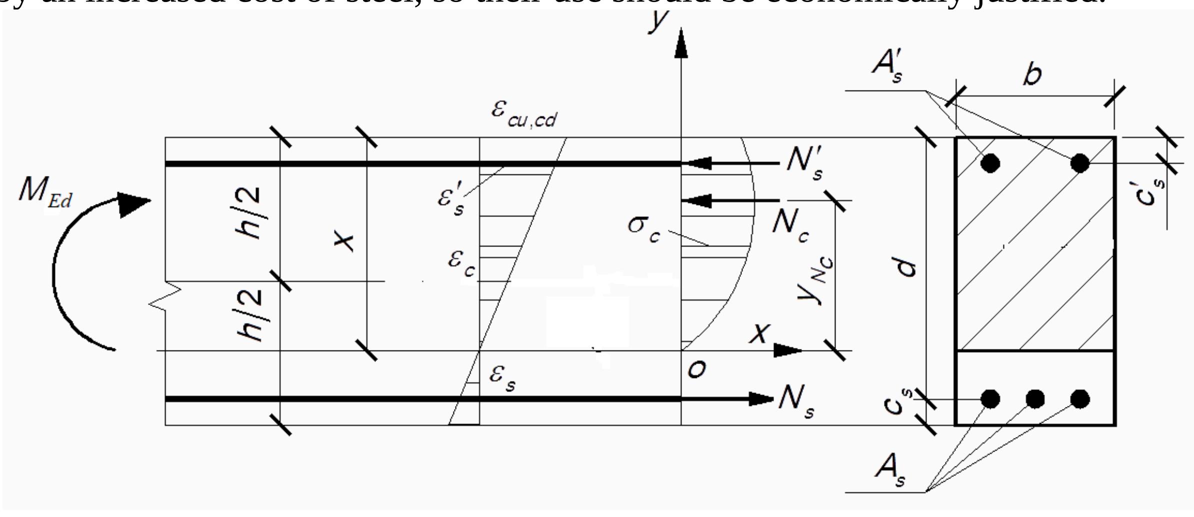


Fig. 7.10. Design scheme of the normal section of flexural RCS of rectangular profile with double reinforcement

Compressed reinforcement is installed by calculation in the case when the condition $\bar{\xi} \leq \bar{\xi}_R$ ($\bar{\alpha}_m \leq \bar{\alpha}_R$) is not fulfilled (i.e., the strength of the concrete in the compressed zone is insufficient to take up the bending moment from the external load), when increasing the effective depth of the cross section is not practical according to architectural requirements, and increasing the class of concrete is impractical for economic and technological reasons. Compressed reinforcement is also installed when elements are subjected to bending moments of two signs (continuous beams, frames girders, etc.).

Reinforcement in a compressed zone is used within the limits of the possible deformability of concrete $\varepsilon_s = \dfrac{\varepsilon_{cu}(\bar{\xi}_R - c_s'/d)}{\bar{\xi}_R} \le \varepsilon_{s0}'$, $\varepsilon_{cu} = \varepsilon_{c1,cd}\eta_u$.

When determining the required area of reinforcement (a task of the second type), use this method [4]:

- count K;
- find $\bar{\alpha}_m = \dfrac{M_{Ed}}{f_{cd}bd^2}$ (at first it is assumed that the element has a single reinforcement);
- according to the tab. A.4 choose $\bar{\alpha}_R$ and $\bar{\xi}_R$;
- check the condition $\bar{\alpha}_m \le \bar{\alpha}_R$;
- if the condition is not fulfilled, the area of the necessary compressed reinforcement $A_s' = \dfrac{M_{Ed} - \alpha_R f_{cd}bd^2}{\varepsilon_s' E_s'(d - c_s')}$ and tensile reinforcement $A_s = \bar{\xi}_R \omega bd \dfrac{f_{cd}}{f_{yd}} + A_s'$ are successively calculated;
- make the detailing of the section.

7.1.2.6. Task № 3. Determination of the area of the longitudinal principal reinforcement of a reinforced concrete beam of a rectangular profile with double reinforcement

Given: a beam of rectangular cross section $b\times h$ = 300×800 mm is made of heavy concrete of class C16/20, reinforcement is made of steel of class A400C with a concrete cover $c_s = c + Ø/2$ = 90 mm. The maximum bending moment in the middle of the beam span from an external load is 780 kNm. It is necessary to determine the area of the longitudinal principal reinforcement.

Determine the strength characteristics of concrete and reinforcement: f_{cd} = 10.5 MPa (γ_{c2}= 0.9), E_{cd} = 20x10^3 MPa, $\varepsilon_{c1,cd}$ = 1.62‰ = 162x10^{-5} (tab. A.1 [4]), f_{yd} = 364 MPa, E_s' = 2,1x10^5 MPa (tab. A.2 [4]).

Calculate the effective depth of the beam section $d = h - c_s =$ 800-90 = 710 mm and $\bar{\alpha}_m = \dfrac{M_{Ed}}{f_{cd}bd^2} = \dfrac{780\times10^6}{10.5\times300\times710^2} = 0,491$.

Find $K = 1.05\,E_{cd}\varepsilon_{c1,cd}\,/\,f_{cd} = 1.05\times20\times10^3\times162\times10^{-5}/10.5 = 3.24$.

Determine $\bar{\alpha}_R$ = 0,34 by the tab. A.4 [4] and compare $\bar{\alpha}_m$ = 0.49 > $\bar{\alpha}_R$ = 0.34. Based on the results of the comparison, we conclude that it is necessary to install longitudinal principal reinforcement in the compressed zone. Its area is

$$A_s{}' = \frac{M_{Ed} - \bar{\alpha}_R f_{cd} bd}{\varepsilon_s{}' E_s{}' (d - c_s{}')} = \frac{780 \times 10^6 - 0.34 \times 10.5 \times 300 \times 710^2}{173 \times 10^{-5} \times 2.1 \times 10^5 (710 - 30)} = 971.92 \text{ mm}^2,$$

where $\varepsilon_s' = \dfrac{\varepsilon_{cu}(\bar{\xi}_R - c_s{}'/d)}{\bar{\xi}_R}$,

$$\varepsilon_s' = \frac{218.9 \times 10^{-5}(0.553 - 30/710)}{0.553} = 202.2 \times 10^{-5} \ ,$$

where $\varepsilon_{cu} = \varepsilon_{c1,cd} \eta_u$ =162x10^{-5}x1.351=218.9x10^{-5},

here η_u = 1.351 (tab. A.3 [4]), $\bar{\xi}_R$ = 0.553.

Check the condition $\varepsilon_s \leq \varepsilon_{s0}{}'$, here $\varepsilon_{s0}{}' = \dfrac{f_{yd}}{E_s}$ = 364/(2.1x10^{-5}) = 173x10^{-5}.

$\varepsilon_s' =$ 202.2x10^{-5} $\geq \varepsilon_{s0}{}' =$ 173x10^{-5} , the condition is not met, accept $\varepsilon_s' = 173 \times 10^{-5}$.

Recommended diameter of compressed reinforcement is 12 mm or more, a concrete cover is equal to $c_s{}'$ = 30 mm. Accept the tab. A.6 [4] 4∅18 A400C (A_s = 1018 mm^2).

Determine the area of principal tensile reinforcement

$$A_s = \bar{\xi}_R \omega bd \frac{f_{cd}}{f_{yd}} + A_s{}' = 0.553 \times 0.801 \times 300 \times 710 \frac{10.5}{364} + 1018 = 3740 \text{ mm}^2.$$

Select according to the tab. A.6 [4] 8∅25 (A_s = 3927 mm^2).

Make the detailing of the section (fig. 7.11).

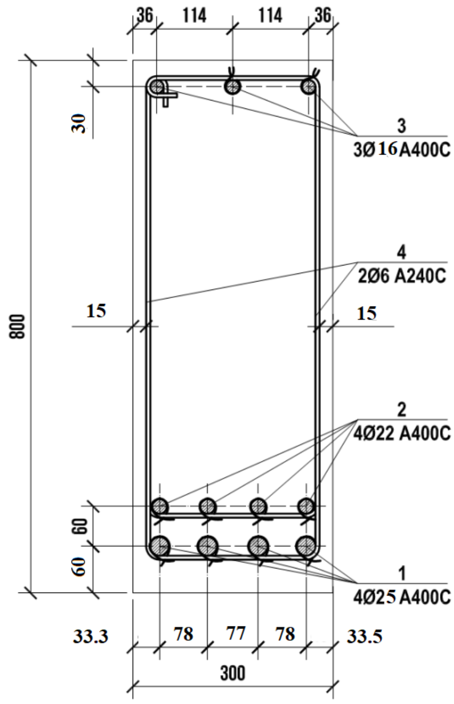


Fig. 7.11. Beam cross-section reinforcement

7.1.2.7. Calculation of the bearing capacity of reinforced concrete members of the T-profile

In practice, flexural members of the T-profile, I-profile, box-shaped and other sections with a flange in the compressed zone are widely used. Cast-in-place beam and girder structures (secondary and main beams) have such sections. Compared to members of a rectangular section, T-profiles with a flange in a compressed zone are more rational, since they have smaller area of tensile concrete that does not work. The flange in the tensile zone (for I-sections) does not affect the strength of the member.

In a compressed flange, the stress with distance from the rib decreases. On the basis of experiments and the practice of using reinforced concrete structures for sections with a flange in the compressed zone, it was found that the width of the overhangs of the flange to each side of the rib, which is taken into account, should not exceed $b_{eff,i} = 0.2b_1 + 0.1l_0 \le 0.2l_0$ and $b_{eff,i} \le b_i$. The effective width of the flange must be taken into account at a distance l_0 between the points of the beam with zero moments, which can be approximately determined from fig. 7.12, the length of the cantilever l_3 should not exceed half of the adjacent span.

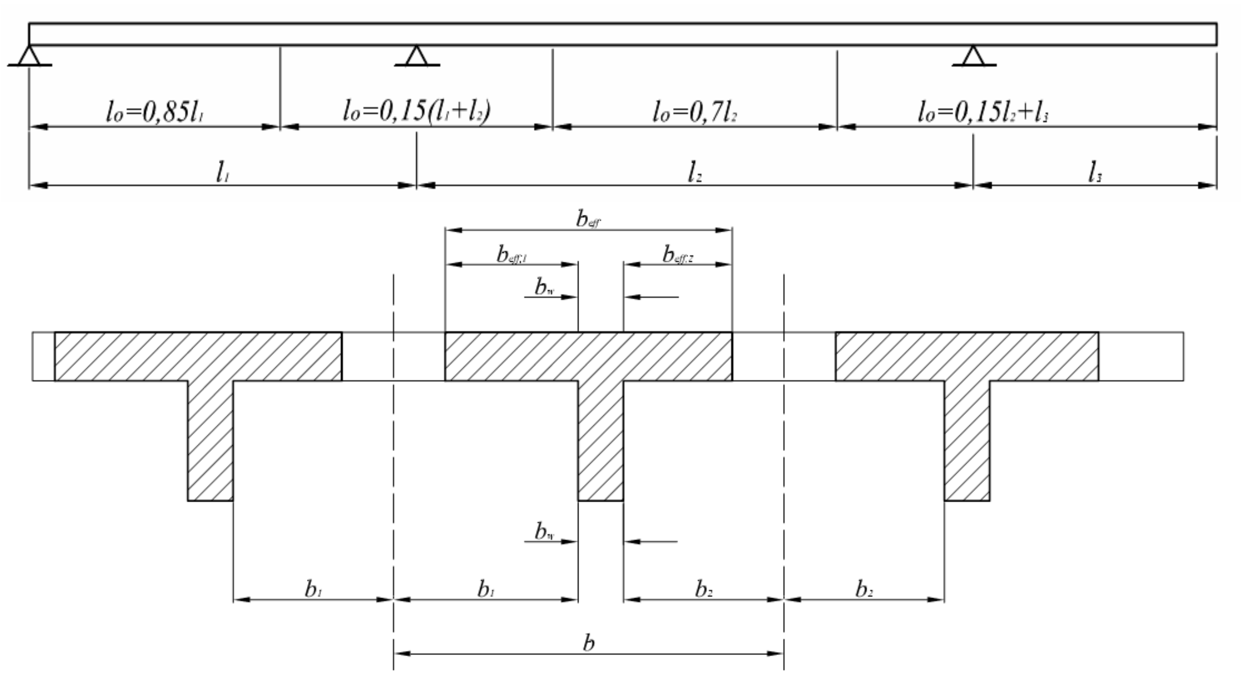
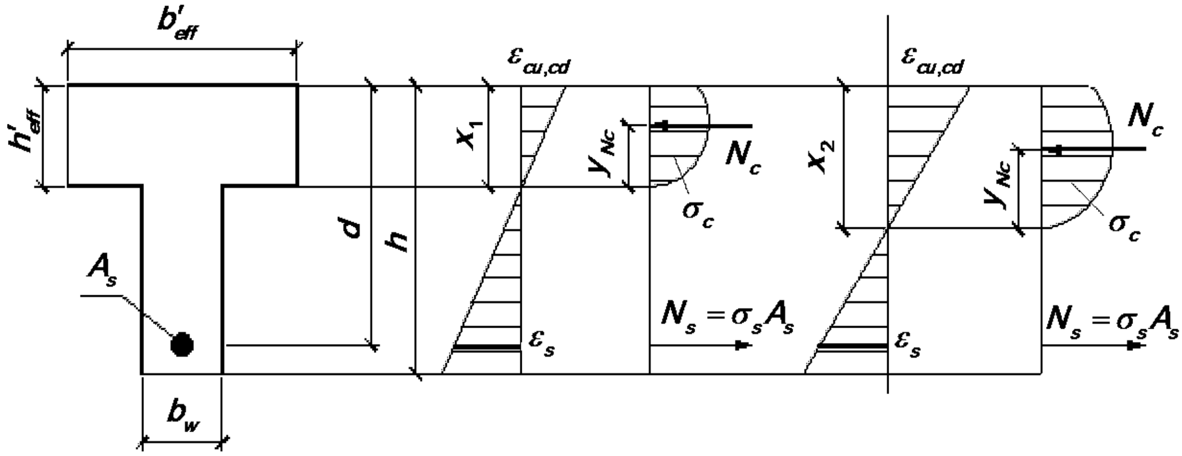


Fig. 7.12. To the determination of the width of the T-section

T-sections with a flange in a compressed zone, as a rule, are designed without compressed reinforcement. The calculation is carried out depending on the position of the neutral axis of the compressed zone in the cross section (fig. 7.13).

Case 1. The neutral axis of the compressed zone is within the flange: $x \le h'_{eff}$. This can be realized in sections with a well-developed flange (precast floor slabs, beams of cast-in-place beam and girder structures). Such sections are calculated as rectangular with the replacement in the corresponding formulas of the section width *b* by $b_{eff}{}'$.



Fig, 7.13 Cases of calculation of the T-section: a – the boundary of the compressed zone passes in the flange; b – in the rib

Case 2. The neutral axis of the compressed zone passes in the rib $x > h'_{eff}$. This option is found in sections with a poorly developed flange (precast beams of floors and roofs, crane beams). In this case, the calculation formulas are significantly complicated.

Cases of calculating T-profile are distinguished by the following qualities:

- when control the strength of the member in the normal section, the neutral axis passes in the flange, if the forces in the tensile reinforcement are less than or equal to the force that the flange can fully accept

$$f_{yd} A_s \le f_{cd} b_{eff}{}' h_{eff}{}' \omega . \tag{7.12}$$

If condition (7.12) is not satisfied, the neutral axis is located in the rib;

- when calculating the required area of the principal longitudinal reinforcement, first determine the moment of internal forces M'_{eff} about axis passing through the center of gravity of the tensile reinforcement, in case when the entire flange is compressed and the neutral axis passes along the lower edge, and then compare M'_{eff} with the moment from the external load. The neutral axis passes in the flange, if the condition is met $M_{Ed} \le M'_{eff}$,

$$M_{eff}{}' = f_{cd} b_{eff}{}' h_{eff}{}' \omega (d - \chi h_{eff}{}' \omega). \tag{7.13}$$

The parameters ω and χ are determined depending on *k*.

7.1.2.8. Task № 4. Determination of the area of the principal reinforcement in the reinforced concrete beam of the T-profile

Given: reinforced concrete beam of T-shaped cross-section with dimensions b'_{eff} = 1500 mm , h'_{eff} = 50 mm , b = 200 mm , h = 400 mm is made of concrete of class C20/25. Maximum bending moment from an external load in the middle of span is M_{Ed} = 150 kNm.

Determine the area of longitudinal principal reinforcement of class A400C.

Determine the strength characteristics of concrete and reinforcement: f_{cd} = 13 MPa (γ_{c2} = 0.9), E_{cd} = 23x10^3 MPa, $\varepsilon_{c1,cd}$ = 1.65‰ = 165x10^{-5} (tab. A.1 [4], f_{yd} = 364 MPa, E_s = 2.1x10^5 MPa (tab. A.2 [4]).

Calculate

$$K = 1.05\,E_{cd}\varepsilon_{c1,cd}\,/\,f_{cd} = 1.05\times 23\times 10^3\times 165\times 10^{-5}/13 = 3.6, \chi = 0.526, \omega = 0.792.$$

The moment, which can be taken by a completely compressed flange, is equal to

$$M'_{eff} = f_{cd}b'_{eff}\,h'_{eff}\omega\left(d-\omega\chi h'_{eff}\right) = 13\times 1500\times 50\times 0.792(350-0.526\times 0.792\times 50) =$$

2541853829 Nmm = 254.18 kNm.

Effective section depth is $d = h - c_s =$ 400 – 50 = 350 mm.

Check the fulfillment of the condition $M_{Ed} \le M'_{eff}$ (150 kNm < 254.18 kNm) and draw a conclusion: the neutral axis passes within the flange, calculate the longitudinal reinforcement area as for a rectangular section with b replaced by b'_{eff} in the formulas.

Find $\bar{\alpha}_m = \dfrac{M_{Ed}}{f_{cd}b'_{eff}d^2} = \dfrac{150\times 10^6}{13\times 1500\times 350^2} = 0.063$.

Determine the value $\bar{\alpha}_R = 0.338$ according to the tab. A.3 [4] and check the condition $\bar{\alpha}_m = 0.063 \le \bar{\alpha}_R = 0.338$, since it is fulfilled by the known K and $\bar{\alpha}_m$ find $\bar{\zeta} = 0.968$.

Calculate the required area of the principal longitudinal reinforcement

$A_s = \dfrac{M_{Ed}}{f_{yd}\bar{\zeta}d} = \dfrac{150\times 10^6}{364\times 0.968\times 350} = 1216.32$ m^2. Select according to the tab. A.6 [4] 4∅20 (A_s = 1257 mm^2).

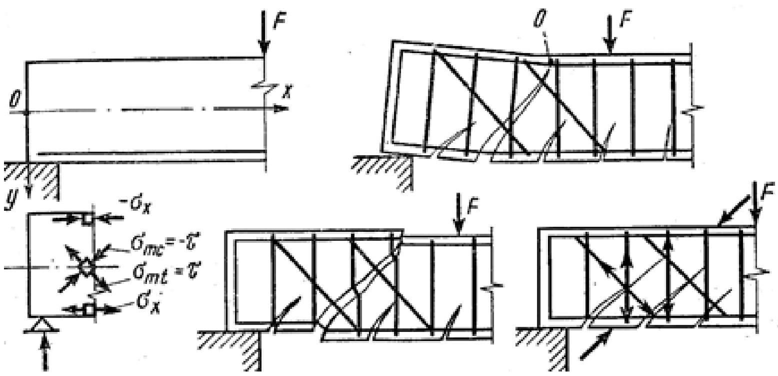
7.1.3. Calculation of the bearing capacity of reinforced concrete members on an inclined section

7.1.3.1 Possible cases of destruction along an inclined section

For reinforced concrete members, the main tensile stress σ_{mt} is of particularly danger; they acquire maximum value near the supports (the maximum shear forces act here) at the level of the neutral axis. If σ_{mt} exceed the design concrete tensile strength, they cause inclined cracks. After their formation, the member is divided into two disks (parts), which in a compressed zone above the crack are interconnected by concrete, and in the tensile zone disks are interconnected by longitudinal, shear and inclined reinforcement crossing the crack. With an increase in load, one of these destruction schemes is possible (fig. 7.14):

- from the overwhelming effect of the bending moment, both parts of the member rotate about the center of gravity of the compressed concrete zone above the inclined crack. While it opens, develops in height, and the height of the compressed concrete zone decreases. When the stress in the entire reinforcement that crosses the crack has reached their limit values (the yield of reinforcement), compressed concrete is crushed and the element is destroyed. This destruction is in nature like the destruction of a normal section from the action of a bending moment. The beam can be destroyed according to this scheme even when the stress in the longitudinal reinforcement is less than the limit, but its anchoring is violated and the reinforcement slips in the concrete;

a *b*



c *d*

Fig. 7.14. Schemes of action of the main stresses (a) and destruction of flexural members in an inclined section (b – d)

- if the cross section of the longitudinal reinforcement is large enough and its anchoring is reliable, which prevents the rotation of both parts of the member, the failure occurs after the limits stress in the shear and inclined reinforcement that crosses the crack, are reached due to shearing of the concrete over the inclined crack; both parts of the member are shift relative to each other. This nature of failure is associated with the predominant shear force;
- when the width of the cross-section of the flexural members (T-beams, I-beams) is quite small, they can be destroyed in the zone of shear forces due to crushing of the concrete rib between inclined cracks from the action of the main compressive stress σ_{mc}.

Although the destruction of flexural members along inclined sections is a consequence of the combined action of the bending moment *M* and the shear force *V*, such sections, in accordance with the listed possible failure schemes, have until recently been calculated for strength separately: for the action of *V* along an inclined crack and along an inclined compressed strip and for the action of M along an inclined crack.

7.1.3.2. Calculation of the bearing capacity of reinforced concrete members along sections inclined to the longitudinal axis

For members that do not require the calculation of shear reinforcement, the calculated value of shear resistance is defined as

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} \right] b_w d \, , \tag{7.14}$$

but should not be less

$$V_{Rd,c} = V_{\min} b_w d \, , \tag{7.15}$$

where $\rho_1 = \dfrac{A_{s1}}{b_w d} \le 0.02$ is the ratio of longitudinal reinforcement;

A_{s1} – the area of the tensile reinforcement, which is wound up at a distance $\ge (l_{bd} + d)$ beyond the section that is considered;

b_w – the smallest cross-sectional width of the member in the tensile zone; it is recommended to take

$$C_{Rd,c} = 0.18 / \gamma_c \, , \tag{7.16}$$

γ_c = 1.3 – coefficient of reliability for concrete in compression;

$$V_{\min} = 0.035 k^{3/2} f_{ck}^{1/2} \, , \tag{7.17}$$

$$k = 1 + \sqrt{\frac{200}{d}} \le 2.0 \, . \tag{7.18}$$

According to the norms, it is recommended to use the “truss” model (fig. 7.15), which is based on an analogy between a reinforced concrete element that works on the perception of shear forces and a truss. The truss top chord is a concrete of the compressed zone; the bottom chord of a truss is a tensile reinforcement. The chords are connected through the struts, where the tensile elements are represented by shear reinforcement, and compressed is imaginary concrete struts, the angle of which can vary. The following notation is used:

α is the angle between the shear reinforcement and the beam axis perpendicular to the shear force;

θ is the angle between the compressed concrete strut and the longitudinal axis of the beam (its value should be limited by the condition $1 \leq \cot\theta \leq 2.5$);

N_{td} – is the design value of the tensile force in the longitudinal reinforcement; N_{cd} – the design value of the compressive force of concrete in the direction of the longitudinal axis of the element;

z – is the inner lever arm, for a member with constant depth, corresponding to the bending moment in the member under consideration. In the shear analysis of reinforced concrete without axial force, the approximate value $z = 0.9d$ may normally be used.

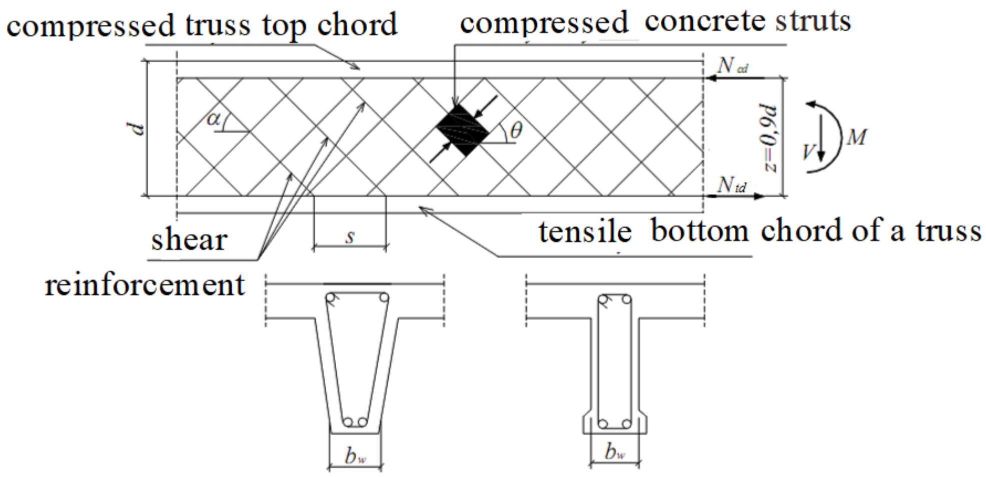


Fig. 7.15. “Truss analogy” for calculating flexural members in an inclined section

For members with vertical shear reinforcement, the smaller of the values should be taken as the shear resistance:

- shear force that shear reinforcement can accept, if the stress in it reach the yield strength

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta ; \tag{7.19}$$

- maximum shear force that can be accepted by the struts

$$V_{Rd,\max} = b_w z \nu_1 f_{cd} / (\cot\theta + \tan\theta) , \tag{7.20}$$

here ν_1 is a strength reduction factor for concrete cracked in shear

$$\nu_1 = 0.6\left[1 - \frac{f_{ck}}{250}\right]. \tag{7.21}$$

If the value of the design stress in the shear reinforcement is less than 0.8 f_{yk}, then it be taken ν_1 = 0.6 at $f_{ck} \leq 60$ MPa and $\nu_1 = 0.9 - f_{ck}/200 > 0.5$ at $f_{ck} \geq 60$ MPa.

For members with an inclined shear reinforcement, the smallest of the following values is accepted as shear resistance

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} (\cot\theta + \cot\alpha)\sin\alpha\,, \tag{7.22}$$

$$V_{Rd,\max} = b_w z \nu_1 f_{cd} \,/\, (\cot\theta + \cot\alpha)(1 + \cot^2\theta)\,. \tag{7.23}$$

The maximum area of the shear reinforcement at $\cot\theta = 1$ should be determined as $\frac{A_{sw,\max} f_{ywd}}{b_w s} \leq \frac{1}{2}\nu_1 f_{cd}$ or $\frac{A_{sw,\max} f_{ywd}}{b_w s} \leq \frac{1}{2\sin\alpha}\nu_1^2 f_{cd}^{\ 2}$.

The maximum shear force provided that the concrete strength is fully used will be achieved provided $V_{Rd,s} = V_{Rd,\max}$. Taking into account the dependences $\sin 2\theta = \frac{2tg\theta}{1 + tg^2\theta}$ and $1 + ctg^2\theta = \frac{1}{\sin^2\theta}$ an expression is obtained for determining the angle θ with a known value of shear force

$$\theta = \frac{1}{2}\arcsin\frac{2V_{Ed}}{z b_w \nu_1 f_{cd}}\,, \tag{7.24}$$

or with known area of shear reinforcement

$$\theta = \arcsin\sqrt{\frac{A_{sw} f_{ywd}}{b_w s \nu_1 f_{cd}}}\,. \tag{7.25}$$

The minimum coefficient of shear reinforcement is determined by the expression $\rho_{w,\min} = 0.08\sqrt{f_{ck}} \,/\, f_{yk}$, and the maximum step $s_{l,\max} = 0.75d(1 + \cot\alpha)$.

The slab in which there is the shear reinforcement should have a thickness of more than 200 mm.

7.1.3.3. Task № 5. Calculation of flexural members in inclined sections to the action of shear force

Given: reinforced concrete beam with a cross-section $b \times h$ = 250×600 mm is made of concrete of class C25/30, reinforcement 2Ø36 A400C ($A_s = 2036$ мм^2) is used for longitudinal principal reinforcement, effective section depth is d = 560 mm, the beam is loaded with a uniformly distributed load $g + v$ = 60 + 40 = 100 kN /m. Design span is 5.2 m.

Perform the calculation of the shear reinforcement and check the strength of the beam in inclined section. Maximum shear force at support is V_{Ed} = 260 kN.

Concrete strength characteristics: f_{cd} = 15.5 MPa (γ_{c2} = 0.9), $f_{ck} = 18.5$ MPa, $f_{ctk,0.05}$ = 1.8 MPa, $f_{ctd} = f_{ctk,0.05} / \gamma_{ct} = 1.2$ MPa (tab. A.1 [4]).

Check the need for the installation of shear reinforcement by calculation. The design shear resistance is defined as

$$V_{Rd,c} = \left[C_{Rd,c}k(100\rho_1 f_{ck})^{1/3}\right]b_w d = \left[0.138\times1.6(100\times0.0145\times18.5)^{1/3}\right]\times250\times560 =$$

92535,2N = 92.5 kN,

$$\rho_1 = \frac{A_{s1}}{b_w d} = \frac{2036}{250\times560} = 0.0145 \le 0.02$$

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.3 = 0.138,$$

$$k = 1+\sqrt{\frac{200}{d}} = 1+\sqrt{\frac{200}{560}} = 1.6 \le 2.0 .$$

$V_{Rd,c}$ should be at least $V_{Rd,c} = V_{min} b_w d = 0.304\times250\times560 =$ 42560 N = 42.56 kN, here $V_{min} = 0.035k^{3/2} f_{ck}^{1/2} = 0.035\times1.6^{3/2}\times18.5^{1/2} = 0.304$ MPa.

Check the condition: $V_{Ed} \le V_{Rd,c}$: 260 kN ≥ 92.5 kN. The condition is not satisfied and the shear reinforcement is necessary for the calculation.

Determine its area.

First, determine the angle of inclination of the compressed concrete strut,

$$\theta = \frac{1}{2}\arcsin\frac{2V_{Ed}}{zb_w v_1 f_{cd}} = \frac{1}{2}\arcsin\frac{2\times260\times10^3}{504\times250\times0.56\times15.5} = 14.19'$$

here $z = 0.9d = 0.9\times560 = 504$ mm, and

$$v_1 = 0.6\left[1-\frac{f_{ck}}{250}\right] = 0.6\left[1-\frac{18.5}{250}\right] = 0.56 .$$

Taking into account the restriction that is set at this angle: 21.8' ≤ θ ≤ 2.5, take θ = 21.8'.

Use 2Ø8 A240C as shear reinforcement (f_{ywd} = 170 MPa, tab. A.2 [4]) and calculate its step $s = \dfrac{A_{sw} z f_{ewd}\cot\theta}{V} = \dfrac{101\times504\times170\times2.5}{260\times10^3}$ =83.21 mm.

Check the condition $s \le s_{\max} = s_{l,\max} = 0.75d = 0,75 \times 560 = 420$ mm (we take a step of 80 mm).

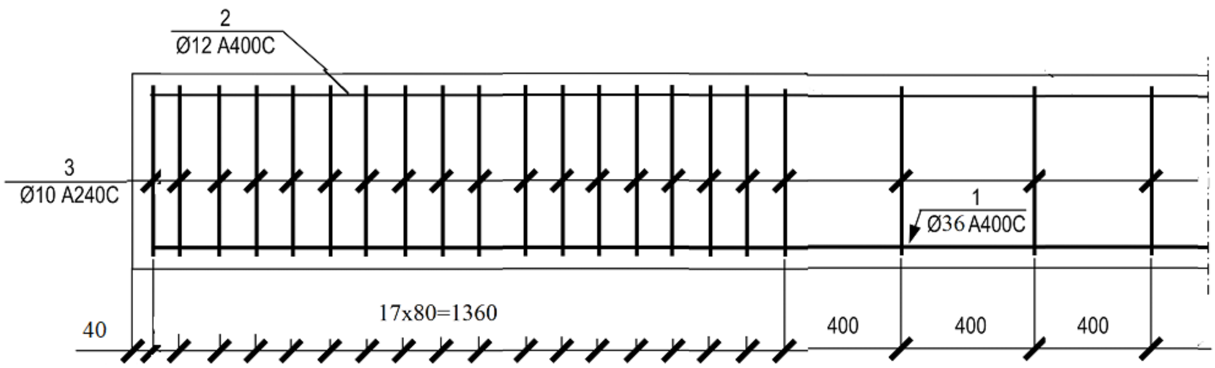


Fig. 7.16. Diagram of shear reinforcement of a beam

7.2 Compressed reinforced concrete members

7.2.1 Features of its design

The standards for the design of reinforced concrete structures [] give general conditions for the equilibrium of elements of a rectangular section under axial compression and bending, considering two forms of equilibrium: the entire section is compressed; in section there is the tensile zone. For compressed RCS, they are shown in fig. 7.17. The case is considered when the diagram of the mechanical state of concrete is given in the form of a polynomial and the section is reinforced with bars without pre-stressing.

For the first form, the equilibrium equations take the form

$$F\left(\aleph, \varepsilon_{cu,cd(1)}\right) - N_{Ed} = 0;$$

$$\frac{bf_{cd}}{\aleph} \sum_{n=1}^{5} \frac{a_n}{n+1} \frac{\varepsilon_{cu,cd(1)}^{n+1} - \varepsilon_{cu,cd(2)}^{n+1}}{\varepsilon_{cu,cd(1)}^{n+1}} + \sum_{i=1}^{n} \sigma_{si} A_{si} - N_{Ed} = 0, \tag{7.26}$$

$$\Phi(\aleph, \varepsilon_{cu,cd(1)}) - M_{Ed} = 0$$

$$\frac{bf_{cd}}{\aleph^2} \sum_{n=1}^{5} \frac{a_n}{n+2} \frac{\varepsilon_{cu,cd(1)}^{n+2} - \varepsilon_{cu,cd(2)}^{n+2}}{\varepsilon_{cu,cd(1)}^{n+1}} + \sum_{i=1}^{n} \sigma_{si} A_{si} (x_1 - z_{si}) - N\,(x_1 - y + e) = 0, \tag{7.27}$$

for the second form, the equilibrium equations take the form

$$\frac{bf_{cd}}{\aleph} \sum_{n=1}^{5} \frac{a_n}{n+1} \gamma^{n+2} + \sum_{i=1}^{n} \sigma_{si} A_{si} - N_{Ed} = 0, \tag{7.28}$$

$$\frac{bf_{cd}}{\aleph^2}\sum_{n=1}^{5}\frac{a_n}{n+2}\gamma^{k+2}+\sum_{i=1}^{n}\sigma_{si}A_{si}(x_1-z_{si})-N_{Ed}(x_1-y+e)=0 \quad , \qquad (7.29)$$

here $\aleph=\frac{1}{\rho}=\frac{\varepsilon_{cu,cd(1)}-\varepsilon_{cu,cd(2)}}{h}$ is the curvature;

$\varepsilon_{cu,cd(1)}$ is strain of concrete of most compressed fiber;

$\varepsilon_{cu,cd(2)}$ is strain of the most tensile fiber of concrete;

$\gamma=\frac{\varepsilon_{cu,cd(1)}}{\varepsilon_{cu,cd(2)}}$;

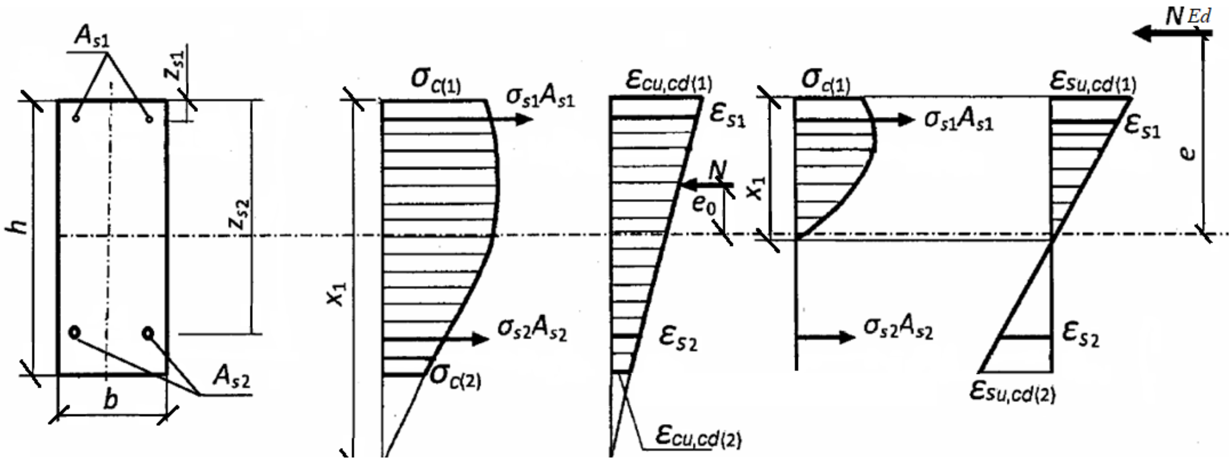
x_1 is neutral axis depth;

z_{si} is the distance between the *i*-th bar (or reinforcing layer) and the most compressed fiber;

e – the eccentricity of the application of an external force N_{Ed} about to the center of gravity of the section.

The stress in the reinforcement is determined by strain diagrams, based on the fact that $\varepsilon_{si}=\aleph x_1-z_{si}$.

Systems of two nonlinear equations with two unknowns are solved by selection with control of criteria for the exhaustion of the bearing capacity at each step of the calculations.



a *b*

Fig. 7.17. To the calculation of a rectangular section under eccentric compression: a – the first form of equilibrium; b – the second form of equilibrium

According to the results of solving the system of equations, the diagrams "normal force – strain of the most compressed concrete fiber" are constructed. The maximum values of *N* is taken as bearing capacity. If certain value of the bearing capacity will be less than external influences, it is necessary to perform a change in section size, reinforcement or concrete strength. The value of external influence and the design bearing capacity should not differ by more than 3%.

7.2.2. Constructional features

The cross-sectional shape of the compressed elements is most often square or rectangular, developed in the plane of action of the moment. With significant bending moments *M*, which act in one direction, it is advisable to take the cross section of precast concrete structures as T-shaped or I-shaped, its dimensions are determined by calculation, for columns they are taken in multiples of 50 mm.

Compressed elements operating under normal conditions are made of concrete of a class not lower than C12 /15, and high-load ones – C20 /25.

In columns, the diameter of the longitudinal bars is assigned at least 16 mm.

Longitudinal and transverse reinforcement are combined into flat or spatial welded or tied cages. In linear eccentrically compressed elements, the distances between axes of the bars of the longitudinal reinforcement should be taken: in the direction perpendicular to the bending plane is not more than 400 mm, and in the direction of the bending plane is 500 mm. If the distance between the axes of the principal bars in the direction of the bending plane exceeds 500 mm, constructive reinforcement with a diameter of at least 12 mm is installed so that between the longitudinal bars there is no more than 400 mm. The principal bars in the cross section of the columns are placed closer to the surface of the element, the minimum thickness of the concrete cover is always greater than the diameter and more than 20 mm (fig. 7.18).

By transverse reinforcement, the compressed bars are secured against loss of stability in any direction. Its diameter is taken not less than $\varnothing_{sw} = 0.25\varnothing$ ($\varnothing$ – the largest diameter of the longitudinal reinforcement and with tied cages not less than 5 mm). In the presence of longitudinal reinforcement, which is taken into account in the calculations, the ties are placed at a distance of not more than 500 mm, and also not more than $20\varnothing$ in welded cages and not more than $15\varnothing$ in tied.

If the extreme plane cages have intermediate longitudinal bars, then they are connected with hairpins through one and not less than 400 mm across the width of the face of the element with longitudinal bars located at the opposite side. A hairpin is not set when the width of the face of the element is ≤ 500 mm, and also if the number of longitudinal bars at this face does not exceed four. In the case of using ties, the longitudinal bars (at least one) are located at the intersection of the ties, and these sections are at a distance of not more than 400 mm along the width of the element face. For compressed elements, a reinforcement ratio of $\rho < 3\%$ is used, for eccentrically compressed elements, $\rho \approx$ 0.5 – 1.2%, the minimum percentage of reinforcement is accepted $\rho = 0.05$ – 0.25.

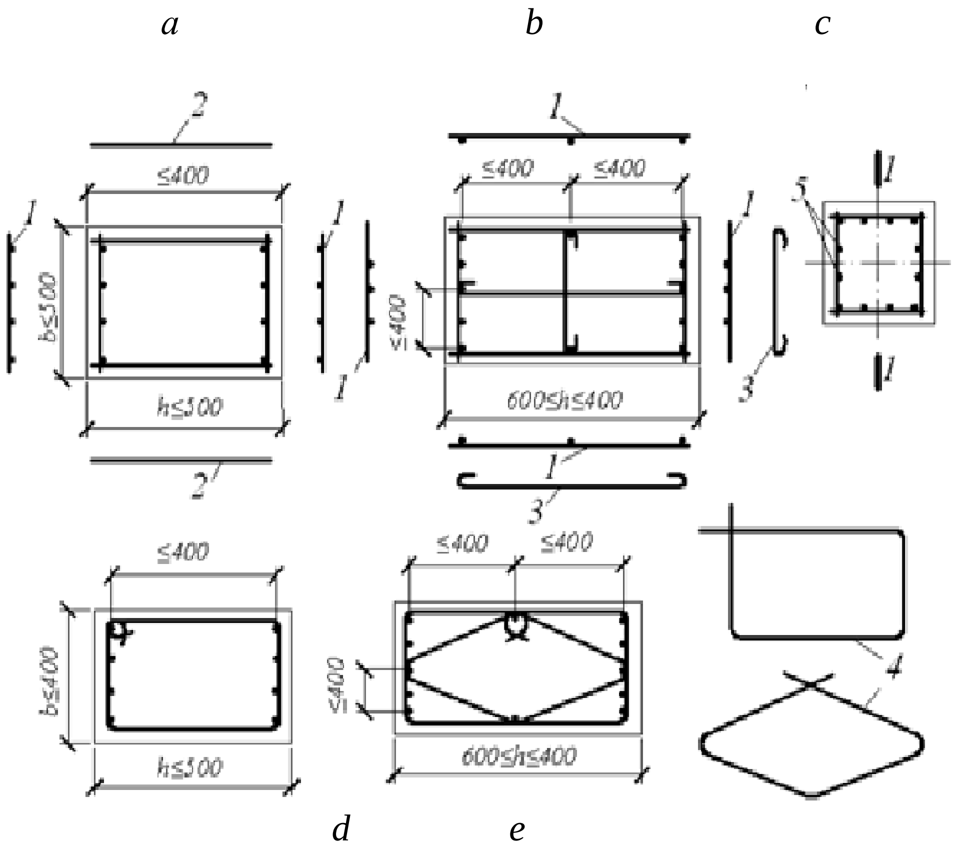


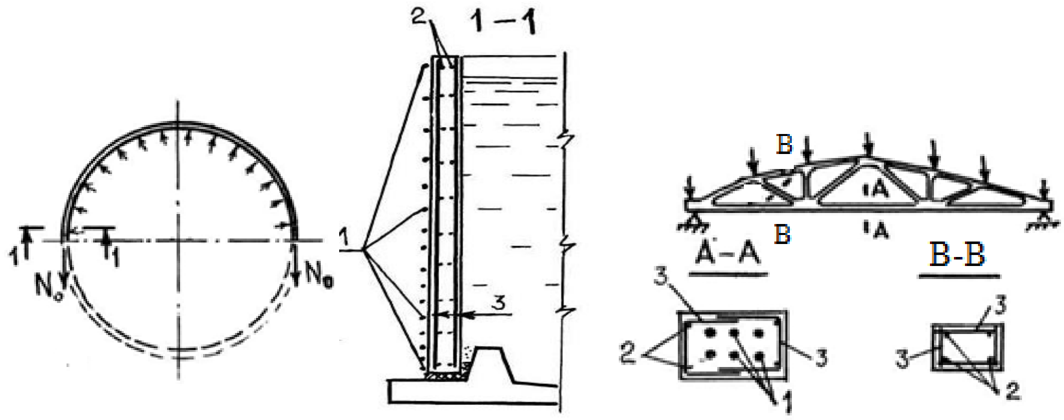
Fig. 7.18. Reinforcement of compressed elements: a, b, c – welded cages; d, e – tied; 1 – welded cages; 2 – connecting bars, 3 – hairpins; 1-1 – bending moment plane; 4 – closed stirrup

7.3. Tensile members

In tensile members (fig. 7.19), as a rule, a longitudinal force N and a bending moment M act simultaneously, and the elements work on eccentric tension with eccentricity $e_o=M/N$. At M = 0, the element operates under axial tension. All these members are pre-stressed, which significantly increases their crack resistance.

In the walls of tanks, silos (bunkers), pre-stressed reinforcement is placed outside and post-tensioned onto hardened concrete.

In the lower truss chords, tensile reinforcement is arranged symmetrically to prevent the element being squeezed in compression while transmitting the compression force. It is advisable to reinforce such structures with a small amount of non-pre-stressed reinforcement (Fig. 7.19), located closer to the outer surface.



a *b*

Fig. 7.19. Tensile reinforced concrete elements: a – walls of a cylindrical tank; b – the lower chord of segmental truss; 1, 2 – tensile reinforcement; 3 – constructive reinforcement

In the case of tensile members, three stages of the stress-strain state appear in them: I – before the formation of cracks in concrete; II – after formation of cracks in concrete; III – failure. At the time of failure, the tensile force N_{Rd} is transmitted only to the reinforcement, and concrete is excluded from work. The strength condition of the axially tensioned member is determined by the strength of the reinforcement

$$N_{Rd} \leq f_{pd} A_p + f_{yd} A_s \tag{7.30}$$

where f_{pd} and f_{yd} are the design resistance of the pre-stressed and non-pre-stressed longitudinal reinforcement, respectively;

and A_s are the area of their cross section.

As a rule, the second component of formula (7.30) is neglected.

7.4. Questions for knowledge control

1. What is $\xi = \frac{x}{d}$?

2. What is $\zeta = 1 - 0,5\xi$?

4. What is $\alpha_m = \xi(1 - 0,5\xi)$?

5. What model is the basis for calculating the strength of inclined sections of reinforced concrete elements?

6. What is single reinforcement?

7. How stress is distributed over the height of the compressed zone of concrete of a flexural member?

8. What ensures the fulfillment of the condition $x \leq h'_{eff}$?

9. What problem is solved when designing reinforced concrete structures?

10. What force the transverse reinforcement is calculated for?

11. What tasks are solved when operating reinforced concrete structures?

12. Which profile is more rational for flexural reinforced concrete members?

REFERENCES

1. DBN B.1.2-2: 2006. Reliability and safety system for construction projects. Loads and effects. Design standards/ Ministry of Regional Development and Construction of Ukraine. - Kyiv: Ministry of Construction of Ukraine, 2006. – 75 p (Ukr).

2. DBN B.1.2-14-2009. Reliability and safety system for construction projects. General principles of ensuring the reliability and structural safety of buildings, structures, building structures and foundations / Ministry of Regional Development of Ukraine. – K., 2009. – 30 p (Ukr).

3. DBN B.2.6-98: 2009. Structures of houses and buildings. Concrete and reinforced concrete structures. Basic provisions / Ministry of Regional Development of Ukraine. – K., 2011. – 71 p. (Ukr).

4. Pavlikov A.M. Calculation of strength of normal cross-sections of beam elements by nonlinear deformation model (based on DBN B.2.6-98: 2009): textbook / A.M. Pavlikov, O.V. Boyko. For order. A.M. Pavlikova. – Poltava: PoltNTU, 2012. – 85 p. (Ukr).

5. DSTU B B.2.6-156: 2010. Concrete and reinforced concrete structures made of heavy concrete. Design rules / Ministry of Regional Development and Construction of Ukraine. – K., 2011. – 118 p. (Ukr).

6. DBN B.2.6-162: 2010 Structures of houses and buildings. Masonry and reinforced masonry structures. Basic provisions/ Ministry of Regional Development of Ukraine. - K ., 2011.– 96 p. (Ukr).

7. European Standart prEN 1992-1-1:2004 Eurocode 2: Design of concrete structures – Part 1: General rules and rules for buildings. – 2012. – 311 p.

APPENDIX A

TABLES OF PARAMETER VALUES AND CHARACTERISTICS WHICH ARE USED IN CALCULATIONS

Table A.1 – Characteristics of strength and deformability of concrete

	Concrete strength class											Analytical dependence / explanation
	C8/10	C12/15	C16/20	C20/25	C25/30	C30/35	C32/40	C35/45	C40/50	C45/55	C50/60	
$f_{ck,cube}$ (MPa)	10	15	20	25	30	35	40	45	50	55	60	
$f_{cm,cube}$ (MPa)	13	19	25	32	38	45	51	58	64	71	77	$f_{cm,cube} = f_{cm,cube}/(1\text{-}1.64V_c)$
$f_{ck,prism}$ (MPa)	7.5	11	15	18,5	22	25.5	29	32	36	39.5	43	
f_{cd} (MPa)	6	8.5	11.5	14,5	17	19.5	22	25	27.5	30	33	$f_{cd} = f_{ck}/\gamma_c$
f_{ctm} (MPa)	1.2	1.6	1.9	2,2	2.6	2.8	3	3.2	3.5	3.8	4.1	
$f_{ctk\ 0.05}$ (MPa)	0.8	1.1	1.3	1,5	1.8	2	2.1	22	2.5	2.7	3	$f_{ctk,0.05}=0.7f_{ctm}$ 5% sample
$f_{ctk\ 0.95}$ (MPa)	1.6	2	2.5	2,9	3.4	3.6	3.9	4.2	4.6	4.9	5.3	$f_{ctk,0.95}=1.3f_{ctm}$ 95% sample
E_{cm} (ГPa)	18	23	27	30	32.5	34.5	36	37.5	39	39.5	40	
E_{ck} (ГPa)	15	20	23	26	29	31	32	34	35	36	37	
E_{cd} (ГPa)	12.6	16.3	20	23	25	27	28.5	30.5	32	33	34	
$\varepsilon_{c\,1,ck}$ (‰)	1.57	1.61	1.66	1.71	1.76	1.81	1.86	1.90	1.94	1.98	2.02	
$\varepsilon_{c\,1,cd}$ (‰)	1.56	1.58	1.62	1.65	1.69	1.72	1.76	1.80	1.84	1.87	1.91	
$\varepsilon_{cu\,1,ck}$ (‰)	4.5	4.4	4.15	3.85	3.55	3.25	3	2.83	2.63	2.5	2.4	
$\varepsilon_{cu\,1,cd}$ (‰)	3.75	3.7	3.59	3.44	3.28	3.1	2.93	2.72	2.57	2.43	2.29	
$\varepsilon_{c\,3,ck}$ (‰)	0.5	0.55	0.65	0.71	0.76	0.82	0.91	0.94	1.03	1.1	1.16	$\varepsilon_{c\,3,ck} = f_{ck,prism}/E_{ck}$
$\varepsilon_{c\,3,cd}$ (‰)	0.48	0.52	0.58	0.63	0.68	0.72	0.77	0.83	0.86	0.91	0.97	$\varepsilon_{c\,3,cd} = f_{cd}/E_{cd}$
$\varepsilon_{cu\,3,ck}$ (‰)	4.05	3.96	3.73	3.46	3.2	2.93	2.7	2.55	2.37	2.25	2.16	$\varepsilon_{cu\,3,ck} = 0{,}9\,\varepsilon_{cu\,1,ck}$
$\varepsilon_{cu\,3,cd}$ (‰)	3.38	3.33	3.23	3.1	3	2.8	2.64	2.45	2.31	2.19	2.06	$\varepsilon_{cu\,3,cd} = 0{,}9\,\varepsilon_{cu\,1,cd}$

*) - value $f_{ck,cube}$ in the table are based on the value of the coefficient of variation V_c, which is equal to 13,5 %

Table A.2 – Characteristics of strength and deformability of reinforcement

Characteristics of reinforcement	A240C	A400C	A500C		B500
			Ø8 – 22	Ø25 – 40	
f_{yk} (MPa)	240	400	500		500
γ_s	1.05	1.10	1.15	1.20	1.20
f_{yd} (MPa)	229	364	435	417	417
E_s (MPa)	2.1x10^5	2.1x10^5	2.1x10^5		1.9x10^5
ε_{ud}	0.025	0.025	0.020		0.012

Table A.3 – Parameter values ω, φ, χ depending on the values K and η_u

	K								
	1.18	1.5	2	2.5	3	3.5	4	4.5	5
η_u	1.075	1.2	1.268	1.309	1.339	1.363	1.382	1.398	1.412
ω	0.587	0.673	0.732	0.767	0.792	0.811	0.826	0.838	0.848
φ	0.3835	0.4214	0.443	0.455	0.462	0.467	0.471	0.474	0.476
χ	0.591	0.555	0.539	0.53	0.526	0.523	0.52	0.518	0.517
φ/ω	0.653	0.626	0.605	0.593	0.583	0.576	0.57	0.566	0.561
$\frac{\omega-\varphi}{\omega}$	0.347	0.374	0.395	0.407	0.417	0.424	0.43	0.434	0.439

Table A.4 – Limit values of coefficients $\bar{\xi}_R$ and $\bar{\alpha}_R$

Class of tensile reinforcement	K							
	2		2.5		3		3.5	
	$\bar{\xi}_R$	$\bar{\alpha}_R$	$\bar{\xi}_R$	$\bar{\alpha}_R$	$\bar{\xi}_R$	$\bar{\alpha}_R$	$\bar{\xi}_R$	$\bar{\alpha}_R$
A240C	0.690	0.368	0.674	0.375	0.666	0.381	0.661	0.386
A400C	0.583	0.329	0.565	0.334	0.556	0.338	0.551	0.342
A500C (Ø8 – 22)	0.539	0.311	0.521	0.315	0.512	0.319	0.507	0.323
A500C (Ø25 – 40)	0.550	0.315	0.532	0.320	0.522	0.324	0.517	0.327
B500	0.525	0.305	0.507	0.309	0.497	0.312	0.492	0.316

Table A.5 – Values of coefficients ξ, $\bar{\xi}$, $\bar{\zeta}$ and $\bar{\alpha}_m$

ξ	К=2			К=2.5			К=3			К=3.5		
	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$
0.01	0.014	0.995	0.010	0.013	0.995	0.010	0.013	0.995	0.010	0.012	0.995	0.010
0.02	0.027	0.989	0.020	0.026	0.989	0.020	0.025	0.989	0.020	0.025	0.990	0.020
0.03	0.041	0.984	0.030	0.039	0.984	0.030	0.038	0.984	0.030	0.037	0.984	0.030
0.04	0.055	0.978	0.039	0.052	0.979	0.039	0.050	0.979	0.039	0.049	0.979	0.039
0.05	0.068	0.973	0.049	0.065	0.973	0.049	0.063	0.974	0.049	0.062	0.974	0.049
0.06	0.082	0.968	0.058	0.078	0.968	0.058	0.076	0.968	0.058	0.074	0.969	0.058
0,07	0.096	0.962	0.067	0.091	0.963	0.067	0.088	0.963	0.067	0.086	0.963	0.067
0.08	0.109	0.957	0.077	0.104	0.958	0.077	0.101	0.958	0.077	0.099	0.958	0.077
0.09	0.123	0.952	0.086	0.117	0.952	0.086	0.114	0.953	0.086	0.111	0.953	0.086
0.10	0.137	0.946	0.095	0,130	0.947	0.095	0.126	0.947	0.095	0.123	0.948	0.095
0.11	0.150	0.941	0.103	0.143	0.942	0.104	0.139	0.942	0.104	0.136	0.943	0.104
0.12	0.164	0.935	0.112	0.156	0.936	0.112	0.151	0.937	0.112	0.148	0.937	0.112
0.13	0.178	0.930	0.121	0.169	0.931	0.121	0.164	0.932	0.121	0.160	0.932	0.121
0.14	0.191	0.925	0.129	0.182	0.926	0.130	0.177	0.926	0.130	0.173	0.927	0.130
0.15	0.205	0.919	0.138	0.195	0.920	0.138	0.189	0.921	0.138	0.185	0,922	0.138
0.16	0.219	0.914	0,146	0.208	0.915	0.146	0.202	0.916	0.147	0.197	0.916	0.147
0.17	0.232	0.908	0.154	0.222	0.910	0.155	0.215	0.911	0.155	0.210	0.911	0.155
0.18	0.246	0.903	0,163	0.235	0.904	0,163	0.227	0.905	0.163	0.222	0.906	0.163
0.19	0.260	0.898	0.171	0.248	0,899	0.171	0.240	0.900	0.171	0.234	0.901	0.171
0.20	0.273	0.892	0.178	0.261	0.894	0.179	0.252	0.895	0.179	0.247	0.895	0.179
0.21	0.287	0.887	0.186	0.274	0.889	0.187	0.265	0.890	0,187	0.259	0.890	0.187
0.22	0.301	0.881	0.194	0.287	0.883	0,194	0.278	0.884	0.195	0.271	0.885	0.195
0.23	0.314	0.876	0.202	0.300	0.878	0.202	0.290	0.879	0.202	0.284	0.880	0,202
0.24	0.328	0.871	0.209	0.313	0.873	0.209	0.303	0.874	0,210	0.296	0.875	0,210
0.25	0.342	0.865	0.216	0.326	0.867	0.217	0.316	0.869	0.217	0.308	0.869	0,217
0.26	0.355	0.860	0.224	0.339	0.862	0.224	0.328	0.863	0.224	0.321	0.864	0,225
0.27	0.369	0.855	0.231	0.352	0.857	0.231	0.341	0.858	0.232	0.333	0.859	0,232
0.28	0.382	0.849	0.238	0.365	0.851	0.238	0.353	0.853	0.239	0.345	0.854	0,239

Continuation of Table A.5

ξ	K=2			K=2.5			K=3			K=3.5		
	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$
0.29	0.396	0.844	0.245	0.378	0.846	0.245	0.366	0.847	0.246	0.358	0.848	0.246
0.3	0.410	0.838	0.252	0.391	0.841	0.252	0.379	0.842	0.253	0.370	0.843	0.253
0.31	0.423	0.833	0.258	0.404	0.835	0.259	0.391	0.837	0.259	0.382	0.838	0.260
0.32	0.437	0.828	0.265	0.417	0.830	0.266	0.404	0.832	0.266	0.395	0.833	0.266
0.33	0,451	0.822	0.271	0.430	0.825	0.272	0.417	0.826	0.273	0.407	0.828	0.273
0.34	0.464	0.817	0.278	0.443	0.820	0.279	0.429	0.821	0.279	0,419	0.822	0.280
0.35	0.478	0.811	0.284	0.456	0.814	0.285	0.442	0.816	0.286	0.432	0.817	0.286
0.36	0.492	0.806	0.290	0.469	0.809	0.291	0.454	0.811	0.292	0.444	0.812	0.292
0.37	0.505	0.801	0.296	0.482	0.804	0.297	0.467	0.805	0.298	0.456	0.807	0.298
0.38	0.519	0.795	0.302	0.495	0.798	0.303	0.480	0.800	0.304	0.469	0.801	0.305
0.39	0.533	0.790	0.308	0.508	0.793	0.309	0.492	0.795	0.310	0.481	0.796	0.311
0.40	0.546	0.785	0.314	0.521	0.788	0.315	0.505	0.790	0.316	0.493	0.791	0.316
0.41	0.560	0.779	0.319	0.534	0.782	0.321	0.518	0.784	0.322	0.506	0.786	0.322
0.42	0.574	0.774	0.325	0.547	0.777	0.326	0.530	0.779	0.327	0.518	0.780	0.328
0.43	0.587	0.768	0.330	0,560	0.772	0.332	0.543	0.774	0.333	0.530	0.775	0.333
0.44	0.601	0.763	0.336	0.573	0.766	0.337	0.555	0.769	0.338	0.543	0.770	0.339
0.45	0.615	0.758	0.341	0.586	0.761	0.343	0.568	0.763	0.343	0.555	0.765	0.344
0.46	0.628	0.752	0.346	0.599	0.756	0.348	0.581	0.758	0.349	0.567	0.760	0.349
0.47	0.642	0.747	0.351	0.612	0.751	0.353	0.593	0.753	0.354	0.580	0.754	0.355
0.48	0.656	0.741	0.356	0.625	0.745	0.358	0.606	0.748	0.359	0.592	0.749	0.360
0.49	0.669	0.736	0.361	0.639	0.740	0.363	0.619	0.742	0.364	0.604	0.744	0.365
0.50	0.683	0.731	0.365	0.652	0.735	0.367	0.631	0.737	0.369	0.617	0.739	0.369
0.51	0.697	0.725	0.370	0.665	0.729	0.372	0.644	0.732	0.373	0.629	0.733	0.374
0.52	0.710	0.720	0.374	0.678	0.724	0.376	0.656	0.726	0.378	0.641	0.728	0.379
0.53	0.724	0.715	0.379	0.691	0.719	0.381	0.669	0.721	0.382	0.654	0.723	0.383
0.54	0.738	0.709	0.383	0.704	0.713	0.385	0.682	0.716	0.387	0.666	0.718	0.388
0.55	0.751	0.704	0.387	0.717	0.708	0.389	0.694	0.711	0.391	0.678	0.713	0.392
0.56	0.765	0.698	0.391	0.730	0.703	0.394	0.707	0.705	0.395	0.691	0.707	0.396

End of Table A.5

ξ	К=2			К=2 .5			К=3			К=3.5		
	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\alpha}_m$	$\bar{\xi}$	$\bar{\zeta}$	$\bar{\xi}$	$\bar{\zeta}$
0.57	0.779	0.693	0.395	0.743	0.697	0.398	0.720	0.700	0.399	0.703	0.702	0.400
0.58	0.792	0.688	0.399	0.756	0.692	0.401	0.732	0.695	0.403	0.715	0.697	0.404
0.59	0.806	0.682	0.402	0.769	0.687	0.405	0.745	0.690	0.407	0.728	0.692	0.408
0.60	0.820	0.677	0.406	0.782	0.682	0.409	0.757	0.684	0.411	0.740	0.686	0.412
0.61	0.833	0.671	0.410	0.795	0.676	0.412	0.770	0.679	0.414	0.752	0.681	0.416
0.62	0.847	0.666	0.413	0.808	0.671	0.416	0.783	0.674	0.418	0.765	0.676	0.419
0.63	0.861	0.661	0.416	0.821	0.666	0.419	0.795	0.669	0.421	0.777	0.671	0.423
0.64	0.874	0.655	0.419	0.834	0.660	0.423	0.808	0.663	0.425	0.789	0.665	0.426
0.65	0.888	0.650	0.422	0.847	0.655	0.426	0.821	0.658	0.428	0.802	0.660	0.429
0.66	0.902	0.644	0,425	0.860	0.650	0.429	0.833	0.653	0.431	0.814	0.655	0.432
0.67	0.915	0.639	0.428	0.873	0.644	0.432	0.846	0.648	0.434	0.826	0.650	0.435
0.68	0.929	0.634	0.431	0.886	0.639	0.435	0.858	0.642	0.437	0.839	0.645	0.438
0.69	0.943	0.628	0.434	0.899	0.634	0.437	0.871	0.637	0.440	0.851	0.639	0.441
0.70	0.956	0.623	0.436	0.912	0.628	0.440	0.884	0.632	0.442	0.863	0.634	0.444
0.71	0.970	0.618	0.438	0.925	0.623	0.442	0.896	0,627	0.445	0.876	0.629	0.447
0.72	0.984	0.612	0.441	0.938	0.618	0.445	0.909	0.621	0.447	0.888	0.624	0.449
0.73	0.997	0.607	0.443	0.951	0.613	0.447	0.922	0.616	0.450	0.900	0.618	0.451
0.74				0.964	0.607	0.449	0.934	0.611	0.452	0.913	0.613	0.454
0.75				0.977	0.602	0.451	0.947	0.606	0.454	0.925	0.608	0.456
0.76				0.990	0.597	0.453	0.959	0.600	0.456	0.937	0.603	0.458
0.77							0.972	0.595	0.458	0.950	0.598	0.460
0.78							0.985	0.590	0.460	0.962	0.592	0.462
0.79							0.997	0.584	0.462	0.974	0.587	0.464
0.80										0.987	0.582	0.465
0.81										0.999	0.577	0.467

Table A.6 – Reinforcement gage

Nominal diameter of the bar, mm	Design cross-sectional area of the bar, mm^2 with the number of bars									Theoretical mass 1 m of reinforcement length, kg	Diameter of reinforcement of classes			
	1	2	3	4	5	6	7	8	9		A240C	A400C	A500C	B500
3	7.1	14.1	21.2	28.3	35.3	42.4	49.5	56.5	63.6	0.056	-	-	-	+
4	12.6	25.1	37.7	50.3	62.8	75.4	88	100.5	113.1	0.099	-	-	-	+
5	19.6	39.3	58.9	78.5	98.2	117.8	137.4	157.1	176.7	0.154	-	-	-	+
6	28.3	57	85	113	141	170	198	226	254	0.222	+	+	+	+
8	50.3	101	151	201	251	302	352	402	452	0.395	+	+	+	+
10	78.5	157	236	314	393	471	550	628	707	0.616	+	+	+	+
12	113.1	226	339	452	565	679	792	905	1018	0.888	+	+	+	+
14	153.9	308	462	616	770	924	1078	1232	1385	1.208	+	+	+	-
16	201.1	402	603	804	1005	1206	1407	1608	1810	1.579	+	+	+	-
18	254.5	509	763	1018	1272	1527	1781	2036	2290	1.998	+	+	+	-
20	314.2	628	942	1257	1571	1885	2199	2513	2827	2.466	+	+	+	-
22	380.1	760	1140	1521	1901	2281	2661	3041	3421	2.984	+	+	+	-
25	490.9	982	1473	1963	2454	2945	3436	3927	4418	3.854	+	+	+	-
28	615.8	1232	1847	2463	3079	3695	4310	4926	5542	4.834	+	+	+	-
32	804.2	1608	2413	3217	4021	4825	5630	6434	7238	6.313	+	+	+	-
36	1017.9	2036	3054	4072	5089	6107	7125	8143	9161	7.991	+	+	+	-
40	1256.6	2513	3770	5027	6283	7540	8796	10053	11310	9.864	+	+	+	-

APPENDIX B

NOTEBOOK FOR SOLVING TASKS

Student	Group	Date	Mark

TASK №1

Basic data

Strength class of concrete	Breadth of the section b, mm	Height of the section h, mm	Reinforcement	Cover c_{nom} , mm	Design value of bending moment M_{Ed}, kNm

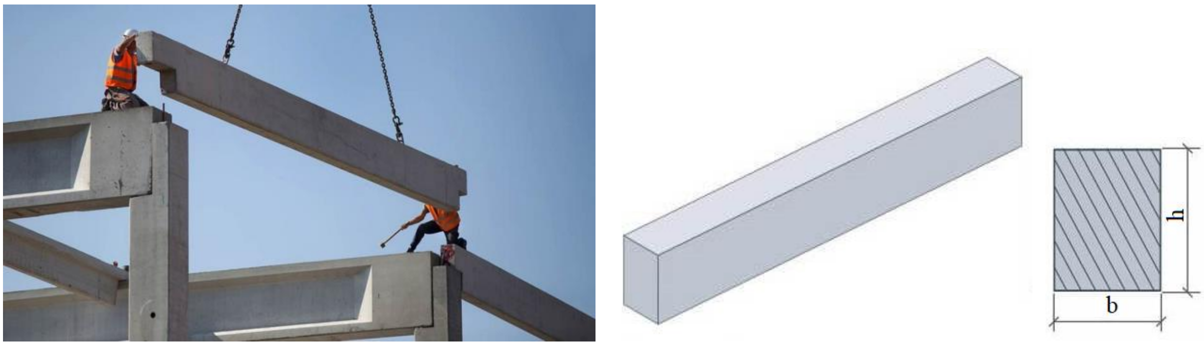


Fig. B.1. Beam with rectangular profile of cross section

Solution

1. Characteristics of materials

Concrete

1.1 Design value of concrete compressive strength f_{cd} = MPa

1.2 Design value of compressive strain in the concrete at the peak stress $\varepsilon_{c1,cd}$ = ‰

1.3 Design value of modulus of elasticity E_{cd} =

GPa

1.4 Coefficient of physical and mechanical properties MPa

$$K = 1.05\, E_{cd}\, \varepsilon_{c1,cd} / f_{cd} =$$

1.5 Characteristic level of strains η_u =

1.6 Coefficient ω =

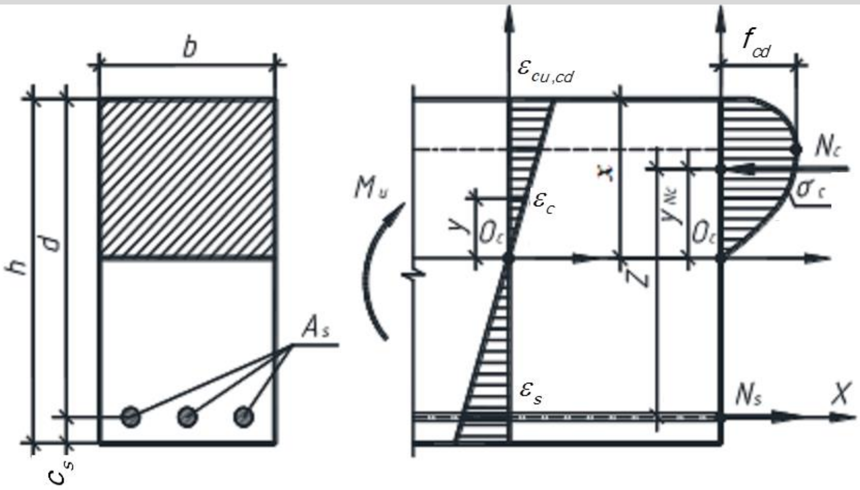
1.7 Coefficient χ =

Reinforcing steel

1.8 Design yield strength of reinforcement f_{yd} = MPa

1.9 Modulus of elasticity of reinforcement E_s = MPa

2. Design model of the normal section



a *b* *c*

Fig. B.2. Design model of the normal section: a – normal section of the element; b – strain diagram; c – stress model

3. Calculation of neutral axis depth

3.1 Effective depth of the section

$$d = h - c_{nom} - \varnothing / 2 \ =$$ mm.

3.2 The relative neutral axis depth for rectangular distribution of stress along compressed zone of concrete is equal

$$\xi = \frac{f_{yd} A_s}{f_{cd} b d} =$$

3.3 The relative neutral axis depth for curvilinear distribution of stress along compressed zone of concrete is equal

$\bar{\xi} =$

.

3.4 The ultimate relative neutral axis depth is equal

$\bar{\xi}_R =$

.

If $\bar{\xi}$ $\bar{\xi}_R$ than the section of the beam is OVERREINFORCED / NOT OVERREINFORCED.

4. Calculation of the bending moment that the beam can take in the normal section

4.1 The relative bending moment according to received K and ξ is equal

$\alpha_m =$.

4.2 The maximal bending moment M_{Rd} that the beam can take in the normal section is

$M_{Rd} = f_{cd} b d^2 \alpha_m =$ Nmm

= kNm.

5. Strength verification of the beam in the normal section

$M_{Ed} =$

MORE / LESS

$M_{Rd} =$

The strength of the beam in the normal section is

PROVIDED / UNPROVIDED

TASK №2

Basic data

Strength class of concrete	Strength class of reinforcement	b, mm	h, mm	M_{Ed}, kNm

Solution

1. Characteristics of materials

Concrete

1.1 Design value of concrete compressive strength f_{cd} = MPa

1.2 Design value of compressive strain in the concrete at the peak stress $\varepsilon_{c1,cd}$ = ‰

1.3 Design value of the modulus of elasticity E_{cd} = GPa

= MPa

1.4 Coefficient of physical and mechanical properties

$$K = 1.05\, E_{cd}\, \varepsilon_{c1,cd} \big/ f_{cd} =$$

Reinforcing steel

1.8 Design yield strength of reinforcement f_{yd} = MPa

1.9 Modulus of elasticity of reinforcement E_s = MPa

2. Necessity for compressed reinforcement

2.1 The cover to reinforcement (previously take diameter of reinforcement bars Ø = mm) is

$$c_{nom} = c_{\min} + \Delta c_{dev} = \quad \text{mm.}$$

2.2 Effective depth of the section is

$$d = h - c_{nom} - \varnothing / 2 = \quad \text{mm.}$$

2.3 Relative value of external bending moment is

$$\alpha_m = \frac{M_{Ed}}{f_{cd} b d^2} =$$

2.4 Ultimate value of relative bending moment that the beam can take with single reinforcement is

$$\bar{\alpha}_R$$

2.5 If α_m $\qquad$ $\bar{\alpha}_R$, than reinforcement in the compressed area is NECESSARY / NOT NECESSARY.

3. Calculation of the area of reinforcement

3.1 Value of relative lever arm is

$$\bar{\zeta} =$$

3.2 The area of reinforcement

$$A_s = \frac{M_{Ed}}{f_{yd} \bar{\zeta} d} = \qquad \text{mm}^2$$

4. Reinforcement bars

Take _______ Ø _________ A_________C. Area of the reinforcement bars $A_{s,\,fact}$ =____ mm^2.

5. Detailing of the cross-section

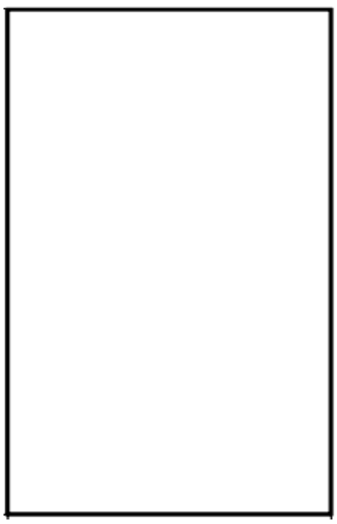


Fig. B.3. The cross-section with single reinforcement

TASK №3

Basic data

Strength class of concrete	Strength class of reinforcement	b, mm	h, mm	M_{Ed}, kNm

Solution

1. Characteristics of materials

Concrete

1.1 Design value of concrete compressive strength f_{cd} = MPa

1.2 Design value of compressive strain in the concrete at the peak stress $\varepsilon_{c1,cd}$ = ‰

1.3 Design value of the secant modulus of elasticity E_{cd} = GPa = MPa =

1.4 Coefficient of physical and mechanical properties

$$K = 1{,}05\, E_{cd}\, \varepsilon_{c1,cd} / f_{cd} =$$

1.5 Characteristic level of strains η_u =

1.6 Coefficient ω =

1.7 Coefficient χ =

Reinforcing steel

1.8 Design yield strength of reinforcement f_{yd} = MPa

1.9 Modulus of elasticity of reinforcement E_s = MPa

2. Necessity for compressed reinforcement

2.1 The cover to reinforcement (previously take diameter of reinforcement bars Ø = mm) is

$$c_{nom} = c_{\min} + \Delta c_{dev} =$$

mm.

2.2 Effective depth of the section is

$$d = h - c_{nom} - \varnothing / 2 =$$

mm.

2.3 Relative value of external bending moment is

$$\alpha_m = \frac{M_{Ed}}{f_{cd} b d^2} =$$

2.4 Ultimate value of relative bending moment that the beam can take with single reinforcement is

$$\bar{\alpha}_R =$$

2.5 If α_m $\bar{\alpha}_R$, than reinforcement in the compressed area is NECESSARY / NOT NECESSARY.

3. Calculation of the area of principal compressed reinforcement

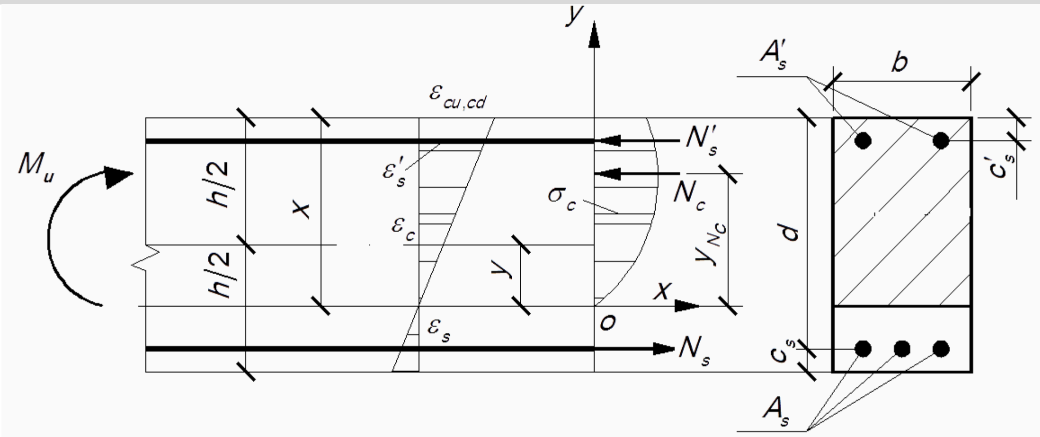


Fig. B.4. Design scheme of the normal section of flexural RCS rectangular profile with double reinforcement

3.1 The area of principal compressed reinforcement

$$A_s' = \frac{M_{Ed} - \alpha_R f_{cd} b d^2}{\varepsilon_s' E_s' (d - c_s')} =$$

mm^2.

where $\varepsilon_s = \dfrac{\varepsilon_{cu}(\bar{\xi}_R - c_s' / d)}{\bar{\xi}_R} \leq \varepsilon_{s0}' =$

$$\varepsilon_{cu} = \varepsilon_{c1,cd}\eta_u =$$

$$\varepsilon_{s0}' = \frac{f_{yd}}{E_s} =$$

3.2 Take _______ Ø ________ A________C. Area of the reinforcement bars $A'_{s,fact} =$___________ mm^2.

4. Calculation of the area of principal tensile reinforcement

4.1 The area of principal tensile reinforcement

$$A_s = \bar{\xi}_R \omega bd \frac{f_{cd}}{f_{yd}} + A_s' = \qquad\qquad\qquad \text{mm}^2.$$

4.2 Take _______ Ø ________ A________C. Area of the reinforcement bars $A_{s,fact} =$____________ mm^2.

5. Construction of the cross-section

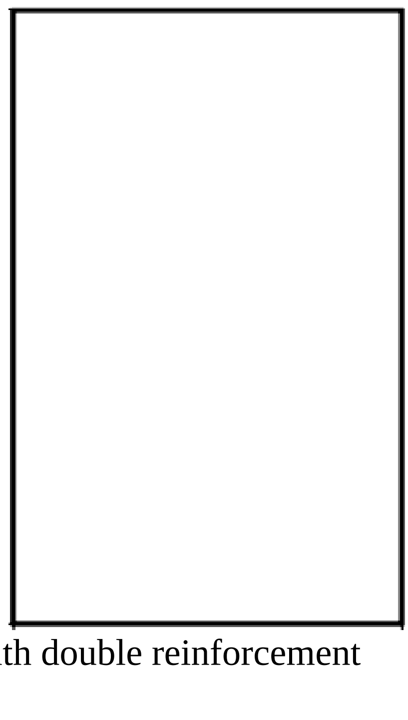


Fig. B.5. The cross-section with double reinforcement

6. Verification of bearing capacity

6.1 The relative neutral axis depth for rectangular distribution
of stress along compressed zone of concrete is equal

$$\bar{\xi} = \frac{f_{yd}A_s - f_{yd}{}'A_s'}{f_{cd}bd}$$

6.2 The relative neutral axis depth for curvilinear distribution
of stress along compressed zone of concrete

$\bar{\xi} =$

.

6.3 The ultimate relative neutral axis depth

$\bar{\xi}_R =$

.

If $\bar{\xi}$ $\bar{\xi}_R$ than the section of the beam is OVERREINFORCED / NOT OVERREINFORCED.

6.4 The relative bending moment according to received K and ξ is equal

$\alpha_m =$

6.5 The maximal bending moment M_{Rd} that the beam can take in the normal section is

$$M_{Rd} = f_{cd}b\bar{\alpha}_m d^2 - A_s' f_{yd}' (d - c_s')$$

or

$$M_{Rd} = f_{cd}b\bar{\alpha}_R d^2 - A_s' f_{yd}' (d - c_s')$$

6.6. The strength of the beam in the normal section is

M_{Ed} =

MORE / LESS

M_{Rd} =

The strength of the beam in the normal section is

PROVIDED / UNPROVIDED

TASK №4

Basic data

Strength class of concrete	Strength class of reinforcement	b_{eff}, mm	h_{eff}, mm	b_w, mm	h, mm	c_{nom}, mm	M_{Ed}, kNm

Fig. B.6. Beam with T- profile of cross section

Solution

1. Characteristics of materials

Concrete

1.1 Design value of concrete compressive strength f_{cd} = MPa

1.2 Design value of compressive strain in the concrete at the peak stress $\varepsilon_{c1,cd}$ = = ‰

1.3 Design value of the secant modulus of elasticity E_{cd} = GPa

= MPa

1.4 Coefficient of physical and mechanical properties

$$K = 1{,}05\, E_{cd}\, \varepsilon_{c1,cd} / f_{cd} =$$

1.5 Characteristic level of strains η_u =

1.6 Coefficient ω =

1.7 Coefficient χ =

Reinforcing steel f_{yd} = MPa

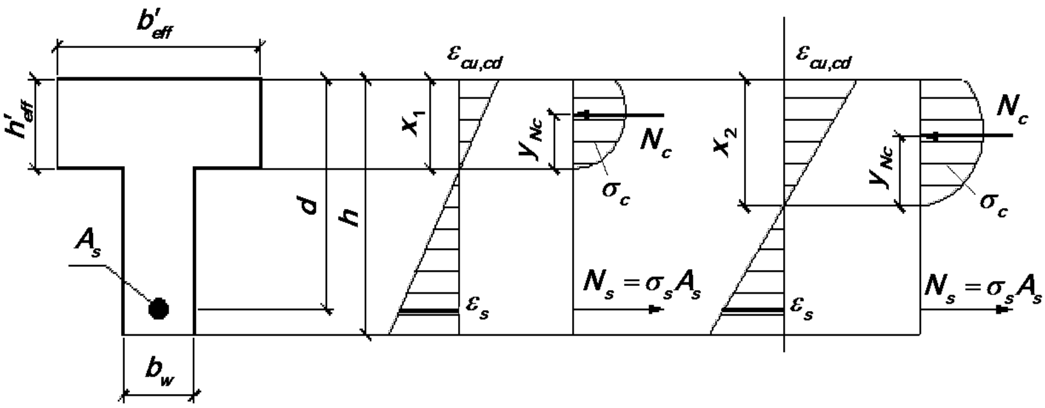
1.8 Design yield strength of reinforcement

E_s = MPa

1.9 Modulus of elasticity of reinforcement

2. Location of the neutral axis

2.1 Design model of cross-section



a b

Fig. B.7. Design model of the normal section: a – normal section of the element; b – stress and strain diagram

2.2 The bending moment that the beam can take with full compressed flange is

$M_{eff}' = f_{cd} b_{eff}' h_{eff}' \omega (d - \chi h_{eff}' \omega)$ - Nmm = kNm.

2.3 If M_{Ed} $M_{Rd,eff}$, than the neutral axis is located

IN THE FLANGE / IN THE RIB.

3. Necessity for compressed reinforcement

3.1 The cover to reinforcement (previously take diameter of reinforcement bars Ø = mm) is

$c_{nom} = c_{\min} + \Delta c_{dev} =$. mm.

3.2 Effective depth of the section is

$$d = h - c_{nom} - \varnothing / 2 \qquad\qquad\qquad\qquad \text{mm.}$$

3.3 Relative value of external bending moment is

$$\alpha_m = \frac{M_{Ed}}{f_{cd} b_{eff}{}' d^2} =$$

3.4 The ultimate relative neutral axis depth is

$$\bar{\xi}_R =$$

3.5 Maximal value of relative bending moment that
the beam can take with single reinforcement

$$\alpha_R =$$

3.6 If α_m $\qquad$ α_R, then reinforcement in the compressed area is NECESSARY / NOT NECESSARY.

4. Calculation of the area of reinforcement

4.1 Value of relative lever arm is

$$\bar{\zeta} =$$

4.2 The area of reinforcement is

$$A_s = \frac{M_{Ed}}{f_{yd} \bar{\zeta} d} =$$

4.3 Take _______ Ø ________ A________C. Area of the reinforcement bars $A_{s,fact}$ =_____________ mm^2.

5. Construction of the cross-section

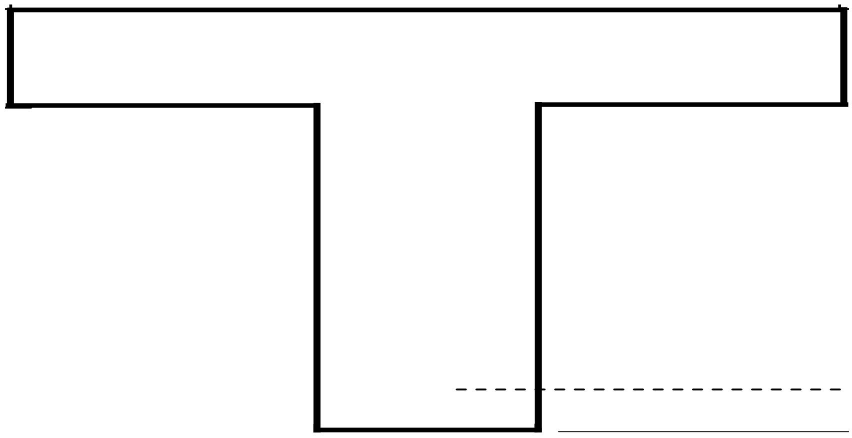


Fig. B.7. The cross T-section

TASK №5

Basic data

Strength class of concrete	Longitudinal reinforcement	Strength class of shear reinforcement	Length of beam, m	b_w, mm	h, mm	c_{nom}, mm	V_{Ed}, kNm

Solution

1. Chracteristics of materials

Concrete

1.1 Design value of concrete compressive strength f_{cd} = MPa

1.2 Characteristic value of concrete compressive strength f_{ck} = MPa

1.3 Safety coefficient for concrete γ_c =

Reinforcement (shear)

1.4 Design yield strength of shear reinforcement f_{ywd} = MPa

1.5 Characteristic value of yield strength of shear reinforcement f_{yk} = MPa

1.6 Modulus of elasticity of reinforcement E_s = MPa

2. Determining the necessity of shear reinforcement

2.1 Effective depth is

$$d = h - c_{nom} - \varnothing / 2$$ mm.

2.2 Determine the estimated value of the beam resistance without shear reinforcement

$$V_{Rd,c} = b_w d \cdot C_{Rd,c} k \left(100 \rho_l f_{ck}\right)^{1/3} =$$ N= kN.

Not less than

$V_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$ N= kN,

where $C_{rd,c} = 0{,}18 / \gamma_c$ =

$k = 1 + (200 / d)^{0.5}$ =

Coefficient of longitudinal reinforcement $\rho_l = A_s / (b_w\, d)$ =
Because

$$V_{Rd,c} \qquad V_{Ed} \,,$$

The reinforcement in the area of the largest transverse forces is NECESSARY / NOT NECESSARY.

3. Determination of the cross-sectional area of the transverse reinforcement

The calculation of the required amount of reinforcement is carried out in accordance with 4.6.3 DSTU B.2.6-156 using the "Truss" model.

3.1 Determine the angle of inclination of the compressed concrete strut to the horizontal

$$\theta = \frac{1}{2} \arcsin \frac{2V_{Ed}}{z b_w \nu_1 f_{cd}} =$$

where $\nu_1 = 0{,}6\left(1 - \dfrac{f_{ck}}{250}\right) =$

$z = 0{,}9d =$ mm.

3.2 Taking into account the restrictions that are set at this angle $1 \le \cot\theta \le 2.5$, we take θ = , $ctg\ \theta$ =

3.3 The required area of transverse reinforcement is determined from the equation (4.44) of DSTU B.2.6-156, taking $V_{Ed} = V_{Ed,s}$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{z\, f_{ywd}\, ctg\, \theta} =$$ mm^2 / mm.

4. Construction of shear reinforcement

4.1 Take shear reinforcement according to constructive requirements ______________________ (A_{sw} = ______________________________ mm^2), the design step of the shear reinforcement is equal to

s = mm.

4.2 According to the constructive requirements of 8.2.6.8 DSTU B.2.6-156, the maximum step of shear reinforcement must not exceed

$s_{l,\max} = 0,75d =$ mm.

4.3 The minimum area of transverse reinforcement in the step $s_{l,\max,\,fact}$ = mm.

4.4 Thus, we finally take on the sites where the maximum shear forces of reinforcement (at distance from the support ______________________ length)

____________________ A________________C step _______________ mm.

On other sections of the beam

____________________ A________________C step _______________ mm.

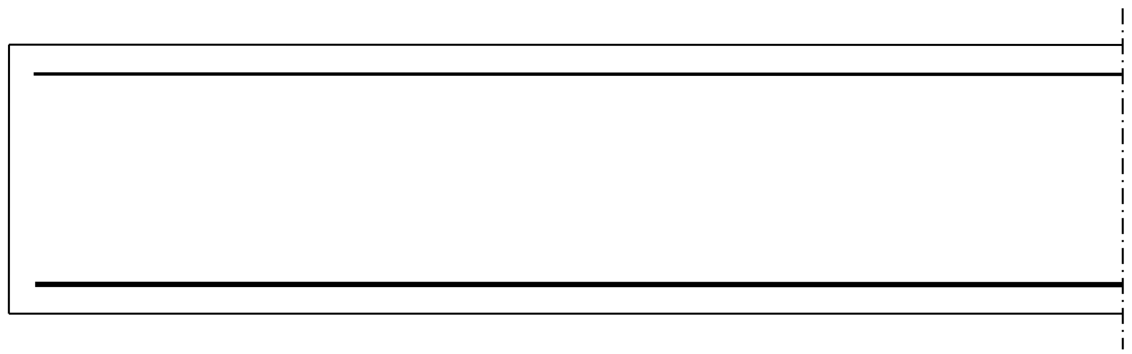


Fig. B.8. Scheme of shear beam reinforcement

APPENDIX C

BASIC DATA FOR TASKS

TASK №1

Variant	Class of concrete	***b***, mm	***h***, mm	Reinforcement	c_{nom}, mm	M_{Ed}, kNm
1	C20/25	200	600	4∅20 A400C	35	207
2	C30/35	300	600	3∅28 A500C	40	344
3	C16/20	400	800	4∅32 A240C	30	494
4	C25/30	500	1000	5∅36 A400C	25	1682
5	C32/40	250	750	3∅32 A500C	30	669
6	C20/25	150	500	2∅25 A240C	40	113
7	C32/40	450	900	4∅36 A400C	45	1043
8	C16/20	550	1000	3∅40 A400C	40	1208
9	C25/30	300	900	4∅32 A500C	30	960
10	C30/35	350	900	4∅36 A240C	35	724
11	C20/25	200	600	4∅20 A400C	35	250
12	C30/35	300	600	3∅28 A500C	40	300
13	C16/20	400	800	4∅32 A240C	30	470

TASK №2

N	Class of concrete	Class of reinforcement	***b***, мм	***h***, мм	M_{Ed}, кНм
1	C25/30	A400C	100	350	20
2	C16/20	A400C	160	460	70
3	C30/35	A240C	140	460	60
4	C16/20	A400C	290	780	360
5	C20/25	A400C	250	550	160
6	C12/15	A400C	140	420	50
7	C20/25	A400C	250	680	240
8	C25/30	A400C	260	760	310
9	C30/35	A240C	200	450	80
10	C12/15	A500C	320	660	290
11	C16/20	A500C	160	460	80
12	C20/25	A500C	250	550	200

TASK №3

№	Class of concrete	Class reinforcement	*b*, мм	*h*, мм	M_{Ed}, кНм
1	C25/30	A400C	100	350	70
2	C16/20	A400C	160	460	140
3	C30/35	A240C	140	460	200
4	C16/20	A400C	290	780	720
5	C20/25	A400C	250	550	350
6	C12/15	A400C	140	420	68
7	C20/25	A400C	250	680	640
8	C25/30	A400C	260	760	900
9	C30/35	A240C	200	450	280
10	C12/15	A500C	320	660	580
11	C25/30	A400C	100	350	**100**
12	C16/20	A400C	160	460	**160**

TASK №4

	Class of concrete	Class of reinforcement	b_{eff}, mm	h_{eff}, mm	b_w, mm	*h*, mm	c_{nom}, mm	M_{Ed}, kNm
1	C16/20	A240C	400	280	180	1000	35	690
2	C30/35	A500C	250	80	200	350	30	71
3	C32/40	A500C	350	260	320	900	35	793
4	C25/30	A400C	500	530	310	1200	40	2043
5	C32/40	A400C	330	140	140	600	40	351
6	C20/25	A240C	450	290	200	1100	30	262
7	C25/30	A500C	540	380	210	800	35	1024
8	C16/20	A400C	480	270	280	1050	30	828
9	C20/25	A400C	300	120	200	400	30	110
10	C30/35	A240C	380	230	230	550	40	408
11	C16/20	A 400C	400	280	180	1000	35	690
12	C16/20	A500C	400	280	180	1000	35	690

TASK №5

	Class of concrete	Longitudinal reinforcement	Class of shear reinforcement	Length of the beam	b_w, mm	h, mm	c_{nom}, mm	V_{Ed}, kNm
1	C16/20	3∅25A400C	A240C	6,0	200	450	30	225
2	C30/35	2∅36A500C	A240C	7,2	300	600	35	254
3	C32/40	2∅32A500C	A240C	6,6	250	600	35	252
4	C25/30	2∅36A400C	A240C	5,4	310	650	50	321
5	C32/40	2∅28A400C	A240C	6,9	220	550	40	208
6	C20/25	2∅20A240C	A240C	6,0	150	450	40	88
7	C25/30	2∅22A500C	A240C	7,2	180	400	50	162
8	C16/20	2∅32A400C	A400C	6,6	270	600	50	494
9	C20/25	3∅32A400C	A400C	5,4	240	700	30	632
10	C30/35	3∅28A500C	A400C	6,9	290	600	40	293

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