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FREQUENCY-SELECTIVE ELECTRIC CIRCUITS

Study manual for students of higher educational institutions of specialities
123 - computer engineering and 172 - telecommunications and radio engineering

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The study manual covers the program material for training students of specialties 123 - computer engineering and 172 - telecommunications and radio engineering from the course "Theory of electric circuits".

The manual analyzes the general properties of single and coupled oscillating circuits, inductive and non-inductive electrical filters, which are most often used in communication technology. The theory of discrete electrical circuits and the construction and algorithms of digital filters are also discussed in detail.

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CONTENTS

INTRODUCTION

The wide use of frequency-selective electric circuits in communication technology requires sufficient attention to the study of patterns that arise during the operation of these devices.

The authors of the manual tried, using the well-known mathematical apparatus (the theory of complex variables, methods of solving differential equations, trigonometric, hyperbolic and special functions) and the basic principles and laws of the theory of electric circuits (TEC), from the most general positions, to analyze the reactions of linear frequency- selective circuits with harmonic input actions

The basic schemes of all types of LC and RC filters and their attenuation characteristics are given. The mathematical apparatus for the synthesis of filters of Batervort and Chebyshev is shown.

Discrete electric circuits in general and digital filters in particular are considered in detail. A lot of attention is devoted to digital filtering algorithms in time and frequency domains. The methods of accelerating the digital convolution and fast Fourier transformation are analyzed.

To simplify the understanding of physical phenomena, many illustrations and examples of the use of these circuits in the apparatus are given.

CHAPTER 1. OSCILLATING CIRCUITS

In communication equipment, electric circuits (EC) containing two reactive elements, an inductance and a capacitance element, are widely used.

These qualitatively new electrical circuits are called oscillating circuits (OC). There are two types of OC: series and parallel, which are shown in fig. 1.1.

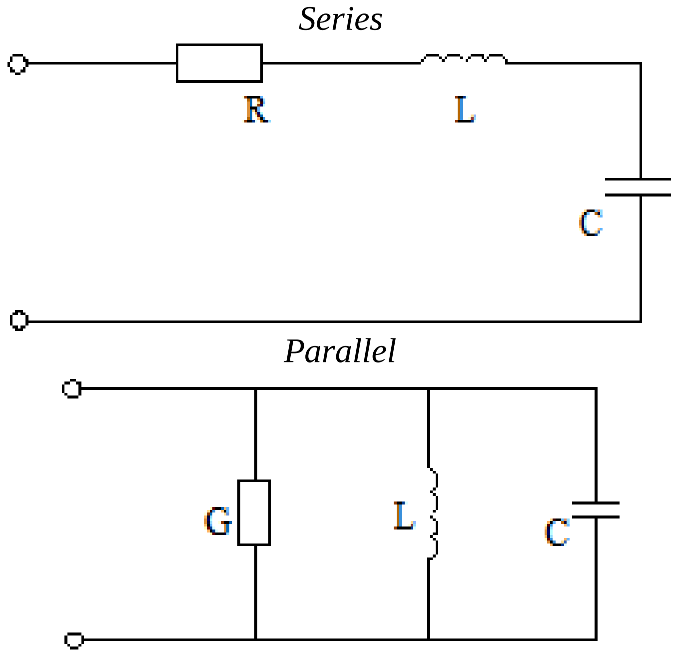


Fig. 1.1. Series and parallel oscillating circuits

The oscillating circuit is characterized by primary and secondary parameters.

Primary: L – inductance; C – capacity; R is active resistance, or G – is active conductivity. Conventional resistance does not exist as a separate element of OC. It takes into account energy losses in the coil wire, connecting wires, shields, dielectric losses in wire insulation, in the dielectric of the capacitor, in the coil frame, etc., and for the convenience of analysis and calculations is depicted on the diagram as a separate element.

Secondary parameters include: f_0 – resonance frequency; R_{res} – resonance resistance; Q – quality factor; ρ – wave resistance; $2\Delta f_p$ – passband; $2\Delta f_m$ – interference band; K_s – the coefficient of squareness.

Let's determine the properties of parallel and series circuits and find expressions for the secondary parameters through the primary ones.

1.1. Properties of parallel and series oscillatory circuits

Let us consider at a qualitative level what processes occur in a parallel oscillating circuit under non-zero initial conditions. If the switch is set to position 1 (Fig. 1.2), the capacitor will be charged to a voltage of E.

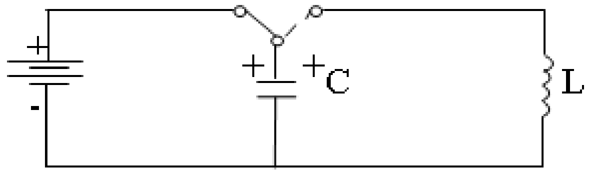


Fig. 1.2. Processes in a parallel oscillating circuit

Let's move the switch to position 2. At the same time, the charged capacitor is discharged through the inductor. The energy of the electric field of the capacitor

$$W_c = \frac{CE^2}{2}$$

is transformed into the energy of the magnetic field of the coil

$$W_L = \frac{LI^2}{2}.$$

Then the energy from the magnetic field of the coil passes into the electric field of the capacitor, etc. In this way, reactive energy is exchanged between the capacitor and the coil in the circuit. In a real oscillating circuit, in the process of this exchange, part of the energy of the circuit is irreversibly lost (due to losses on the element R), turning into heat. As a result, the amount of energy in the circuit continuously decreases and free oscillations die out.

In modern radio technology, undamped oscillations are used. To obtain such oscillations, it is necessary to continuously replenish the energy supply of the circuit to compensate for the losses. Such oscillations occurring in the circuit are called forced.

In Fig. 1.3 an electric circuit is shown consisting of a harmonic current generator and an inductor and a capacitor are connected in parallel.

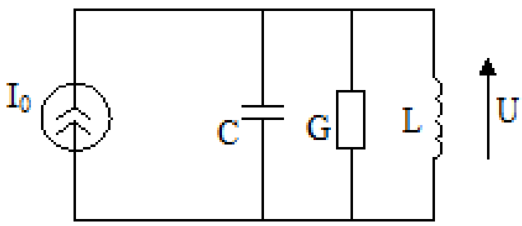


Fig. 1.3. An inductor and a capacitor are connected in parallel

Under the action of the generator, an alternating current flows through the elements of the oscillating circuit. Let's consider the dependence of reactive conductivity of inductance and capacitance on frequency.

When

$$\omega = 2\pi f \to 0; \ \frac{1}{X_C} \to 0; \ \frac{1}{X_L} \to \infty.$$

As frequency increases, the capacitance of the capacitor increases (Fig. 1.4).

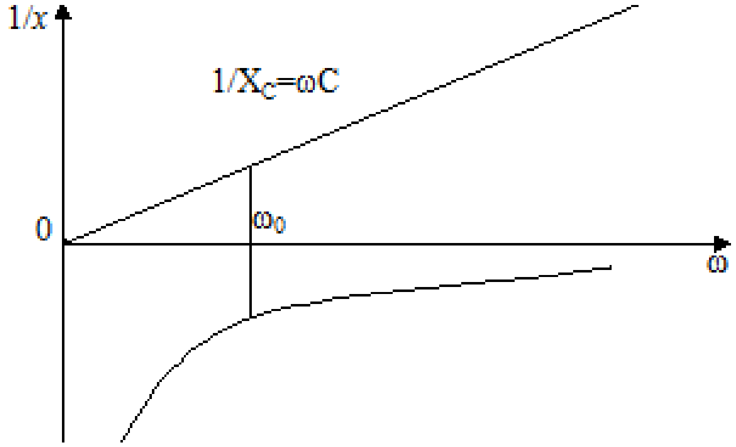


Fig. 1.4. Dependence of reactive conductivity of inductance and capacitance on frequency

Inductive conductivity decreases with increasing frequency of the generator. At some frequency – ω_0, the capacitive conductivity of the capacitor and the inductive conductivity of the coil become numerically equal.

That is,

$$\omega_o C = \frac{1}{\omega_o L}.$$

Hence

$$\omega_o = \frac{1}{\sqrt{LC}}$$

is the resonant frequency. The phenomenon that occurs in an electric circuit consisting of a generator and an inductor and a capacitor are connected in parallel, with the equality of capacitive and inductive conductivity, is called parallel resonance. It should be noted that the capacitive and inductive conductivity are equal in the case when the frequency of the generator is equal to the frequency of free oscillations (ω_0) of the circuit. At the same time, the currents of the inductive and capacitive branches are also equal – $I_L = I_C$. The current in the branch with the capacitor is ahead of the voltage by – $\frac{\pi}{2}$, and the current flowing through the inductor is behind the voltage by – $\frac{\pi}{2}$. Therefore, in a common unbranched circuit, these currents appear in antiphase. In an ideal circuit, the current in the general unbranched circuit is equal to 0. The absence of current in the general part of the circuit allows us to assume that the resistance of an ideal parallel circuit at resonance is infinitely large.

In a real circuit, part of the electrical energy is spent in active resistance and an active current – I_0 flows in the general circuit, which coincides in phase with the circuit voltage (Fig. 1.5).

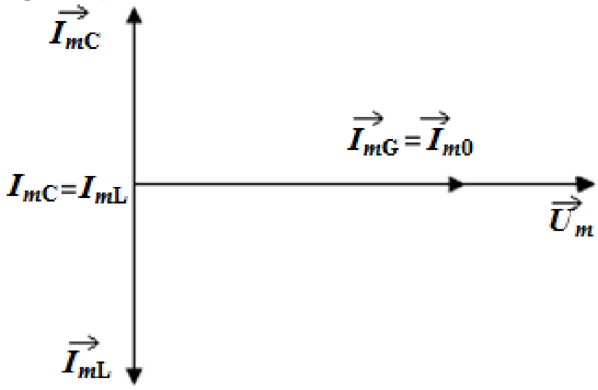


Fig. 1.5. Vector diagram of currents in a parallel oscillating circuit

The presence in the general circuit of the current – I_0, which coincides in phase with the circuit voltage, indicates that the resistance of a real parallel circuit at resonance is not infinitely large, but has a certain value and is active in nature. It should be noted that the current – I_0 in the general part of the circuit is much smaller in amplitude than the current in the reactive branches of the circuit.

The gradual rocking of an oscillating system to large amplitudes, with the condition that the rocking force acts in time with the period of its own oscillations, which is familiar from the experience of swinging a swing. Initially, under the action of a weak link, the system deviates from equilibrium very little, but when the efforts are repeated (in time with the period of its own oscillations), the system constantly oscillates. It is on such an accumulation of the action of weak forces that the swaying of the circuit is based. The energy coming from the source for each period is partially dissipated in the resistance of losses, and partially goes to increase the energy reserve of the oscillating system.

In the stable mode, the oscillations stop increasing and the total power of the source at each moment is equal to the power dissipated in the loss resistance. The same can be considered in relation to a series OC, which is acted upon by a source of harmonic electromotive force (HEF) (Fig. 1.6).

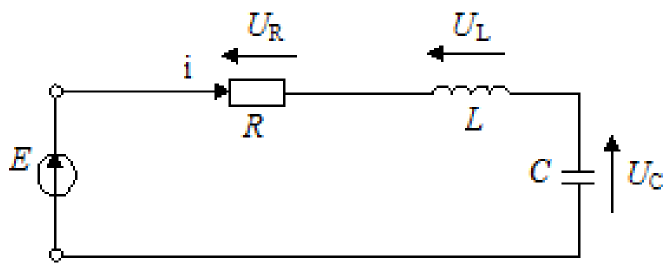


Fig. 1.6. A series oscillating circuit under the action of a HEF source

The phenomenon of resonance here consists in a sharp increase in the amplitude of the alternating current when the oscillation frequency of the generator coincides with the natural frequency of the oscillating circuit. At the resonance frequency, voltages – U_L and U_C mutually compensate each other due to the fact that at any frequency they are in antiphase, and at the resonance frequency, the amplitudes of these voltages are equal. For an ideal oscillating circuit ($R=0$, no losses) at resonance, the conductivity and, accordingly, the current amplitude are equal to infinity. For an oscillating circuit with losses ($R\neq 0$), the current amplitude is limited. The vector diagram of currents and voltages in a series OC at resonance is shown in Fig. 1.7.

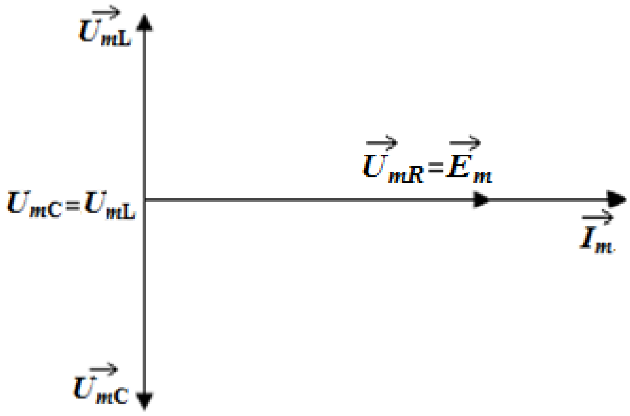


Fig. 1.7. The vector diagram of voltages in a series oscillating circuit

Using the symbolic method, we will calculate the parameters of the parallel oscillating circuit.

1. Complex circuit resistance

$$\dot{Z}=\frac{1}{\dot{Y}}=\frac{1}{G+j\omega\omega+\frac{1}{j\omega\omega}}=\frac{1}{G+j\left(\omega\left(-\frac{1}{\omega L}\right)\right.}=\frac{1}{\sqrt{G^2+\left(\omega\omega-\frac{1}{\omega L}\right)^2}}e^{-jarctg\frac{\omega C-\frac{1}{\omega L}}{G}}$$

2. The voltage on the circuit

$$\mathring{U}_m=\dot{I}_{m0}\dot{Z}=\frac{\dot{I}_{m0}}{G+j\left(\omega\left(-\frac{1}{\omega L}\right)\right)}=\frac{I_m e^j I_0}{\sqrt{G^2+\left(\omega\omega-\frac{1}{\omega L}\right)^2}}e^{-jarctg\frac{\omega C-\frac{1}{\omega L}}{G}}=\frac{I_m}{\sqrt{G^2}+\left(\omega\omega-\frac{1}{\omega L}\right)^2}e^{j\left(i_0-^{-jarctg\frac{\omega C-\frac{1}{\omega L}}{G}}\right)}$$

$$=U_m e^{j\varphi_u},$$

where

$$U_m=\frac{I_m}{\sqrt{G^2+\left(\omega\omega-\frac{1}{\omega L}\right)^2}};$$

$$\varphi_u = \varphi_{io} - arctg\frac{\omega C - \dfrac{1}{\omega L}}{G}.$$

3. The current through the inductance (taking into account the condition

$$\exp\left(\frac{j\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j,$$

$$\dot{I}_{mL} = \mathring{U}_m \frac{1}{j\omega\omega} = \frac{U_m e^{jU}}{\omega L e^{\frac{j\pi}{2}}} = \frac{U_m}{\omega L} e^{j\left(U - \frac{\pi}{2}\right)} = I_{mL} e^{jIL},$$

where

$$I_{mL} = \frac{U_m}{\omega L} = \frac{I_m}{\omega L\sqrt{G^2 + \left(\omega\omega - \dfrac{1}{\omega L}\right)^2}};\ \varphi_{IL} = \varphi_U - \frac{\pi}{2}.$$

4. The current through the capacitor:

$$\dot{I}_{mC} = \dot{U}_m\ j\omega\omega = U_m e^{jU} \times \omega C_e^{\frac{j\pi}{2}} = U_m \omega C_e^{j\left(U + \frac{\pi}{2}\right)} = I_m C_e^{jIC},$$

where

$$I_{mC} = U_m \omega C = \frac{I_m \omega C}{\sqrt{G^2 + \left(\omega\omega - \dfrac{1}{\omega L}\right)^2}};$$

$$\varphi_{IC} = \varphi_U + \frac{\pi}{2}.$$

5. The current through active conduction

$$\dot{I}_{mG} = \dot{U}_m G = U_m e^{jU} \times G = I_{mG} e^{jU},$$

where

$$I_{mG} = U_m G = \frac{I_m G}{\sqrt{G^2 + \left(\omega\omega - \dfrac{1}{\omega L}\right)^2}}.$$

Let us find out what $Z, U_m, I_{mL}, I_{mC}, I_{mG}$ are equal to at the resonant frequency. As noted, the frequency

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

is called resonant, while

$$\frac{1}{X_c} + \frac{1}{X_L} = 0.$$

1. Resistance of parallel oscillating circuit

$$Z = \frac{1}{\sqrt{G^2 + \left(\omega_0 C - \dfrac{1}{\omega_0 L}\right)^2}} = \frac{1}{G} = R_{res},$$

that is, at the resonance frequency, the resistance of the parallel oscillating circuit is maximum, it is equal to R_{res} and has a purely active character. The dependence of resistance on frequency is shown in Fig. 1.8. At frequencies $\omega < \omega_0$, the resistance has an active-inductive character, and at frequencies $\omega > \omega_0$ is inductive-active.

2. The amplitude of the current through to inductance

$$I_{mL} = \frac{U_m}{\omega_0 L} = \frac{I_m}{\omega_0 L \sqrt{G^2 + \left(\omega_0 C - \frac{1}{\omega_0 L}\right)^2}} = \frac{I_m}{\omega_0 LG} = I_m \frac{R_{res}}{\rho_L} = I_m Q,$$

where

$$\rho_L = \omega_0 L$$

is the wave resistance of the inductance;

$$Q = \frac{R_{res}}{\rho_L}$$

is the quality factor (the ratio of the resonant resistance to the wave resistance of the inductance).

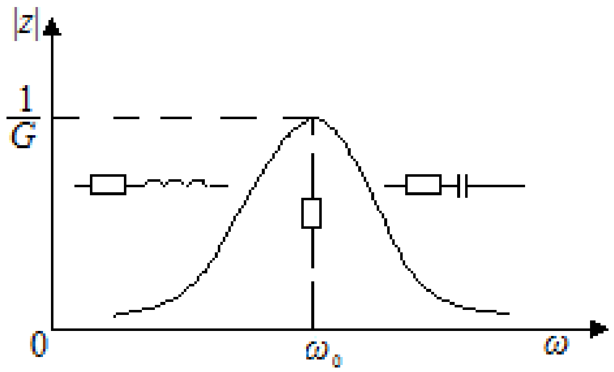


Fig. 1.8. Dependence of the resistance of the parallel oscillating circuit on the frequency

3. The amplitude of the current through the capacitance

$$I_{mc} = U_m \omega_0 C = \frac{I_m \omega_0 C}{\sqrt{G^2 + \left(\omega_0 C - \frac{1}{\omega_0 L}\right)^2}} = I_m \frac{R_{res}}{\rho_c} = I_m Q$$

where

$$\rho_c = \frac{1}{\omega_0 C}$$

is wave resistance of the capacity.

4. The amplitude of the voltage on the circuit

$$U_m = \frac{I_m}{\sqrt{G^2 + \left(\omega_0 C - \frac{1}{\omega_0 L}\right)^2}} = \frac{I_m}{G} = I_m R_{res},$$

that is, at resonance, the voltage on the oscillating circuit is maximum and is equal to $I_m R_{res}$. The dependence of the voltage on the parallel oscillating circuit on the frequency is shown in Fig. 1.9.

Let's analyze the concept of wave resistance in more detail ρ_L and ρ_C

$$\rho_L = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}},$$

$$\rho_C = \frac{1}{\omega_0 C} = \frac{1}{C\frac{1}{\sqrt{LC}}} = \frac{1}{\frac{C}{\sqrt{LC}}} = \sqrt{\frac{L}{C}}.$$

It turns out that $\rho_L = \rho_C = \rho$ and are simply called wave or characteristic resistance. It is numerically equal to the ratio of the voltage amplitude to the current amplitude of the reactive element at resonance.

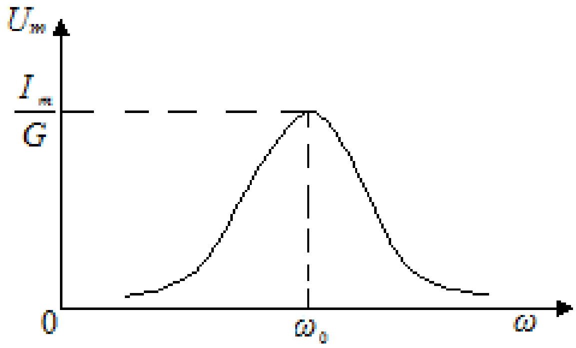


Fig. 1.9. Dependence of the voltage on the parallel oscillating circuit on the frequency

Conclusions.

1. At resonance, the resistance of the parallel oscillating circuit is maximum and purely active.

2. The voltage amplitude on the parallel oscillating circuit at resonance is maximal and equal to $I_m R_{res}$.

3. The value

$$\sqrt{\frac{L}{C}} = \rho$$

is called wave or characteristic resistance.

4. The ratio of the resonant resistance of a parallel oscillating circuit to the wave (characteristic) resistance is called the quality factor.

5. At resonance, the currents through the reactive elements of the parallel oscillating circuit are maximum. They exceed the source current by Q times, so this phenomenon is called current resonance.

After carrying out similar calculations, it is possible to draw appropriate conclusions for a series oscillatory circuit.

Resistance of the series oscillating circuit

$$Z=\sqrt{R^2+\left(\omega_0 L-\frac{1}{\omega_0 C}\right)^2}=R=R_{res},$$

this is, at the resonance frequency, the resistance of the series oscillating circuit is maximum, it is equal to R_{res} and has a purely active character. The dependence of resistance on frequency is shown in Fig. 1.10.

6. The amplitude of the voltage on the inductance

$$U_{mL}=I_m\omega_0 L=\frac{E_m\omega_0 L}{\sqrt{R^2+\left(\omega_0 L-\frac{1}{\omega_0 C}\right)^2}}=\frac{E_m\omega_0 L}{R}=E_m\frac{\rho_L}{R}=E_m Q,$$

where

$$Q=\frac{\rho_L}{R}$$

is quality factor (ratio of the wave resistance of the inductance to the resonant resistance).

7. The amplitude of the voltage on the capacitor

$$U_{mC}=\frac{I_m}{\omega_0 C}=\frac{E_m}{\omega_0 C\sqrt{R^2+\left(\omega_0 L-\frac{1}{\omega_0 C}\right)^2}}=E_m\frac{\rho_C}{R}=U_m Q.$$

At resonance, the voltages on the reactive elements of the series oscillating circuit are maximum. They exceed the source voltage by Q times, so this phenomenon is called voltage resonance.

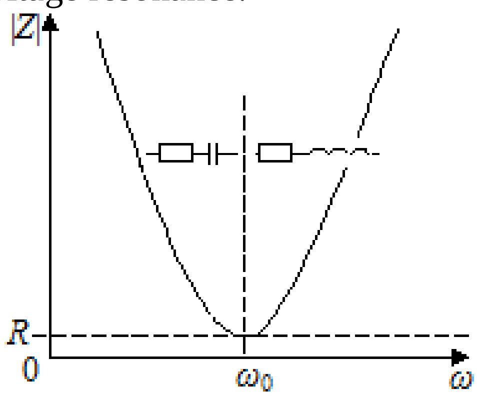


Fig. 1.10. Dependence of the resistance of the parallel oscillating circuit on the frequency

8. The amplitude of the current through the circuit

$$I_m=\frac{E_m}{\sqrt{R^2+\left(\omega_0 L-\frac{1}{\omega_0 C}\right)^2}}=\frac{E_m}{R},$$

that is, at resonance, the current through the oscillating circuit is maximum and is equal to $\frac{U_m}{R_{res}}$. The dependence of the current amplitude through the successive oscillating circuit on the frequency is shown in fig. 1.11.

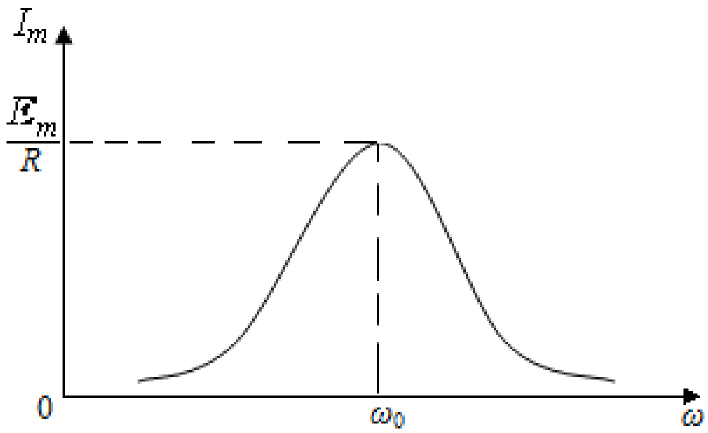


Fig. 1.11. Dependence of the current amplitude through a series oscillating circuit on the frequency

1.2. Frequency characteristics of parallel and series oscillating circuit

Let's examine the amplitude-frequency characteristic (AFC) is $\left|T(j\omega)\right|$ and the phase-frequency characteristic (PFC) is $\theta_{(\omega)} = \arg\left[T(j\omega)\right]$ for the transfer function of the parallel oscillating circuit

$$T_{(j\omega)} = \left|T(j\omega)\right| e^{j\theta(\omega)} = \frac{\dot{U}_m}{\dot{I}_{0m}} = Z = \frac{U_m e^{j\psi_u}}{I_{0m} e^{j\psi_{I_0}}} = \frac{U_m}{I_{0m}} e^{j\left(\psi_u - \psi_{I_0}\right)}.$$

Accordingly

$$\left|T(j\omega)\right| = |z| = \frac{1}{\sqrt{G^2 + \left(\omega\omega - \dfrac{1}{\omega L}\right)^2}},$$

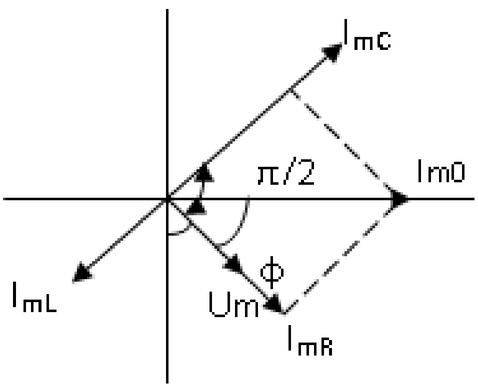
$$\theta_{(\omega)} = \arg[z] = arctg\left(\frac{\omega C - \dfrac{1}{\omega L}}{G}\right).$$

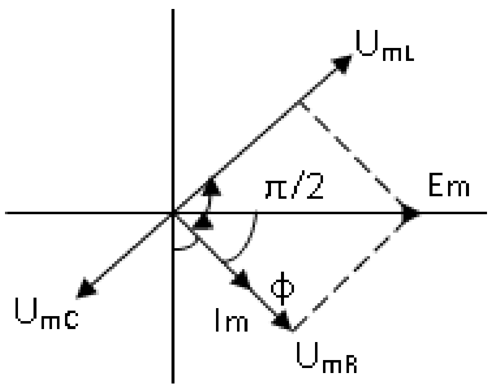
When $\omega \to 0$; $z_L = \omega L \to 0$; $|z| \to 0$ and all the setting current will flow through the inductance, the circuit voltage will lead the setting current by $\frac{\pi}{2}$, so $\theta_{(\omega)} = \psi_u - \psi_{I_0} \to \frac{\pi}{2}$.

When $\omega \to \infty$; $z_C = \frac{1}{\omega C} \to 0$; $|z| \to 0$. Thus, the voltage on the circuit will lag behind the current by $\frac{\pi}{2}$, a $\theta_{(\omega)} = \psi_u - \psi_{I_0} \to -\frac{\pi}{2}$.

When $\omega = \omega_0$ the circuit resistance is maximum and purely active $|z| = \frac{1}{G}$, the voltage on the circuit coincides in phase with the current, i.e $\theta_{(\omega)} = \psi_u - \psi_{I_0} = 0$.

Examples of vector diagrams of currents and voltages at $\omega \neq \omega_0$ (in this case $\omega > \omega_0$) are given for a parallel oscillating circuit in fig. 1.12,a and the series oscillating circuit in fig. 1.12,b.





a *b*

Fig. 1.12. Examples of vector diagrams of currents and voltages

Graphs of AFC and PFC for $T_{(j\omega)} = \frac{\dot{U}_m}{\dot{I}_0}$ are shown in fig. 1.13.

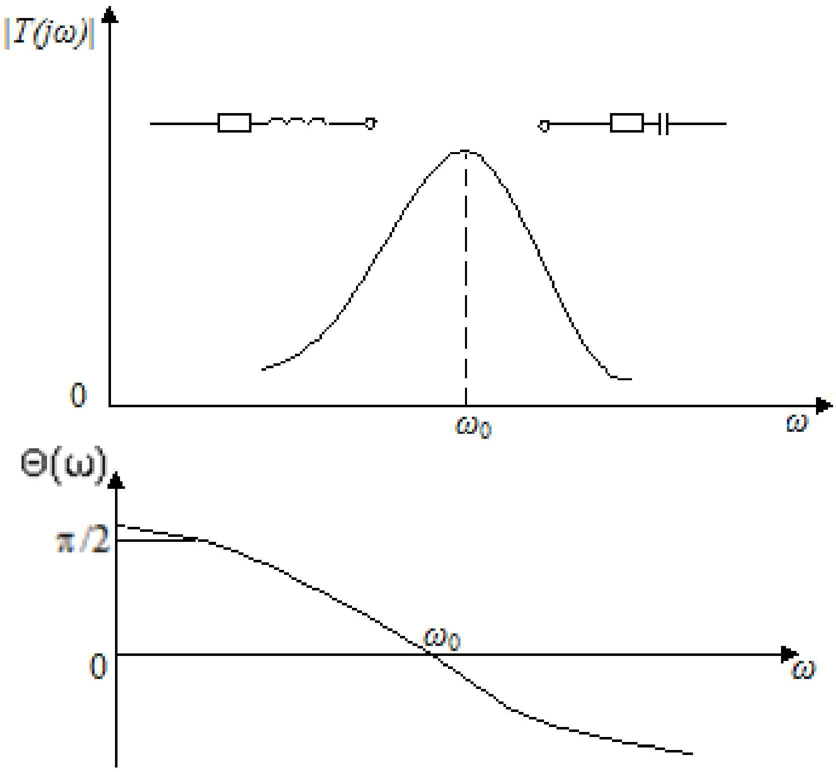


Fig. 1.13. Graphs of AFC and PFC

For a series oscillating circuit, the form of dependences of AFC and PFC is similar, considering that for him the transfer function $T_{(j\omega)} = \frac{\dot{I}_m}{\dot{E}_m}$.

Let us derive the expressions for frequency response and frequency response of a parallel oscillating circuit using parameters – Q and ω_0 (the same expressions can be obtained for a series oscillatory circuit).

$e^{jarctgQv}$

$$T_{(j\omega)} = \frac{\dot{U}_m}{\dot{I}_0} = Z_{(j\omega)} = \frac{1}{Y_{(j\omega)}} = \frac{1}{G + j\omega\omega + \frac{1}{j\omega\omega}} = \frac{1}{G\left[1 + j\left(\frac{\omega C}{G}\cdot\frac{\omega_0}{\omega_0} - \frac{1}{\omega L G}\cdot\frac{\omega_0}{\omega_0}\right)\right]} =$$

$$= \frac{1}{G\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]} = \frac{1}{G(1 + jQv)} = \frac{1}{G\sqrt{1 + (Qv)^2}\cdot e^{jarctgQv}},$$

where

$$v = \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

is a relative imbalance, hence

$$\left|T(j\omega)\right| = \frac{1}{G\sqrt{1+(Qv)^2}};$$

$$\theta(\omega) = -arctgQv.$$

Such an amplitude-frequency characteristic is called a <u>resonance characteristic</u> of a parallel oscillating circuit. The AFC has the maximum value at the resonance frequency

$$\omega = \omega_0 \left|T(j\omega)\right|_{\max} = \left|T(j\omega_0)\right| = \frac{1}{G}, \text{ PFC } \theta_{(\omega_0)} = 0.$$

The resonant characteristic of the circuit is usually normalized with respect to its maximum value. The normalized AFC

$$\left|T(j\omega)\right| = \frac{\left|T(j\omega)\right|}{\left|T(j\omega)\right|_{\max}} = \frac{1}{\sqrt{1+Q^2v^2}}$$

has a maximum value that is equal to unity regardless of the values G. The normalized AFC is convenient for comparing oscillating circuits with different quality factors: the higher the quality factor of the oscillating circuit, the sharper its resonance curve (Fig. 1.14).

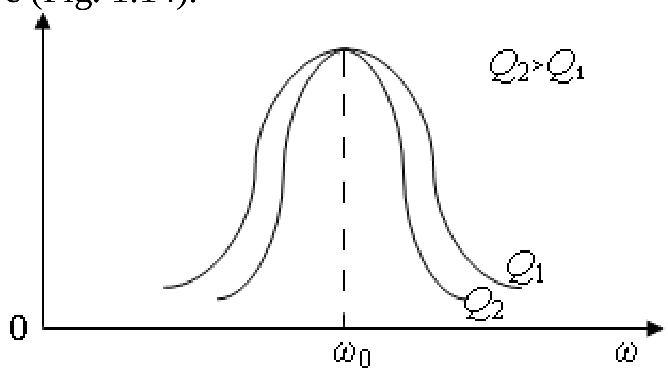


Fig. 1.14. Resonance characteristics of oscillating circuits

AFC has the property of geometric symmetry, i.e

$$\left|T(j\alpha\omega_0)\right| = \left|T\left(j\frac{\omega_0}{\alpha}\right)\right|.$$

In other words, the resonance frequency of the circuit is the geometric mean for any pair of frequencies – ω_α and $\omega_{-\alpha}$, at which the AFC of the circuit takes equal values $\omega_\alpha\omega_{-\alpha} = \omega_0^2$. On the other hand, the PFC values for the two frequencies – ω_α and $\omega_{-\alpha}$ differ only in signs

$$\theta(\alpha\omega_0) = -\theta\left(\frac{\omega_0}{\alpha}\right).$$

An important characteristic of the oscillating circuit is the bandwidth, which is defined as the frequency band at the boundaries of which the AFC value of the circuit is $0,707\cdot\left(\frac{1}{\sqrt{2}}\right)$ of its maximum value. It is denoted by

$$2\Delta\omega_n\left(or 2\Delta f_n\right).$$

According to fig. 1.15 $2\Delta\omega_n = \omega_1 - \omega_{-1}$.

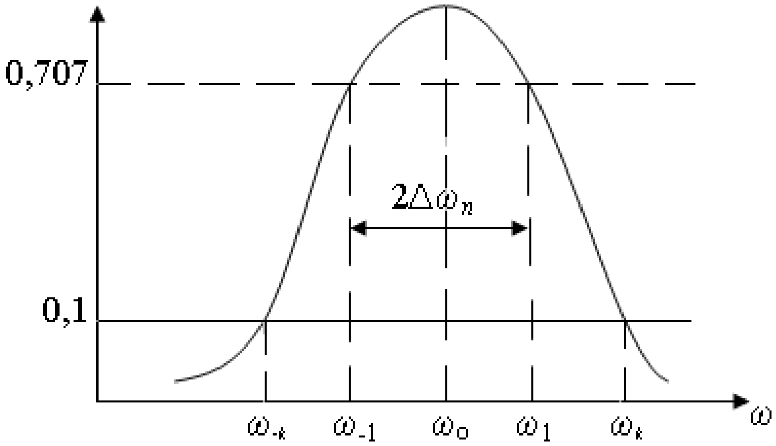


Fig. 1.15. AFC of the oscillating circuit

Let's derive the dependence between the bandwidth, the quality factor and the resonant frequency of the circuit.

$$\left|T\left(j\omega_1\right)\right| = \left|T\left(j\omega_{-1}\right)\right| = 0,707 = \frac{1}{\sqrt{2}};$$

$$\left|T\left(j\omega_1\right)\right| = \frac{1}{\sqrt{1+Q^2 v_1^{\,2}}} = \frac{1}{\sqrt{2}};$$

$$Q^2 v_1^2 = 1;$$

$$Q v_1 = 1;$$

$$Q\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = 1;$$

$$\frac{\omega_1^2 - \omega_0^2}{\omega_0 \omega_1} = \frac{1}{Q}.$$

On the property of geometric symmetry – $\omega_1\omega_{-1} = \omega_0^2$. Let's replace – ω_0^2 in the numerator of the previous equality on – $\omega_1\omega_{-1}$, we will get

$$\frac{\omega_1^2 - \omega_1\omega_{-1}}{\omega_0\omega_1} = \frac{1}{Q},$$

$$\frac{\omega_1\left(\omega_1 - \omega_{-1}\right)}{\omega_0\omega_1} = \frac{1}{Q},$$

$$\omega_1 - \omega_{-1} = \frac{\omega_0}{Q},$$

or

$$f_1 - f_{-1} = \frac{f_0}{Q}.$$

Thus, the bandwidth of the oscillating circuit is directly proportional to the value of the resonant frequency and inversely proportional to its quality factor

$$Q = \frac{\omega_0}{\omega_1 - \omega_{-1}},$$

$$Q = \frac{f_0}{f_1 - f_{-1}}.$$

This expression is also used to determine the quality factor of the circuit based on the results of measuring its resonant frequency and bandwidth.

The frequency band at the borders of which the AFC value is 0,1 (0,01; 0,001) from its maximum value is called the interference band.

The selective properties of the oscillating circuits are estimated by the squareness factor. The squareness factor is the ratio of the interference band to the transmission band

$$K_n = \frac{\omega_k - \omega_{-k}}{\omega_1 - \omega_{-1}}.$$

It can be shown that – K_n of a single OC when counting the interference band at the level of 0,1 from the maximum value is approximately equal to 10.

In order to improve the selectivity of oscillating systems in modern radio engineering equipment, along with single OCs, coupled circuits are used.

The most widely coupled OCs are used as amplifier loads in intermediate frequency amplifiers, where it is necessary to obtain an amplitude-frequency response close to a rectangular one.

In this connection, the task arises: on the basis of knowledge about single oscillating circuits, to determine the types and properties of connected oscillating circuits, to characterize these properties and to obtain a mathematical ratio of parameters.

1.3. Types of connection between circuits, their comparative assessment

Two circuits are said to be connected if energy from one circuit passes to the other. The circuit fed directly from the generator is called primary, and the circuit in which oscillations occur under the action of the primary circuit is called secondary.

The connection between circuits can be through a common electric or magnetic field or through a common resistance.

Depending on the nature of the connection, there are schemes:

1) with capacitive connection (external and internal);

2) with magnetic connection (transformer and autotransformer);

3) with mixed connection.

The degree of interaction of the circuits is estimated by the coefficient of connection, which is generally determined by the ratio

$$K = \frac{\chi_{con}}{\sqrt{\chi_1 \chi_2}},$$

where χ_{con} – resistance of the element of connection; χ_1, χ_2 – reactive resistances of the primary and secondary circuits have the same name as the element of connection.

This formula is suitable for circuits with transformer, autotransformer and internal capacitive connection.

The coefficient of connection shows what part of the electromotive force actually induced in the secondary circuit is of the limiting value of the electromotive force that could be induced by the primary circuit in the secondary

$$K = \frac{E_2}{E_{2\max}}.$$

The coefficient of connection can take values from 0 to 1 and is often expressed as a percentage.

Consider the scheme with an external capacitive connection (Fig. 1.16).

Such schemes find the most practical application in BT3, because they make it possible to seal the primary and secondary oscillating circuits, and to change the connection between them using C_{con}.

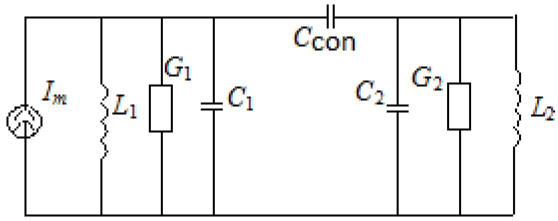


Fig. 1.16. AFC of the oscillating circuit

The degree of influence of one circuit on another is determined by the capacity of this capacitor. The C_{con} capacitor and the secondary circuit are connected in series. When the capacity of the capacitor C_{con} increases, its resistance decreases and its current, that supplies the secondary circuit, increases, that is, when C_{con} increases, the connection between the circuits

$$k = \frac{C_{con}}{\sqrt{(C_1 + C_{con})(C_2 + C_{зв})}},$$

increases and when

$$C_1 = C_2 = C_k = \frac{C_{con}}{C + C_{con}}.$$

The resonant frequency of the connected circuits

$$\omega_0 = \frac{1}{\sqrt{L(C + C_{con})}}$$

will change when the connection between the circuits changes, which is not desirable. Therefore, the capacity – C_{con} is chosen much smaller than the capacity of the circuit – $C_{con} \ll C$.

In the scheme with an internal capacitive of the connection (Fig. 1.17), the connection between the circuits is carried out through the capacitor – C_{con} of the connection.

Due to the primary circuit, a variable potential difference occurs on the capacitor – C_{con}, under the action of which a current is created in the secondary circuit.

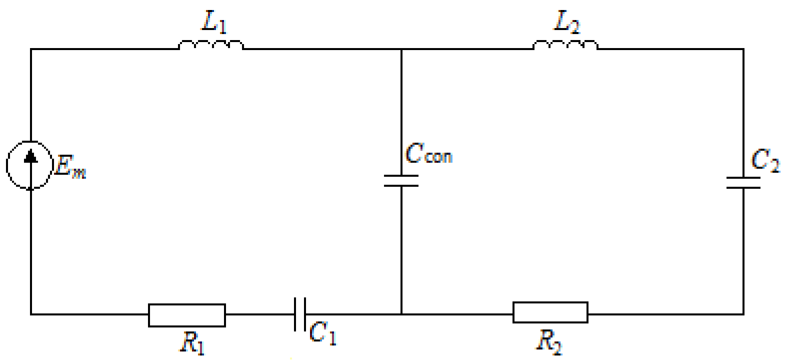


Fig. 1.17. Connected oscillating circuits with internal capacitive of the connection

In coupled circuits with internal capacitive of the connection, C_{con} is connected in series with the capacitors of the circuits and the equivalent capacity of each of the circuits will be determined by the smaller of the capacities. In order for the circuit tuning frequency to be determined by the circuit capacity and practically independent of – C_{con}, the connection capacity is chosen much larger than the circuit capacity. At the same time

$$k = \frac{C_1' C_2'}{C_{con}},$$

where

$$C_1' = \frac{C_1 C_{con}}{C_1 + C_{con}};$$

$$C_2' = \frac{C_2 C_{con}}{C_2 + C_{con}}.$$

Consider connected circuits with a transformer connection (Fig. 1.18).

The connection between the circuits is carried out through the magnetic flux common to the coils, that is, due to the mutual induction M.

The current I_1 creates a magnetic flux around the coil, part of which crosses the turns of the coil of the secondary circuit, creating an electromotive force of mutual induction in it. Under its action, a mutual induction current I_2 is created in the secondary circuit, which releases some active power in the active resistance.

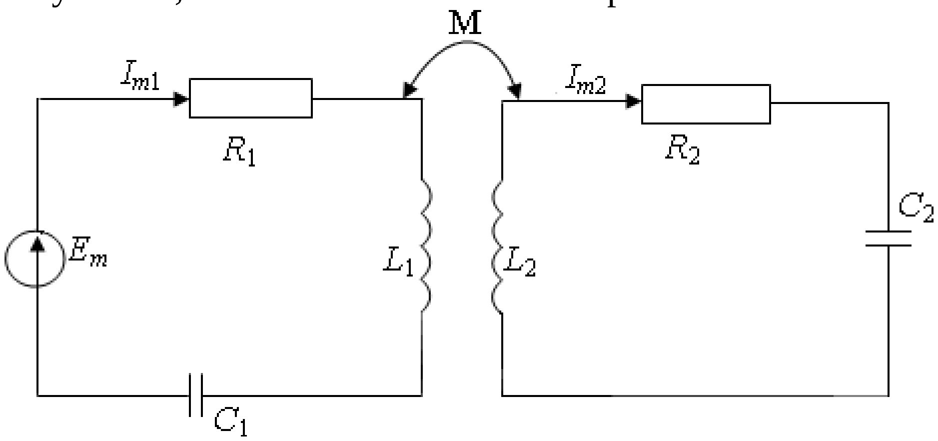


Fig. 1.18. Connected oscillating circuits with a transformer connection

Thus, it is possible to talk about the transfer of energy from the primary circuit to the secondary circuit. The degree of mutual influence of the circuits depends on the distance between the coils and their mutual location. The smaller the distance between the coils, the stronger the connection between the circuits.

When the coils are arranged perpendicular to each other, the electromotive force induced in the coil of the secondary circuit is zero even with a small distance between the coils. By turning one of the coils, it is possible to change the value of the connection between the circuits.

$$k = \frac{X_{зз}}{\sqrt{X_1 X_2}} = \frac{\omega M}{\sqrt{\omega L_1 \omega L_2}} = \frac{M}{\sqrt{L_1 L_2}}.$$

With autotransformer connection (Fig. 1.19), the part of the coil of the primary circuit, common to the primary and secondary circuits, serves as a communication element. Due to the currents of the primary circuit, a variable potential difference occurs on the coil, which creates a current in the secondary circuit. Accordingly, the correlation coefficient is equal to

$$k = \frac{L_{con}}{L_1 + L_{зоп}} = \frac{1}{1 + \frac{L_1}{L_{зв}}}.$$

The considered schemes do not exhaust the variety of systems of connected circuits: by combining different types of communication and parameters of the circuits themselves, it is possible to create different types of systems of connected circuits. For example, an inductive-capacitive connection between circuits.

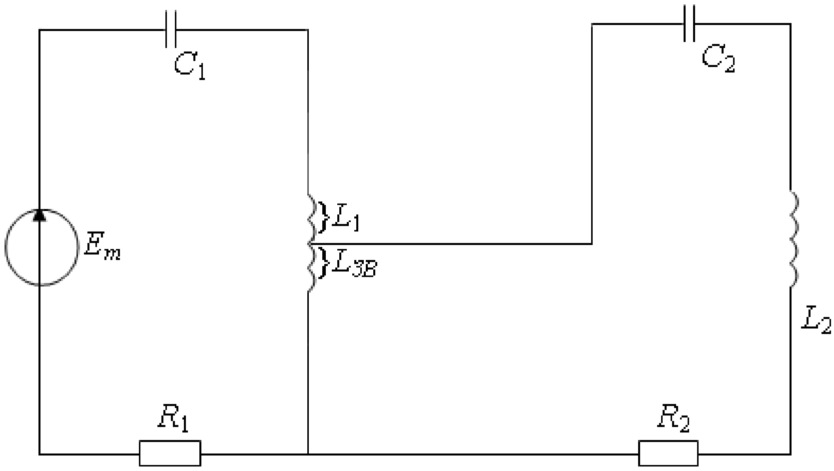


Fig. 1.19. Connected oscillating circuits with an autotransformer connection

Despite the difference in connection types, the processes in connected oscillatory circuits are subject to general laws.

1.4. Frequency characteristics of connected oscillatory contours at different degrees of connection

When considering physical processes in connected OC's, we talked about the transfer of energy from the primary circuit to the secondary circuit. It turns out that not only the primary circuit affects the secondary circuit, excites electromotive force in it, but also the secondary circuit affects the primary circuit, changing its mode. Let's consider in more detail the influence of the secondary circuit on the primary one using the example of circuits connected inductively (Fig. 1.18).

We will have for the intrinsic resistances of the primary – $\left(\dot{Z}_1\right)$ and secondary – $\left(\dot{Z}_2\right)$ circuits

$$\dot{Z}_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1} = R_1 + \left(\omega L_1 - \frac{1}{\omega C_1}\right) = R_1 + jX_1\,;$$

$$\dot{Z}_2 = R_2 + j\omega L_2 + \frac{1}{j\omega C_2} = R_2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right) = R_2 + jX_2\,,$$

Where – X_1 i X_2 are reactive resistances of the primary and secondary circuits.

For the primary and secondary circuits, we will compile equations using the method of circuit currents, taking into account the markings made

$$\dot{Z}_1 \dot{I}m_1 - j\omega M \dot{I}m_2 = \dot{E}m\,;\; -j\omega M \dot{I}m_1 + \dot{Z}_2 \dot{I}m_2 = 0\,.$$

Let's find the solution of this system by the method of determinants

$$\Delta = \begin{pmatrix} \dot{Z}_1 & -j\omega M \\ -j\omega M & \dot{Z}_2 \end{pmatrix} = \dot{Z}_1 \dot{Z}_2 + \omega^2 M^2.$$

$$\Delta_1 = \begin{pmatrix} \dot{E}_m & -j\omega M \\ 0 & \dot{Z}_2 \end{pmatrix} = \dot{E}_m \dot{Z}_2,\; \Delta_2 = \begin{pmatrix} \dot{Z}_1 & \dot{E}_m \\ -j\omega M & 0 \end{pmatrix} = j\omega M \dot{E}_m\,.$$

Therefore,

$$\dot{I}_{m_1} = \frac{\Delta_1}{\Delta} = \frac{\dot{E}_m \dot{Z}_2}{\dot{Z}_1 \dot{Z}_2 + \omega^2 M^2} = \frac{\dot{E}_m}{\dot{Z}_1 + \dfrac{\omega^2 M^2}{Z_2}}\,;$$

$$I_{m_2} = \frac{\Delta_2}{\Delta} = \frac{j\omega M E_m}{\dot{Z}_1 \dot{Z}_2 + \omega^2 M^2}\,.$$

In practice, the following designations are used:

$$\omega M = X_{con}$$

is reactive resistance of the connection,

$$\frac{X_{con}^2}{Z_2} = Z_{\text{int}}$$

is reactive resistance introduced into the circuit, which reflects the influence of the second circuit on the first.

Let us consider in more detail the nature of the introduced resistance

$$Z_{\text{int}} = \frac{X_{con}^2}{Z_2} = \frac{X_{con}^2}{R_2 + jX_2} = \frac{X_{con}^2 R_2 - jX_{con}^2 X_2}{R_2^2 + X_2^2} = \frac{X_{con}^2 R_2}{R_2^2 + X_2^2} - j\frac{X_{con}^2 \cdot X_2}{R_2^2 + X_2^2} = R_{\text{int}} - jX_{\text{int}}.$$

In this way, the introduced resistance is complex:

– its active component increases the losses of the primary circuit, worsens its quality factor, and expands the bandwidth;

– the reactive component of the introduced resistance is opposite in nature to the own reactive component of the primary circuit and leads to its disorder.

The magnitude of the applied resistance can be changed by adjusting the circuits or changing the connection between them.

Let's investigate the dependence of the active component of the introduced resistance on the coefficient of the connection and the frequency of the input signal. We consider that the primary parameters of both circuits are the same. Then

$$R_{\text{int}} = \frac{X_{con}^2 R}{R^2 + X^2}.$$

The dependence of this type is a bell-shaped curve, which reaches a maximum at the frequency – $\omega = \omega_0$, where – $x = 0$. Then,

$$R_{\text{int max}} = \frac{X_{con}^2 R}{R^2} = \frac{X_{con}^2}{R}$$

or

$$X_{con}^2 = R_{\text{int max}}.$$

It can be seen from this expression that the introduced resistance – $R_{int\ max}$ is always less than – R. At a certain value of – X_{int} at the resonance frequency

$$\omega_0 R_{int} = R.$$

Let's determine the condition for the fulfillment of this equality

$$X_{int} = \omega_0 M = \omega_0 kL = R; \omega_0 kL / R = 1; k\rho / R = 1; kQ = 1.$$

The product kQ is called the connection parameter. The condition $kQ = 1$ is a critical connection condition. If $kQ < 1$, then the connection is weak, if $kQ > 1$, then the connection is strong.

The graphs of the dependence of R_{int} on ω for different connection parameters are shown in Fig. 1.20,a. Let's investigate the dependence

$$X_{\text{int}} = \frac{X_{con}^2 X}{R^2 + X^2}$$

on – ω for different connection parameters (X_{con}). At any X_{con} at the frequency $\omega = \omega_0 x = 0$, therefore, $X_{int}(\omega_0) = 0$. Near the frequency $\omega_0 X << R$. Thus, a simplified formula

$$X_{\text{int}} = \frac{X_{con}^2 X}{R^2} \approx (kQ)^2 X$$

can be derived.

It can be seen from this expression that for small connection parameters $kQ < 1$ (weak connection) X_{int} is always less than X . When $kQ = 1$ (critical connection) X_{int} can reach X , and when $kQ > 1$ (strong connection) there are two frequencies at which $X_{int} = X$.

The graph of the dependence of X_{int} on the frequency for various connection parameters is shown in Fig. 1.20,b. The dependence of the secondary circuit current on the frequency of the input signal at various connection parameters is shown in Fig. 1.20,c.

It is not difficult to explain the course of the resonance curves using the theorem on the maximum average power in the load. At the same time, the internal resistance of the generator should be understood as the value of the input resistance, and the load should be understood as the value of the resistance of the primary circuit.

It is necessary to pay attention to the fact that the second condition of the theorem is always fulfilled at the resonance frequency at any degree of connection

$$X_n = -X\left(X_{int} = -X = 0\right).$$

With a weak connection $R_{int} < R$ and at the frequency $\omega = \omega_0$, one of the two conditions for transmitting the maximum average power is not fulfilled. Therefore, the current I_2 at the resonant frequency is less than the maximum possible (Fig. 1.20,c).

At the critical connection $R_{int} = R$, at the frequency $\omega = \omega_0$ (Fig. 1.20,a), therefore, at the resonance frequency $I_2 = I_{2max}$ (Fig. 1.20,c).

With a strong connection at the resonant frequency $R_{int} > R$ (Fig. 1.20,a), that is, one of the conditions for transmitting the maximum average power is not fulfilled at this frequency. However, both conditions are fulfilled at two frequencies ω_{-m} and ω_m, at which the current I_2 becomes maximally possible. This leads to the appearance of the so-called two-humped frequency characteristic (Fig. 1.20,c).

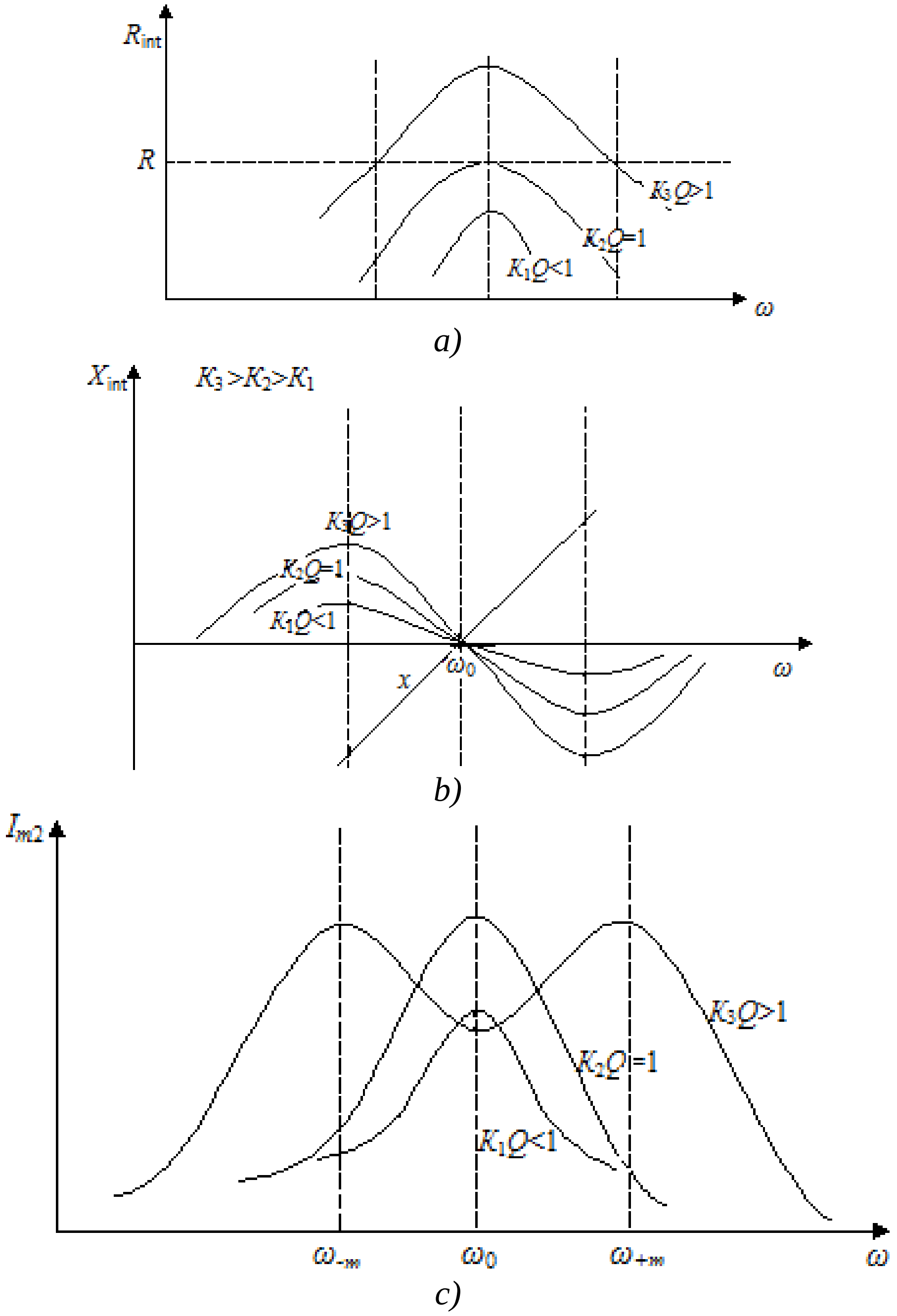


Fig. 1.20. AFC of connected oscillatory circuits for different degrees of connection

At $kQ = 0,67$, the bandwidth of connected circuits, calculated at $0,707$ of the maximum value, will be the same as that of a single circuit having the same quality factor.

At a critical connection ($\kappa Q = 1$), the bandwidth is $\sqrt{2}$ times wider than that of a single oscillating circuit. With strong connection ($\kappa Q > 1$the AFC of the connected oscillating circuits will be double-humped, and the dip of the double-humped curve is greater, the more κQ differs from unity. Usually, the connection is increased until the dip of the curve reaches the level of $0,707$ from the maximum AFC value (κQin this case is equal to $2,41$). In this case, the bandwidth of the connected circuits is $3{,}11$ times greater than the bandwidth of a single oscillatory circuit (when $Q_{зкк} = Q_{окк}$), and the coefficient of squareness $K_n = 2{,}32$, which is significantly better than that of a single oscillatory circuit and connected oscillating circuits with a weak or critical connection.

1.5. The method of setting connected circuits

In practice, they often strive to obtain the maximum value of the current I_2 or the maximum power P_2 in the secondary circuit. Such a problem can arise, for example, when the transmitter is connected to the transmitting antenna. This is achieved by setting the connected circuits accordingly. There are different ways to set it up.

I method. The setting of connected circuits is carried out by changing the parameters of only the primary circuit (for example, the capacity C_1); the resulting resonance is called the first resonance.

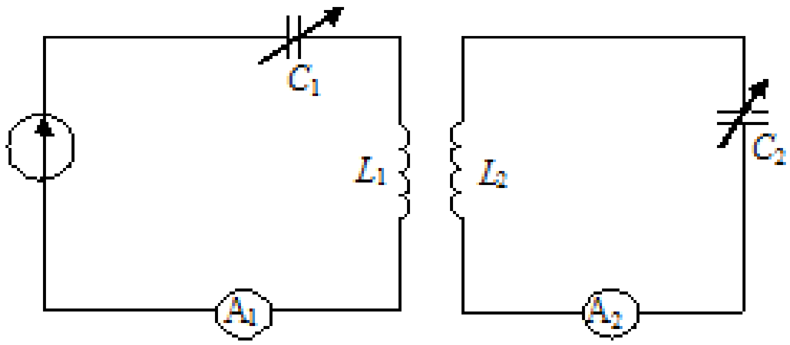


Fig. 1.21. Setting up connected circuits by changing the parameters of only the primary circuit

This resonance can be determined by the maximum current in the primary or secondary circuit. In fig. 1.22 shows the dependence of the currents in the primary and secondary circuits on the setting of the primary circuit.

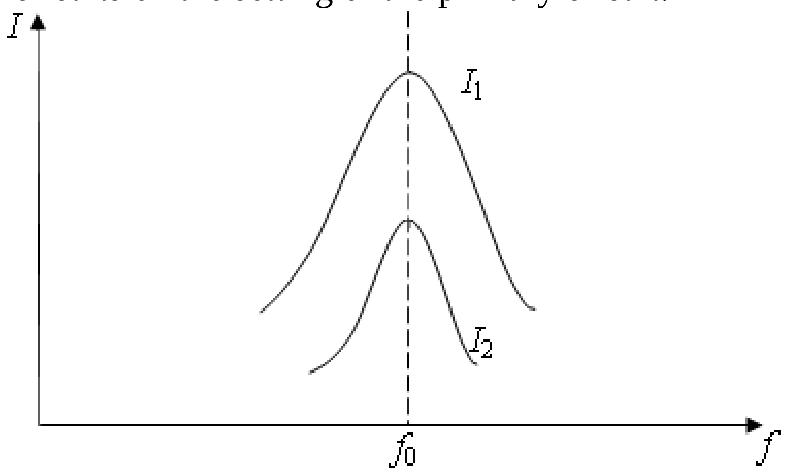


Fig. 1.22. Dependence of currents in the primary and secondary circuits on the setting of the primary circuit

Method II. The setting of connected circuits is carried out by changing the parameters of only the secondary circuit, for example, by changing the capacity C_2. In this case, a secondary self-resonance occurs. This resonance can be determined by the maximum current in the secondary circuit.

In fig. 1.23 shows the dependence of currents in the primary and secondary circuits on the setting of the secondary circuit.

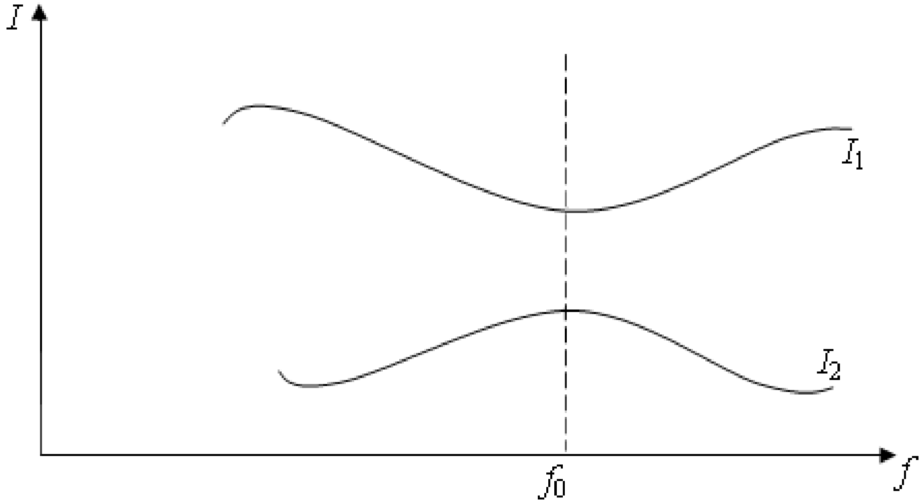


Fig. 1.23. Setting up linked circuits by changing the parameters of only the secondary circuit

Method III. The setting of connected circuits is carried out by changing the parameters of one of the circuits and the connection resistance. Resonance in this case is called complex.

It can be shown that with complex resonance, the maximum of the secondary current does not depend on which of the circuits is set to resonance. This method is used in low-power radio stations.

Method IV. The setting of the connected circuits is carried out by changing the parameters of both circuits and the connection resistance. In this case, the resonance is called complete. At the same time, the primary circuit is first configured with the secondary circuit open. Then the secondary circuit is set up. Finally, the optimal connection resistance is selected.

Although the maximum secondary current with full resonance tuning is the same as with complex resonance tuning, full resonance setting has the advantage that the absolute value of the connection resistance turns out to be smaller than when tuning to a complex resonance, and is calculated in units of resistance (Ohm).

Oscillating circuits are used in various radio technical devices, but they are mainly used in radio transmitting and receiving devices.

<u>In transmitters</u>, the values of capacitance and inductance of the circuit determine the frequency of oscillations at which the transmitter operates. By changing the capacitance or inductance of the circuit, the frequency of oscillation can be changed.

<u>In the receiving device</u>, OC determines the selectivity of the receiver, that is, its ability to single out the signal of one specific transmitter from the set of signals received by the antenna. By changing the capacitance or inductance of the

oscillating circuit, we can select the desired one from the signals received by the antenna, that is, we can tune the receiver to one or another frequency.

Oscillating circuits also solve a number of other radio engineering problems: measuring frequency, coil inductance and capacitor capacity, filtering currents of different frequencies, etc.

Coupled oscillating circuits are widely used in engineering, improve the selectivity of the resonant system, expand the bandwidth and allow for its adjustment. At the same time, they are characterized by the following disadvantages: an increase in the number of elements of the circuit, that reduces its reliability, the complexity of tuning, and a decrease in quality factor.

1.6. Oscillating circuits of II and III types

Oscillating circuits of the so-called II and III types are used for coordination without changing the quality factor of the oscillating circuit with radio engineering devices. In fig. 1.24 shows the oscillating circuit of the II type.

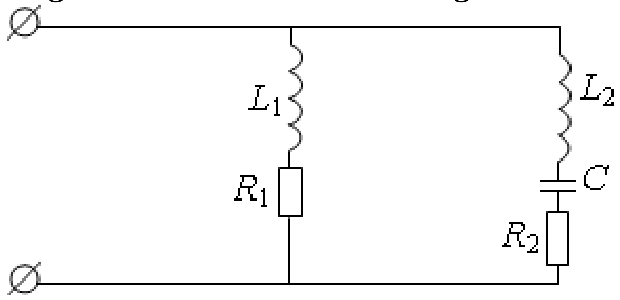


Fig. 1.24. Oscillatory circuit of the II type

Let's determine the resonant frequency of this oscillating circuit. For the solution, we calculate the complex conductivity of the circuit:

$$Y = \frac{1}{R_1 + j\omega_1 L_1} + \frac{1}{R_2 + j(\omega_1 L_2 - \frac{1}{\omega_1 C})} = \frac{R_1}{R_1^2 + (\omega_1 L_1)^2} + \frac{R_2}{R_2^2(\omega_1 L_2 - \frac{1}{\omega_1 C})^2} -$$

$$-j(\frac{\omega_1 L_1}{R_1^2 + (\omega_1^2 L_1^2)} + \frac{\omega_1 L_2 - \frac{1}{\omega_1 C}}{R_2^2(\omega_1 L_2 - \frac{1}{\omega_1 C})^2}).$$

At resonance, the reactive component of the conductivity turns into 0. Considering that active resistances are small relative to reactive resistances, we obtain a condition for determining ω_1 in the form

$$1/\omega_1 L_1 + [1/(\omega_1 L_2 - 1/\omega_1 C_2)] = 0.$$

Hence

$$\omega_1 = 1/|H(j\omega(| = \frac{1}{\sqrt{d_0\omega^{2m} + d_1\omega^{2m-2} + ... + d_{m-1}\omega^2 + d_m}}.$$

We will also determine the frequency of series resonance ω_2 in the oscillating circuit L_2C on the condition that

$$\omega_2 L_2 - 1/\omega_2 C = 0.$$

Then

$$\omega_2 = 1/\sqrt{L_2 C}.$$

Thus, the AFC of the oscillating circuit (the dependence of the voltage on the oscillating circuit on the frequency) has a clearly defined maximum (at the frequency ω_1) and minimum (at the frequency ω_1) (Fig. 1.25).

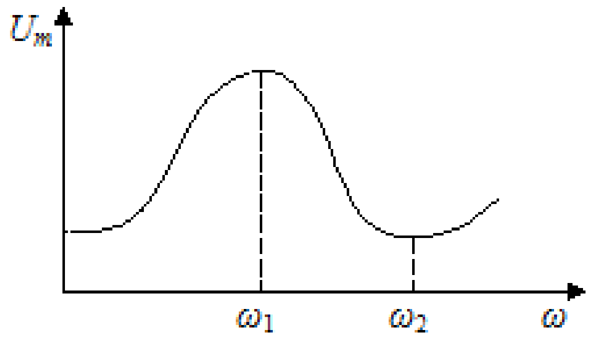


Fig. 1.25. AFC of the oscillating circuit of the II type

The oscillating circuit of type III is shown in fig. 1.26.

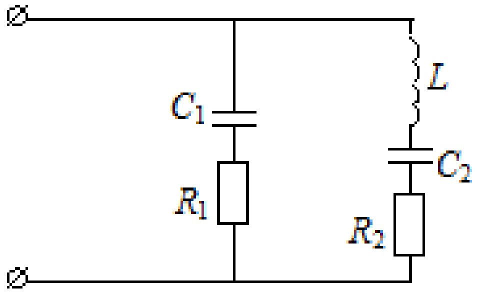


Fig. 1.26. oscillating circuit of the III type

Accordingly, its AFC is shown in fig. 1.27.

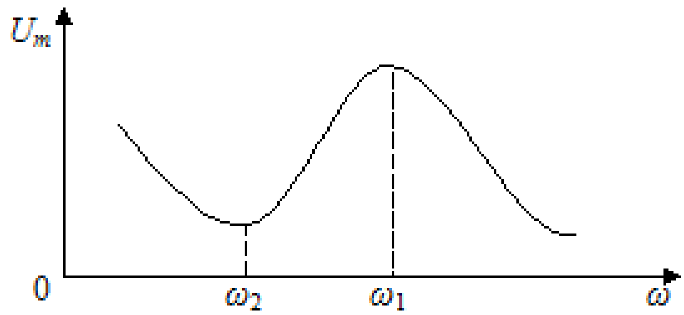


Fig. 1.27. AFC of the oscillating circuit of type III

The series resonance frequency

$$\omega_2 = 1/\sqrt{LC_2}\,,$$

and the parallel resonance frequency

$$\omega_1 = \sqrt{(C_1 + C_2)/LC_1C_2}\,.$$

Control questions and tasks

1. In which mode of the circuit in the parallel circuit is the resonance of the currents observed ?

2. How does the resonant impedance of the circuit behave when its quality factor changes ?

3. How will the transparency band of the parallel circuit change if the internal resistance of the electromotive force source decreases ?

4. How to calculate the resonant frequency of a parallel circuit with:

a) small losses;

b) big losses ?

5. How is the quality factor determined: a) of the loaded circuit; b) of an unloaded circuit ?

6. At what frequencies does the input resistance of the parallel circuit have:

a) a capacitive character;

b) an inductive nature;

c) an active character ?

7. How much should the circuit be unbalanced so that the output voltage is halved ?

8. Describe the processes that occur during free electromagnetic oscillations in a circuit. How to determine the period of oscillations ?

9. Derive the equations of damping oscillations in an oscillatory circuit.

10. What circuit resistance is called critical ?

11. What is logarithmic decrement fading ?

12. How do oscillations occur in an electrical circuit ?

13. Under what conditions does voltage resonance occur ?

14. Derive the formula for the resonant frequency.

15. What is the quality factor of the circuit ? What does it depend on ?

16. Why are the oscillations damped in a real circuit ?

CHAPTER 2. ELECTRICAL FILTERS

One of the most common devices in communication and radio technology are electrical filters. They are used to isolate or suppress certain oscillations, separate channels, and form the spectrum of signals. The need to create filters arose in connection with solving the problems of the condensed use of communication lines, which transmit messages on several channels at the same time with the distribution of these channels by frequency. At the same time, each channel has its own frequency band.

An inductor and a capacitor, which are used to separate the frequency band from the upper frequency band, turned out to be the simplest filtering device.

In the thirties of the 20th century, the development of the modern theory of the construction of electrical filters began, based on the use of strict mathematical methods of the best approximation of functions developed by the mathematician P.L. Chebyshev, his students and followers. The use of these methods made it possible to ensure the construction of electrical filters with the required characteristics with the minimum required number of elements. Today, the practical use of electrical filters is quite diverse. They are used in radio communication equipment, radio relay communication, transmission systems with frequency distribution of channels, automation, instrument building and other areas of technology where the principle of frequency distribution is used.

An electric filter (EF) is a quadripole that passes without attenuation (*attenuation or damping will be denoted as working attenuation in the theory of quadrupoles* A_p) or with a small attenuation of oscillations of certain frequencies and passes with a large attenuation of oscillations of other frequencies.

The band of frequencies in which the attenuation is small is called the passband. The frequency band in which the attenuation is large is called the stopband (delay). Between these bands is a transitional area.

According to the location on the frequency scale of the passband, the following filters are distinguished:

– low-pass filter (LPF), in which the passband is located on the frequency scale from $\omega = 0$ to some limiting frequency $\omega = \omega_p$, and the stopband from the frequency $\omega = \omega_s$ to infinitely high frequencies;

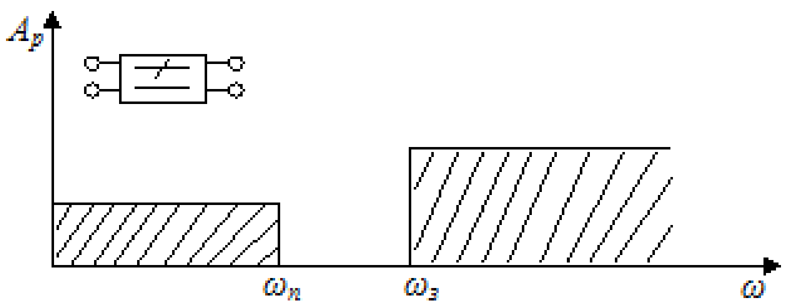


Fig. 2.1. Bandwidth and delay for LPF

– high-pass filter (HPF) with a passband from the frequency $\omega=\omega_p$ to infinitely high frequencies with a stopband from $\omega=0$ to $\omega=\omega_s$;

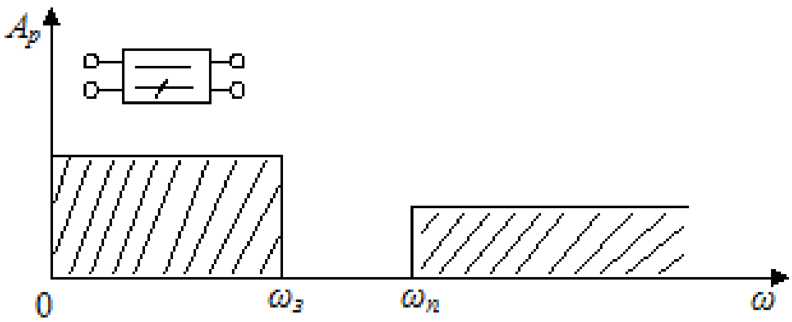


Fig. 2.2. Bandwidth and delay for HPF

– bandpass filter (BF), in which the passband $\omega_{p1}-\omega_{p2}$ is located between the stopband $0-\omega_{s1}$ i $\omega_{s2}-\infty$;

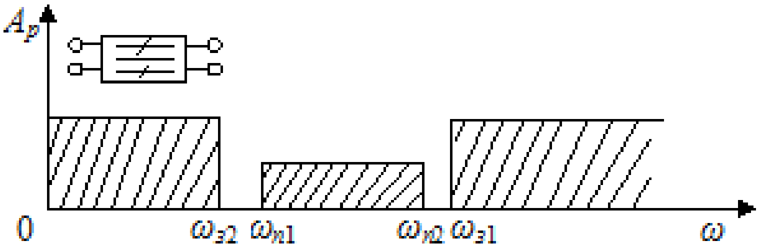


Fig. 2.3. Bandwidth and delay for BF

– blocking (rejector) filter (BF or RF), in which there is a stopband $\omega_{s1}-\omega_{s2}$ between the passband $\omega_{p1}-\omega_{p2}$;

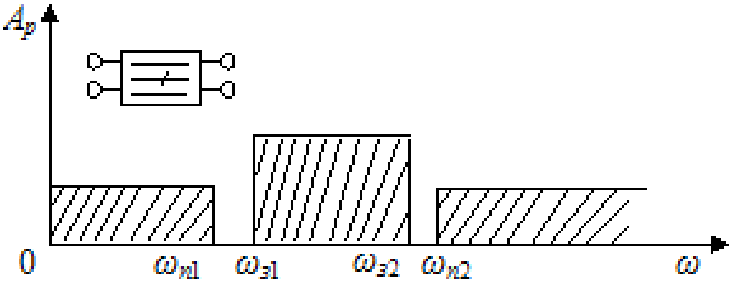


Fig. 2.4. Bandwidth and delay for RF

– comb filter (GF), which have several passband and stopband

According to the used elemental base, several classes of filters have been distinguished in the modern period.

Historically, the first are passive filters containing elements L and C. They are called LC-filters.

In many cases, quite high selectivity was required in practice (the difference in the attenuation in the passband and stopband is tens of thousands of times). This led to the appearance of filters with mechanical resonators: quartz, magnetostrictive, electromechanical.

The most significant achievements in the field of the theory and design of filters are connected with the successes of microelectronics. The requirements for the microminiaturization of radio-electronic equipment led to the rejection of the use of inductors that have large dimensions. Thus, active RC-filters appeared, consisting of resistors, capacitors and active devices. These filters can be made in the form of a micromodule design or an integrated microcircuit. The creation of digital communication systems and achievements in the field of digital computers stimulated the formation of filters based on elements of digital and computing technology - digital filters.

2.1. Conditions for passing a reactive filter

Let's find out the conditions under which a reactive quadrupole will be an electric filter, that is, a device that has a passband in some frequency range, and a stopband in another. To do this, let's turn to the theory of quadripoles and consider the simplest L-shaped quadripole (Fig. 2.5).

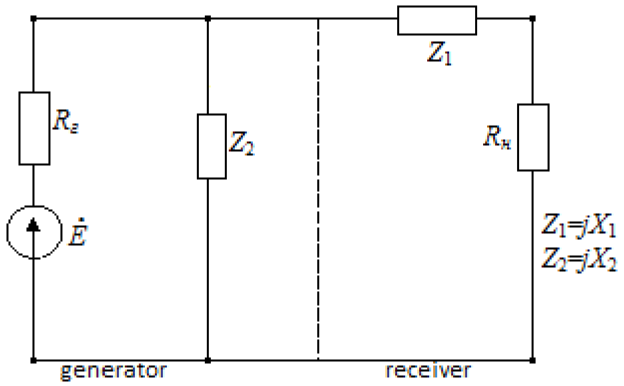


Fig. 2.5. The simplest L-shaped quadripole

Assume that Z_{1} and Z_2 are purely reactive resistors. According to the theorem about the maximum average power, $P_{\max}$ in the load will be in the case when the resistance of the generator and the resistance of the load are complex conjugate, i.e

$$R_H + jX_1 = \frac{1}{\frac{1}{R_Г} - \frac{1}{jX_2}} = \frac{1}{\frac{1}{R_Г} + j\frac{1}{X_2}}.$$

Transforming this expression, we get

$$\frac{R_H}{R_Г} - \frac{X_1}{X_2} + j\left(\frac{X_1}{R_Г} + \frac{R_H}{X_2}\right) = 1 + j0.$$

Hence we have two equalities:

$$R_H R_Г = -X_1 X_2$$

and

$$\frac{R_H}{R_Г} = 1 + \frac{X_1}{X_2}.$$

It can be seen from the first expression that X_1 and X_2 must have opposite signs, that is,

$$\frac{Z_1}{Z_2} < 0.$$

Accordingly, it follows from the second that

$$\frac{X_1}{X_2} > -1$$

or

$$\frac{Z_1}{Z_2} > -1 .$$

Combining these inequalities together, we get

$$-1 < \frac{Z_1}{Z_2} < 0 .$$

Since Z_1 and Z_2 are frequency dependent, the resulting inequality will hold for a specified band of frequencies called the passband.

To obtain a filter, it is necessary to include reactivities of different signs in the step structure in the longitudinal and transverse branches, that is, capacitance and inductance (Fig. 2.6, a, b), because with the same reactivities, it will always be

$$\frac{Z_1}{Z_2} > 0$$

and the transmission condition will not be fulfilled.

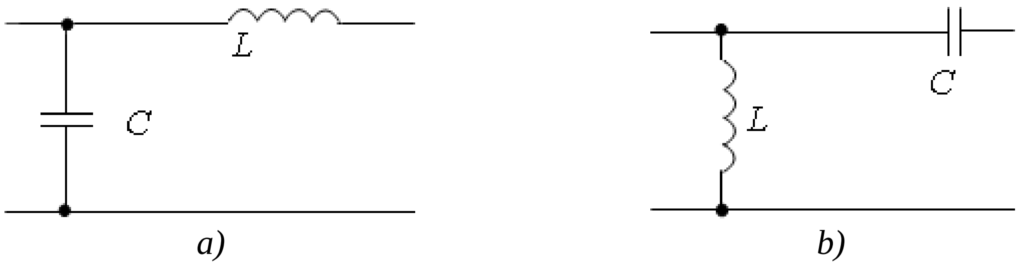


Fig. 2.6. Links of electric filters

2.2. Requirements for electrical characteristics of filters

The selectivity of the filter (the degree of demarcation of passing and non-passing bands) is determined by the steepness of the operating attenuation characteristic. The greater the steepness of this characteristic and the smaller the attenuation in the passband, the better the selectivity of the filter and thus the lower the level of interference from damped oscillations. In an ideal case, the characteristic of the operating attenuation, for example, for a low-pass filter, has the form shown in Fig. 2.7.

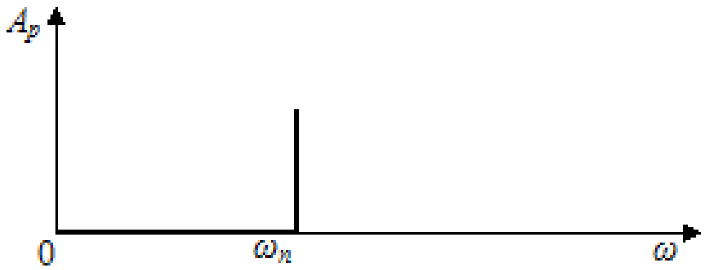


Fig. 2.7. Characteristics of the working attenuation of an ideal low-pass filter

The operational amplitude-frequency characteristic

$$\left|H_p(j\omega)\right| = \exp\left\{-A_p(\omega)\right\}$$

is associated with the operational attenuation. It is obvious that the amplitude-frequency characteristic of an ideal low-pass filter has the form shown in Fig. 2.8.

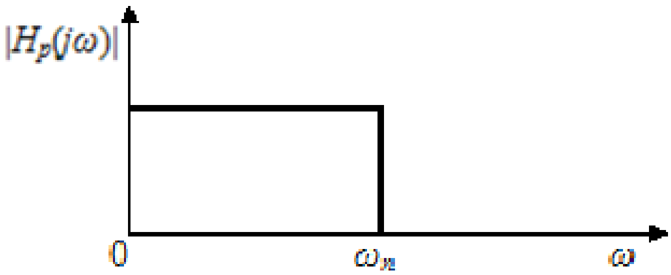


Fig. 2.8. AFC of an ideal LPF

Real filters (filters that consist of real elements) have operating attenuation characteristics and amplitude-frequency response that differ from ideal ones.

The requirements for the electrical characteristics of the filters are set in the form of restrictions imposed on these characteristics. Thus, the working attenuation in the passband should not exceed some maximum permissible value $A_{p\max}$, and in the stopband it should not be below some minimum permissible value $A_{p\min}$. Thus, knowing the requirements for A_p, you can list these requirements for AFC, where:

$$\left|H_p(j\omega)\right|^2 = \exp(-2A_{p\backslash\max}),$$

$$0 < \omega < \omega_p ;$$

$$\left| H_p \left(j\omega \right) \right|^2 = \exp(-2A_{p\min}) ,$$

$$\omega > \omega_s .$$

The characteristics of the designed filters must comply with these requirements.

In addition to the requirements for the frequency dependence of the operating attenuation, there may also be requirements for the phase-frequency characteristic of the filter (permissible deviations from the linear law), nonlinear distortions, and other characteristics and parameters of the filter.

The ideal frequency characteristics of a filter cannot be realized, the real frequency characteristics can only approximate them with one or another degree of accuracy depending on the complexity of the filter scheme.

Electric filters with a transfer function of the form

$$H(p) = \frac{1}{b_m p^m + b_{m-1} p^{m-1} + ... + b_1 p + b_0} \qquad (2.1)$$

are called polynomial. The amplitude-frequency characteristic of such filters looks like this

$$\left| H \left(j\omega \right) \right| = \frac{1}{\sqrt{d_0 \omega^{2m} + d_1 \omega^{2m-2} + ... + d_{m-1} \omega^2 + d_m}} . \qquad (2.2)$$

And, thus, the working attenuation can meet the specified requirements with the proper choice of the degree of the polynomial (order of the filter) and coefficients d_k.

$$A_p = 10 \lg \left[d_0 \omega^{2m} + d_1 \omega^{2m-2} + ... + d_{m-1} \omega^2 + d_m \right] \qquad (2.3)$$

Among the polynomial filters, the Butterworth and Chebyshev filters are the most widely used. Note that in the theory of filters, it is customary to use the normalized frequency for the low-pass filter

$$\Omega = \frac{\omega}{\omega_П} .$$

2.3. Butterworth filters

If in formulas (2.2) and (2.3) we accept the coefficients $d_1 = d_2 = ... = d_{m-1} = 0$, $d_0 = d_m = 1$, then taking into account the normalized frequency we get

$$\left(H_p\left(j\Omega\right)\right)^2 = \frac{1}{\left(1 + d_0\Omega^{2m}\right)} \qquad (2.4)$$

$$A_p = \frac{1}{2}\ln\left(1 + d_0\Omega^{2m}\right). \qquad (2.5)$$

Polynomials

$$B_m\left(\Omega\right) = \Omega^{2m}$$

are known as Butterworth polynomials.

Therefore, the filters in which the square of the AFC is described by the expression (2.4) and the working attenuation by the expression (2.5) were named Butterworth filters.

It follows from formulas (2.4) and (2.5) that at the frequency $\Omega = 0$ the value of the square of AFC is equal to one, and the working attenuation is zero. As the frequency increases, the square of the AFC of the Butterworth filter decreases and falls to zero, and the operating attenuation increases smoothly to infinity. Thus, expressions (2.4) and (2.5) approximately reflect the characteristics of an ideal filter.

In order for these characteristics to satisfy the requirements of the filter, it is necessary to have an operating attenuation (2.5) in the passband less than $A_{p\max}$, and in the stopband is greater than $A_{p\min}$. The first condition can be satisfied if at the limit frequency $(\Omega = 1)$ we put equality

$$A_p\left(\Omega\right)|_{\Omega=1} = A_{p_{\max}},$$

Then

$$1 + d_0 = exp\left(2A_{p\,max}\right); d_0 = e^{2Ap\,max} - 1.$$

$$E = \sqrt{d_0} = \sqrt{e^{2Ap\max} - 1}$$

is called the attenuation unevenness coefficient in the passband of the filter. If $A_{p\max}$ is measured in decibels, then the correct ratio is

$$E = \sqrt{10^{0,1Ap\max} - 1}.$$

Taking into account the above designations

$$\left(H_p\left(j\Omega\right)\right)^2 = \frac{1}{1 + E^2\Omega^{2m}}, \qquad (2.6)$$

$$A_p = \frac{1}{2}\ln(1 + E^2\Omega^{2m}) \text{ [Нп]}, \qquad (2.7)$$

$$A_p = 10lg\left(1 + E^2\Omega^{2m}\right) \text{ [Дб]}. \qquad (2.8)$$

Let us present the graphic dependences of the obtained functions (Fig. 2.9).

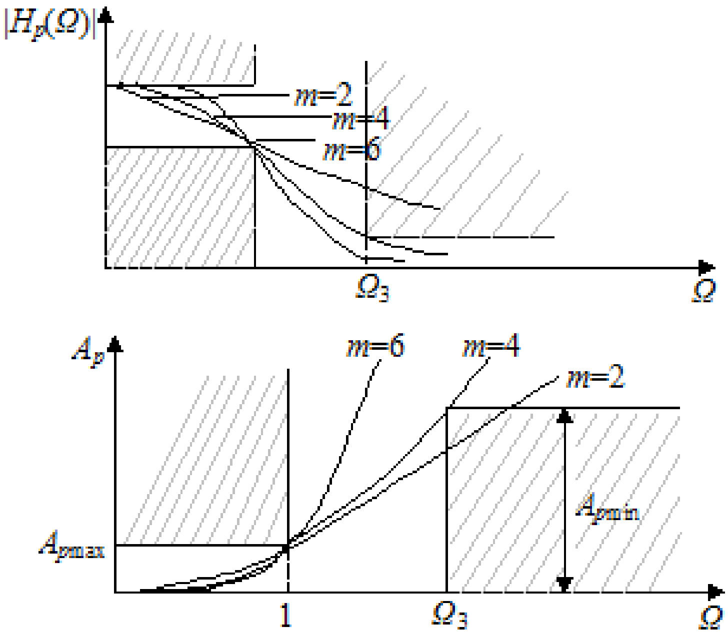


Fig. 2.9. AFC and operational attenuation of Butterworth filters

Note that the steepness of the frequency characteristics depends on the power of m, i.e. the greater the m, the greater the steepness of the characteristics.

Thus, to meet the requirements in the passband, it is necessary to choose the appropriate order of the filter m, it can be determined under the condition

$$A_p(\Omega_3) \geq A_{pmin};$$

$$1 + E^2\Omega_3^{2m} > e^{2Apmin};$$

$$\Omega_3^{2m} \geq \frac{1}{E^2}(e^{2Apmin} - 1).$$

After logarithmization, we get

$$2 \cdot m \cdot ln\Omega_3 \geq ln\frac{e^{2Apmin} - 1}{E^2}.$$

Finally we have

$$m \geq \ln(\frac{e^{2Apmin} - 1}{E^2})/(2\ln\Omega_3), \qquad (2.9)$$

$$m \geq \lg(\frac{10^{0,1Apmin} - 1}{E^2})/(2\lg\Omega_3). \qquad (2.10)$$

The transfer function of the Butterworth filter can be obtained from formula (2.6), if $j\Omega = p$, then

$$|H_p(p)|^2 = H_p(p)H_p(-p) = \frac{1}{1 + E^2(-p^2)^m}. \qquad (2.11)$$

Let's determine the roots of the denominator, that is, the poles of the function $H_p(p) \cdot H_p(-p)$ separately for even and odd values of m.

For even values of m

$$1 + E^2 p^{2m} = 0$$

and

$$p_к = \frac{1}{\sqrt[m]{E}} \sqrt[2m]{-1},$$

$$k = 1,\ 2, \ldots 2m.$$

Because

$$-1 = exp\left[j\left(2k-1\right)\pi \right] = cos\left(2k-1\right)\pi + jsin\left(2k-1\right)\pi,$$

then

$$p_к = \frac{1}{\sqrt[m]{E}} \sqrt[2m]{e^{j(2k-1)\pi}} = \frac{1}{\sqrt[m]{E}} e^{j\frac{(2k-1)\pi}{2m}} = \frac{1}{\sqrt[m]{E}} (\cos\frac{2k-1}{2m}\pi + j\sin\frac{2k-1}{2m}\pi).$$

For odd values m

$$1 - E^2 p^{2m} = 0.$$

Because

$$1 = exp\left[j2k\pi \right] = cos\left(2k\pi\right) + jsin\left(2k\pi\right),$$

Then

$$p_к = \frac{1}{\sqrt[m]{E}} \sqrt[2m]{e^{j2k\pi}} = \frac{1}{\sqrt[m]{E}} e^{j\frac{2k\pi}{2m}} = \frac{1}{\sqrt[m]{E}} (\cos\frac{k}{m}\pi + j\sin\frac{k}{m}\pi),\ k = 1, 2 \ldots 2m.$$

After defining the roots, the expression (2.11) becomes

$$H_p\left(p\right) \cdot H_p\left(-p\right) = \frac{1}{E^2 (p-p_1)(p-p_2)..(p-p_{2m})}.$$

Having chosen the poles located in the left half-plane of the complex variable p, we obtain the transfer function of the physically realized Butterworth filter of the form

$$H_p\left(p\right) = H \frac{1}{E^2 (p-p_1)(p-p_2)..(p-p_{2m})}, \qquad (2.12)$$

Where

$$H = \frac{1}{E}.$$

Using the value $B_m\left(\Omega\right) = \Omega_m$ of the Butterworth polynomial, it is possible to represent the frequency characteristics of the Butterworth filter in the following form:

$$\left|H_p\left(j\Omega\right)\right|^2 = 1 / \left[1 + E^2 B_m^2\left(\Omega\right)\right]; \qquad (2.13)$$

$$A_p\left(\Omega\right) = \frac{1}{2} \ln\left[1 + E^2 B_m^2\left(\Omega\right)\right] \text{ [Нп]}; \qquad (2.14)$$

$$A_p\left(\Omega\right) = 10 \lg\left[1 + E^2 B_m^2\left(\Omega\right)\right] \text{ [Дб]}. \qquad (2.15)$$

Butterworth filters are also called filters with maximally flat attenuation in the passband.

2.4. Chebyshev filters

Formulas of the type (2.13)-(2.15) are universal in their structure. It is enough to replace the Butterworth polynomial in them with some other polynomial and you can get a new type of filter. For example, if instead of the polynomial $B_m(\Omega)$ we use the so-called Chebyshev polynomial, then we get

$$\left|H_p(j\Omega)\right|^2 = 1/\left[1+E^2T_m(\Omega)\right]; \tag{2.16}$$

$$A_p(\Omega) = \frac{1}{2}\ln\left[1+E^2T_m^2(\Omega)\right] \text{ [Нп]}; \tag{2.17}$$

$$A_p(\Omega) = 10\lg\left[1+E^2T_m^2(\Omega)\right] \text{ [Дб]}. \tag{2.18}$$

where $T_m(\Omega)$ is a Chebyshev polynomial of degree m, E is the unevenness coefficient in the passband of the filter.

Filters with characteristics (2.16)-(2.18) are called Chebyshev filters. Consider six Chebyshev polynomials: $T_0(\Omega)=1$, $T_1(\Omega)=\Omega$, $T_2(\Omega)=2\Omega^2-1$, $T_3(\Omega)=4\Omega^3-3\Omega$, $T_4(\Omega)=8\Omega^4-8\Omega^2+1$, $T_5(\Omega)=16\Omega^5-20\Omega^3+5\Omega$.

Any Chebyshev polynomial for $m \geq 2$ can be calculated by the recurrent formula $T_m(\Omega)=2\Omega T_{m-1}(\Omega)-T_{m-2}(\Omega)$, therefore relations (2.16)-(2.18) satisfy general expressions (2.1) - (2.3) characteristics of polynomial filters. There is a single trigonometric form of writing Chebyshev polynomials in the interval $-1 \leq \Omega \leq 1$:

$$T_m(\Omega) = cos(m \cdot arccos\Omega). \tag{2.19}$$

Actually: $T_0(\Omega)=cos(0\cdot arccos\Omega)=1$, $T_1(\Omega)=cos(1\cdot arccos\Omega)=\Omega$, $T_2(\Omega)=2cos(2\cdot arccos\Omega)-1=2\Omega^2-1$. Outside the interval $-1 \leq \Omega \leq 1$ polynomials $T_m(\Omega)$ are also given in trigonometric form

$$T_m(\Omega) = ch(m \cdot arcch\Omega). \tag{2.20}$$

The analysis of the behavior of Chebyshev polynomials shows that in the interval $-1 \leq \Omega \leq 1$ the angle $\theta = \arccos\Omega$ changes from $-\pi(at\Omega=-1)$ to a value equal to 0 $(at\Omega=1)$, and $T_m(\Omega) m+1$ times reaches values equal to “+1” or “-1”. Outside the interval $-1 \leq \Omega \leq 1$ $T_m(\Omega)$ increases monotonically according to formula (2.20).

According to the formula (2.18), the operating attenuation of the Chebyshev filter is zero at those frequencies where the polynomial $T_m(\Omega)$ turns to zero. At frequencies at which $T_m(\Omega)=\pm1$, the operating attenuation reaches the value

$$A_p = 10lg\left(1+E^2\right) = 10lg\left(1+10^{0,1Apmax}-1\right) = A_{pmax}.$$

With increasing values of the polynomial $T_m(\Omega)$ at frequencies $\Omega>1$, the working attenuation $A_p(\Omega)$ also increases monotonically.

In fig. 2.10 shows the working attenuation graph of the fourth-order Chebyshev filter.

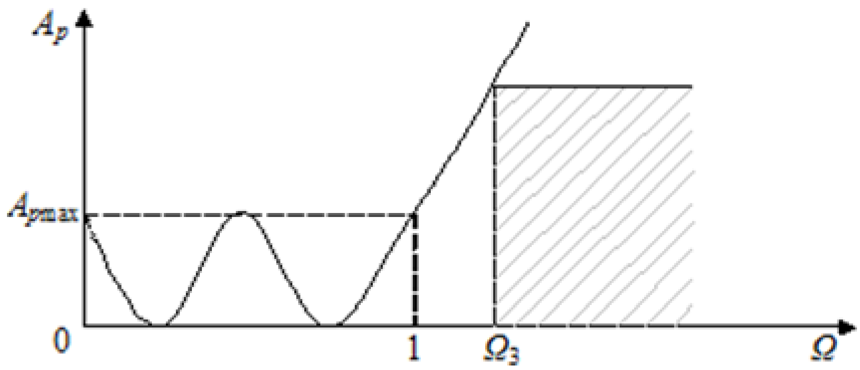


Fig. 2.10. The working attenuation graph of the Chebyshev filter

Chebyshev filters are also called filters with equal-wave characteristics in the passband.

Consider the frequency dependence of the square AFC of the Chebyshev filter for different values of *m* (Fig. 2.11).

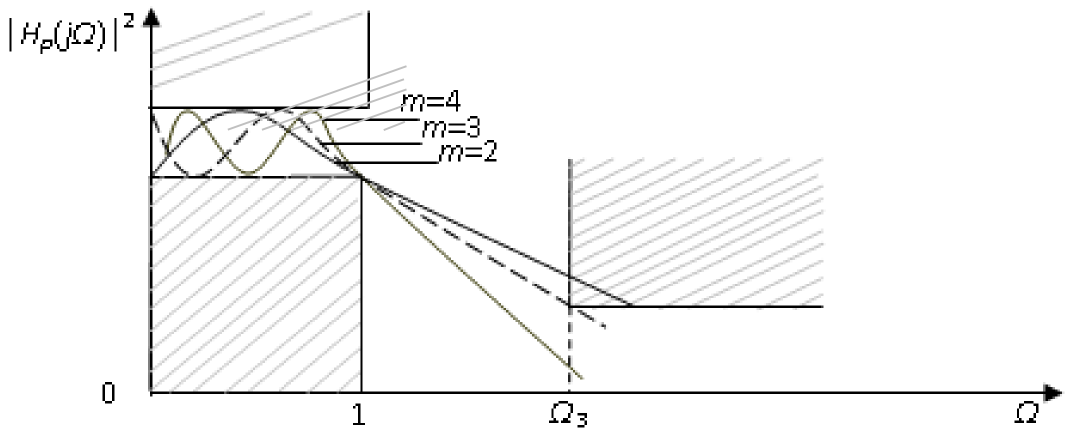


Fig. 2.11. AFC of the Chebyshev filter of different orders

In order for the filter characteristics to meet the requirements in the passband, it is necessary to choose the order of the filter m from the condition

$$\left|H_p\left(j\Omega\right)\right| \le exp\left[-2A_{p\ min}\right],$$

taking into account the expression (2.20) at $\Omega = \Omega_3$ we get

$$m \ge arch\sqrt{\frac{e^{-2Ap\min}-1}{E^2}}/2arch\Omega_3\left(A_p in\ \left[Np\right]\right), \qquad (2.21)$$

or

$$m \ge arch\sqrt{\frac{10^{0,1-Ap\min}}{E^2}}/2arch\Omega_3\left(A_p in\ \left[dB\right]\right). \qquad (2.22)$$

Comparing the frequency characteristics of the Butterworth and Chebyshev filters, it can be seen that the Chebyshev polynomials are the polynomials of the best approximation. This means that for the same values of m, the Chebyshev filter in the passband has the greatest attenuation than the Butterworth filter. However, the working attenuation characteristic of the Butterworth filter in the passband has a monotonic character and is therefore easier to adjust to eliminate distortions of the transmitted signals. The choice of the type of polynomial filters is determined by the specific conditions of their use in communication equipment and radio technical devices.

To obtain the transfer function of the Chebyshev filter, we replace the operator $j\Omega$ with the operator p and pass from the function $\left|H_p(j\Omega)\right|^2$ to the function

$$\left|H_p(p)\right|^2 = H_p(p)H_p(-p) = \frac{1}{1+E^2T_m^2(p/j)}.$$

Taking into account formula (2.19), we find the poles of the function $\left|H_p(p)\right|^2$ by solving the equation

$$E^2cos^2\left[m\cdot arccos(p/j)\right]+1=0. \qquad (2.23)$$

The roots of this equation:

$$p_\kappa = sh\gamma sin\frac{2\kappa-1}{2m}\pi + jch\gamma\cos\frac{2\kappa-1}{2m}\pi,\ k=1,2, \qquad (2.24)$$

where $\gamma = \frac{1}{m}arsh\frac{1}{E}$.

The roots in the left half-plane form multipliers of the type (p-p_k), and the transfer function of the Chebyshev filter is constructed based on them

$$H_p(p) = H\frac{1}{p^m + b_{m-1}p^{m-1}+\ldots+b_1p+b_0},$$

where $H = E^{-1}/2^{m-1}$.

2.5. Realization of electric filters

Any filters, both polynomial and others, depending on the specifics of their application, can be implemented either in the form of passive LC-circuits or in the form of active RC-circuits.

Passive LC-filters generally represent a reactive stepped four-pole connected between the generator $E_г$ with an active internal resistance $R_г$ and a load with an active resistance $R_н$. The input resistance of a reactive four-pole, loaded with resistance $R_н$, is indicated in Fig. 2.12 $Z_{вх1}(p)$.

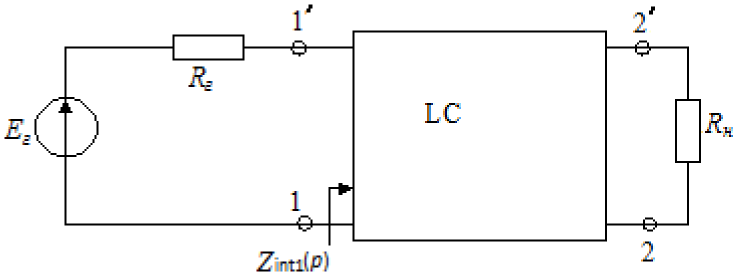


Fig. 2.12. Passive LC-filters

If the filter from the side of clamps 1-1´ is considered as a two-pole formed by a reactive four-pole and a load $R_н$, then, knowing the expression $Z_{\text{int}1}(p)$, it is possible to realize this two-pole by one of the methods of synthesis of two-poles known in the theory of electric circuits.

For example, the polynomial LPF of the fifth order $(m=5)$ is implemented in the form of one of the two schemes shown in Fig. 2.13.

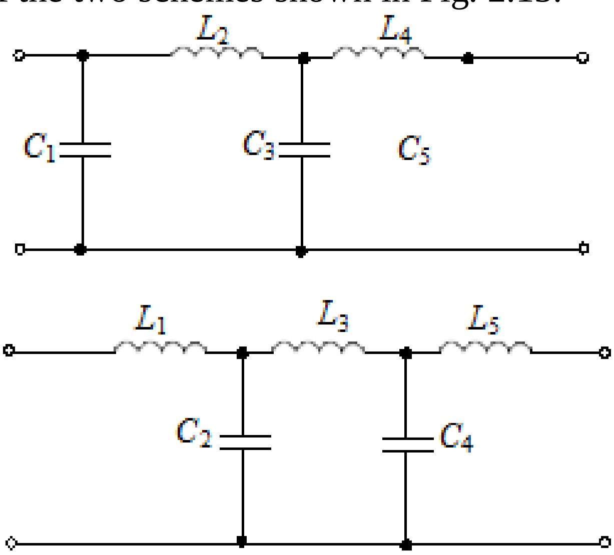


Fig. 2.13. Polynomial AFCs of the fifth order

The number of reactive elements is determined by the order of the filter m. The Butterworth filter will differ from the Chebyshev filter by different values of the reactive elements obtained in the process of implementing the corresponding transfer functions.

Low-pass filters with fading bursts

Schemes of stepped filters with damping bursts contain inductances or parallel oscillating circuits in the longitudinal branches and capacitors or series oscillating circuits in the transverse branches. Examples of such filters are shown in fig. 2.14, a and 2.14, b.

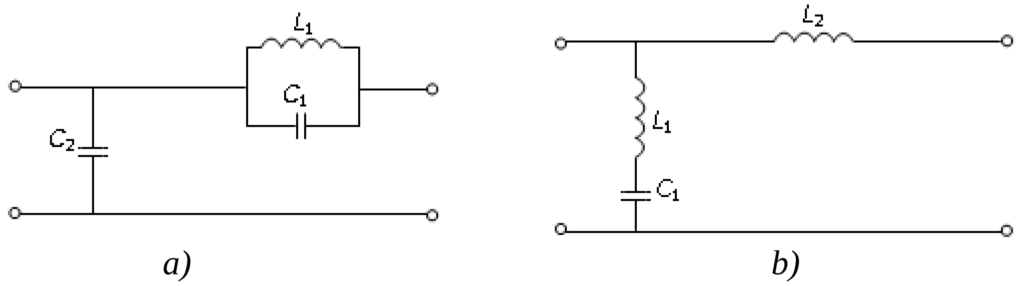


Fig. 2.14. Burst attenuation filter circuits

The number of attenuation bursts is determined by the number of oscillating loops in the filter scheme. For the filters shown in fig. 2.14, the attenuation characteristic can have the form shown in fig. 2.15.

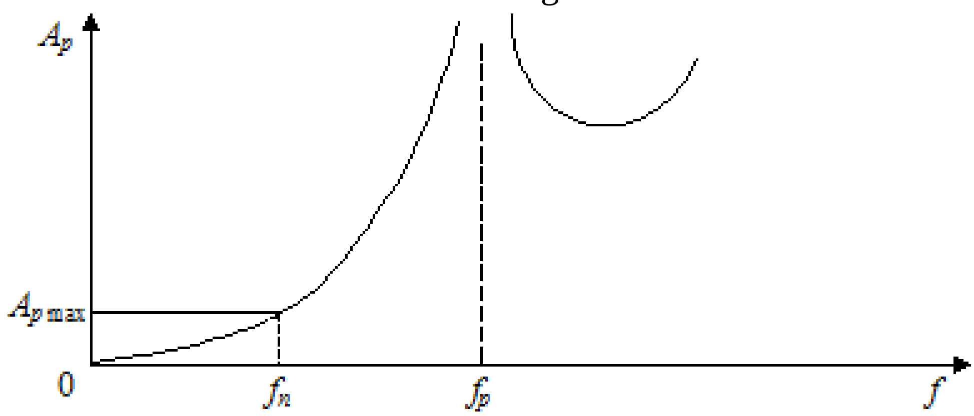


Fig. 2.15. Attenuation characteristics of filters with bursts

At the resonant frequency f_p, the resistance of the parallel oscillating circuit included in the longitudinal branch (Fig. 2.14,a) is infinitely large (the attenuation of the filter goes to infinity (Fig. 2.15)). The resistance of the series oscillating circuit, included in the transverse branch of the filter (Fig. 2.14, b), at the resonant frequency is practically zero, and the circuit completely shunts the load (the attenuation of the filter at this frequency is infinite).

The analytical recording of the attenuation of the low-pass filter with the given placement of the attenuation bursts is as follows:

$$A_p = \frac{1}{2}\ln\left[1+(e^{2A_{p\max}}-1)\hat{F}^2(\Omega)\right] \qquad (2.25)$$

Here is $\hat{F}^2(\Omega)$ the so-called Chebyshev fraction. It is an odd fractional rational function of normalized frequency.

In many practical problems, the requirements for the attenuation characteristics of LPF with attenuation bursts are formulated as follows: the attenuation of the filter in the passband $0 \le \Omega \le 0$ should not exceed the specified value $A_{p\max}$, and in the passband, starting from the frequency $\Omega_{\kappa} = \frac{\omega_{\kappa}}{\omega_0}$, should not be less than some constant value $A_{p\min}$ min (Fig. 2.16).

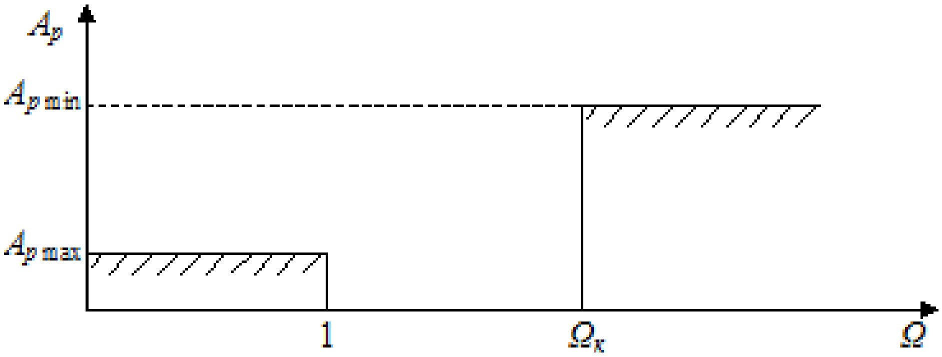


Fig. 2.16. LPF attenuation characteristics requirements

To solve the corresponding approximation problem, one of the problems of the best approximation of functions is used, formulated and solved by E.I. Zolotaryov, a student of P.L. Chebyshev, and above all, the problem of a rational function of the given nth order, the absolute value of which in the interval $-1 \le \Omega \le 1$ did not exceed unity, and in the interval $|\Omega| > 1$ the smallest absolute value of its value would be the maximum possible.

In the passband, the attenuation characteristic of a low-pass filter with Zolotaryov characteristics will have an equal-wave character, and in the stopband, starting with the frequency Ω_k, the smallest attenuation value of such a filter will be the maximum possible compared to all other filters with the same values of n and $A_{p\max}$. The graph of the frequency dependence of the attenuation of the filter with Zolotaryov characteristics for $n = 5$ is shown in Fig. 2.17.

A distinctive feature of these filters is the equality of the minima of the attenuation characteristic of the filter in the stopband, and the values of these minima are equal to the value of the attenuation of the filter at the limit of its stopband.

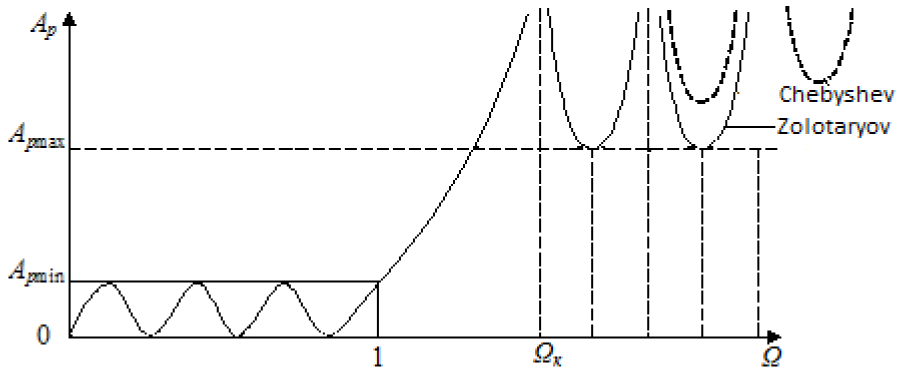


Fig. 2.17. Frequency dependence of filter attenuation with Zolotaryov characteristics

Filters with the Zolotaryov characteristic can be considered as a special case of filters with the Chebyshev characteristic, when the minimum values of the attenuation characteristic of the filter in the stopband are aligned, and the number of attenuation bursts is the maximum possible at the selected value of n.

High-pass filter

We will begin the study of the principles of constructing high-pass filters by considering the essence of the frequency variable conversion method.

From practice, it is clear that high-pass filters can be obtained from low-pass filters, if in the latter each inductance is changed to a capacitance, and each capacitance to an inductance (Fig. 2.18).

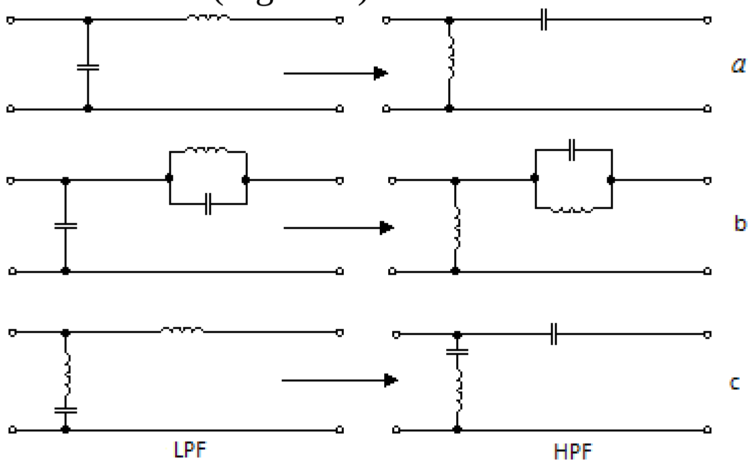


Fig. 2.18. Conversion of LPF to HPF

The attenuation of the low-pass filter is an even function of the frequency Ω, accordingly, the attenuation curve will be symmetrical with respect to the ordinate axis (Fig. 2.19)

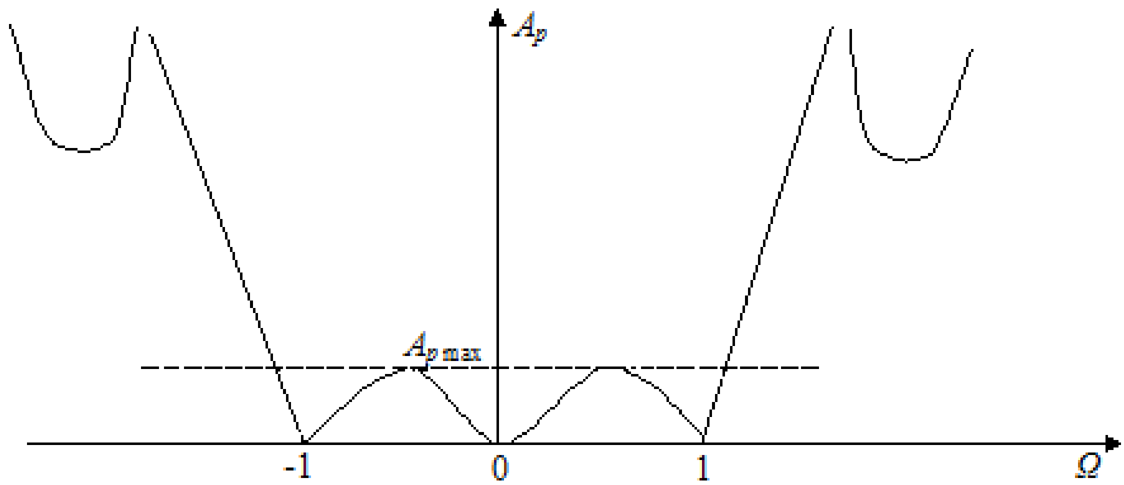


Fig. 2.19. The attenuation characteristic of the low-pass filter is symmetrical with respect to the ordinate axis

The left part of this graph corresponds to the relative placement of the stopband and the passband for the high-pass filters (Fig. 2.19).

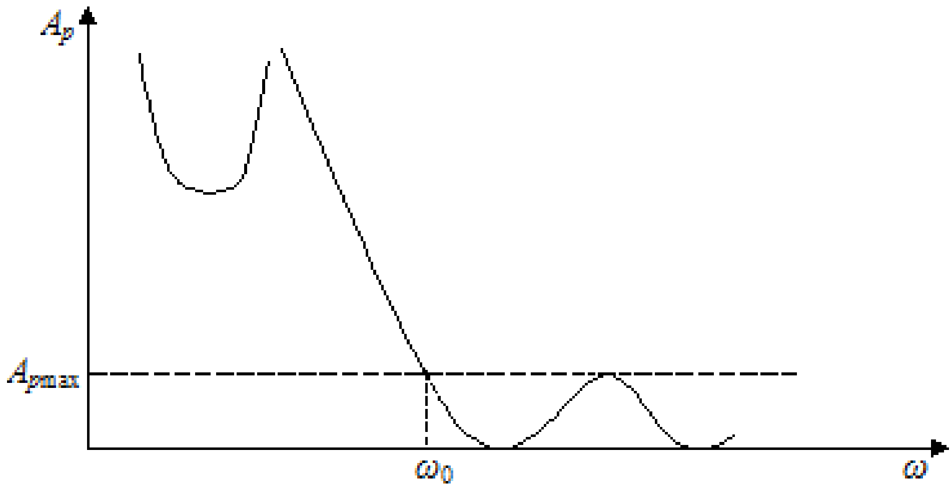


Fig. 2.20. Characteristics of attenuation of the high-pass filter

Let’s choose such a function that would turn the negative semi-axis Ω (Fig. 2.18) into a positive one (Fig. 2.19). At the same time, the point “$-\infty$” (Fig. 2.18) should correspond to the point “0” (Fig. 2.19), “-1”-“ω_0”, “0”-“∞”. It is clear that such a function will be

$$\Omega = -\omega / \omega_0 \qquad (2.26)$$

Thus, all expressions obtained earlier for low-pass filters are also correct for high-pass filters, if in these expressions Ω is determined from relation (2.26).

Normalization by frequency allows you to fully use all methods, formulas, nomograms and data tables obtained for low-pass filters when calculating high-pass filters with Butterworth, Chebyshev, Zolotaryov characteristics or with arbitrary placement of attenuation bursts. For example, in fig. 2.21 shows the schemes and characteristics of some high-pass filters with equal-wave attenuation characteristics.

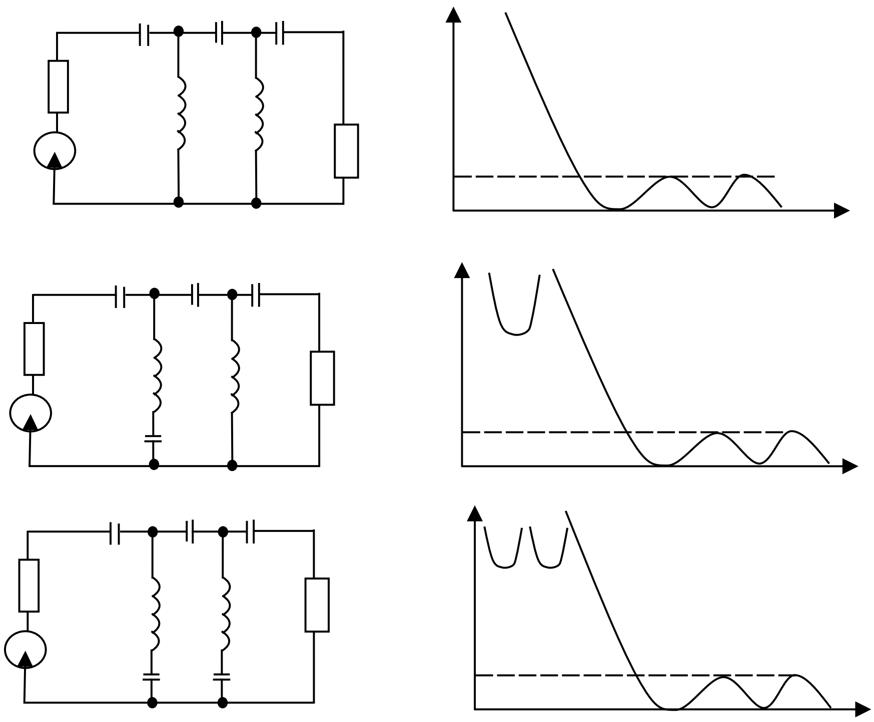


Fig. 2.21. Attenuation characteristics of high-pass filters with equal-wave attenuation characteristics

Bandpass filters and other types of induction filters

A bandpass filter can be formed from a low-pass filter, if in the latter each inductance is replaced by a series oscillating circuit without losses, and each capacitance is replaced by a parallel one, while the resonant frequencies of all circuits are taken to be the same (Fig. 2.22)

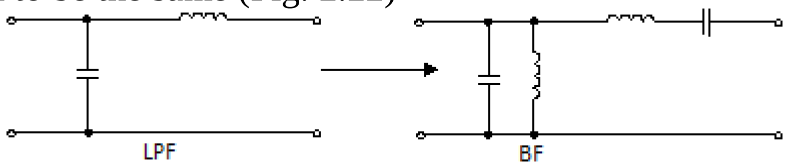


Fig. 2.22. LPF to BF conversion

Then, before the resonance frequency, the nature of the resistance of the branches of the resulting filter will be the same as that of the high-pass filter, and after the resonance frequency, it will be the same as that of the low-pass filter. In general, the filter will be a bandpass filter, while the resonant frequency of the circuits will be in the passband of the filter.

Let's select a function that would transform the frequency axis $\Omega(-\infty;\infty)$ (рис. 2.19) into the semi-axis $\omega(0\div\infty)$ (рис. 2.23).

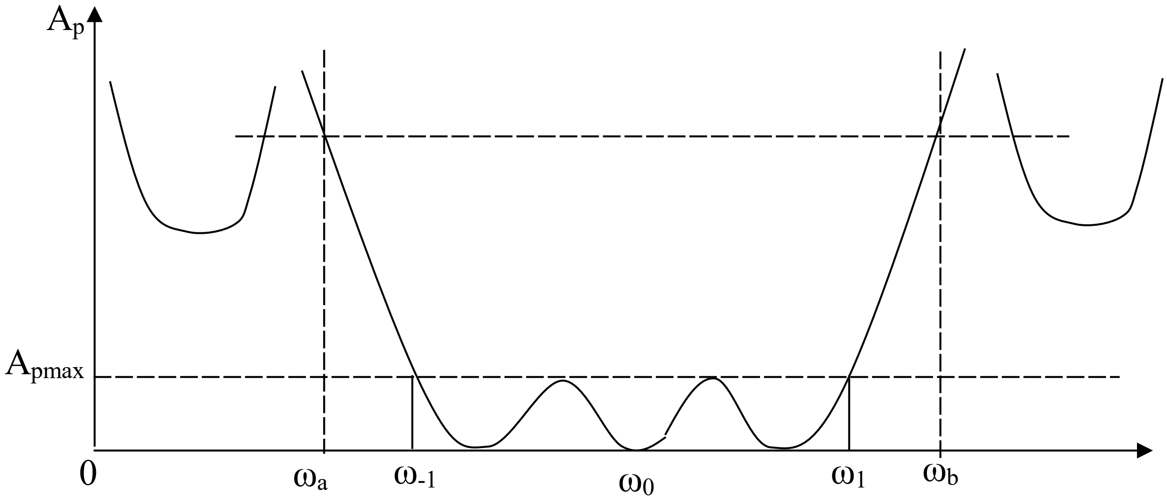


Fig. 2.23. Characteristics of BF attenuation

Thus:
The point «–∞» (Fig. 2.19) should be the point «0» (Fig. 2.23).
The point «–1» (Fig. 2.19) should be the point «ω_{-1}» (Fig. 2.23).
The point «0» (Fig. 2.19) should be the point «ω_0» (Fig. 2.23).
The point «1» (Fig. 2.19) should be the point «ω_1» (Fig. 2.23).
The point «∞» (Fig. 2.19) should be the point «∞» (Fig. 2.23).
Such a transformation can be performed by a function

$$\Omega = \frac{\omega_0}{\omega_1 - \omega_{-1}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \quad (2.27)$$

Thus, all the expressions obtained earlier for low-pass filters remain valid for the considered bandpass filters, if in these expressions Ω is determined from relation (2.27).

It is important to note that the frequency characteristics of the filters under consideration have one peculiarity, which is due to the fact that the function (2.27) takes values equal in absolute value and opposite in sign for any pair of frequencies ω_a and ω_b, which are related by the relation $\omega_a \cdot \omega_b = \omega_0^2$ that is, for any pair of frequencies placed symmetrically with respect to the frequency ω_0, which in turn is the geometric mean frequency of the filter passband. Accordingly, the attenuation of the filter at frequencies ω_a and ω_b will be the same, that is, the attenuation characteristic of any bandpass filter obtained by frequency transformation (27) will always be geometrically symmetric with respect to the frequency ω_0. The graphic illustration is shown in fig. 2.24. That is why such filters were called filters with symmetrical (geometric) attenuation characteristics.

The practical use of the filters described earlier is limited only to those cases where the requirements for the attenuation characteristics of the filter on both sides of its passband are close to symmetrical. Otherwise, in some frequency areas, there will be an unreasonably large margin of attenuation, which indicates the existence of a more economical solution in terms of the number of elements. Accordingly, in the general case, a bandpass filter can have any number of bursts to the left and right of its passband, different steepness of the attenuation characteristic at $\omega \to 0$ and at $\omega \to \infty$, which differ in the width of the transition band, that is, an attenuation characteristic that is significantly differs from symmetrical.

In addition to such low-pass filters, high-pass filters, bandpass filters, and rejection filters are also often used in technology.

The attenuation characteristic of the rejection filter is shown in Fig. 2.24.

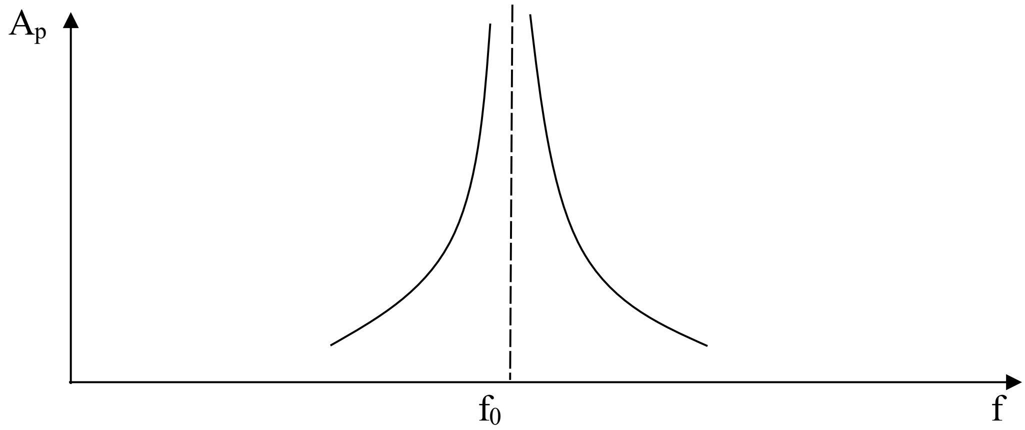


Fig. 2.24. Attenuation characteristics of the bandpass filter

If you compare this characteristic with the characteristic of a bandpass filter, you can see that the rejection filter has an attenuation characteristic opposite to that of a bandpass filter, so it can also be obtained from a low-pass filter prototype, but in this case, in the low-pass filter, each capacitor needs to be replaced by a series oscillating circuit , and the inductance on is parallel (Fig. 2.25), while the resonant frequency of all circuits must be the same, then such a filter will freely pass (with little attenuation) all frequencies below and above the resonant frequency of the circuits, and at frequencies close to resonance, the attenuation of the filter will be large because the series circuits will have a shunting effect on the input signal and the parallel circuit will have a higher resistance to it.

In radar technology, comb filters are widely used, in which passbands alternate with stopbands. There are many ways to obtain a comb-like attenuation characteristic. The simplest of them is the method of creating such a characteristic using a set of bandpass or rejection filters.

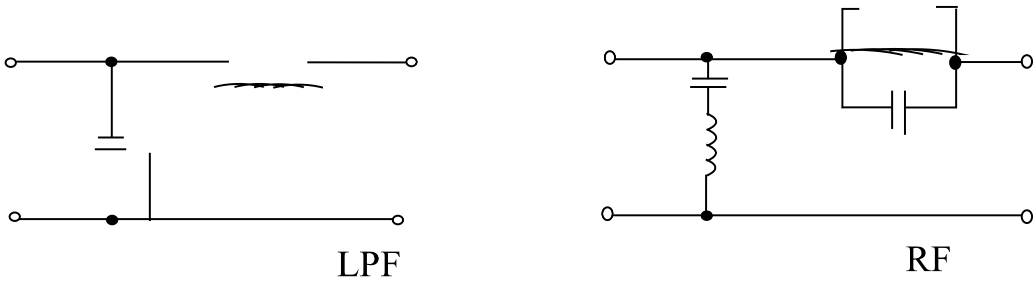


Fig. 2.25. Transition to the rejection filter

In fig. 2.26 shows the method of obtaining a comb filter and its attenuation characteristics.

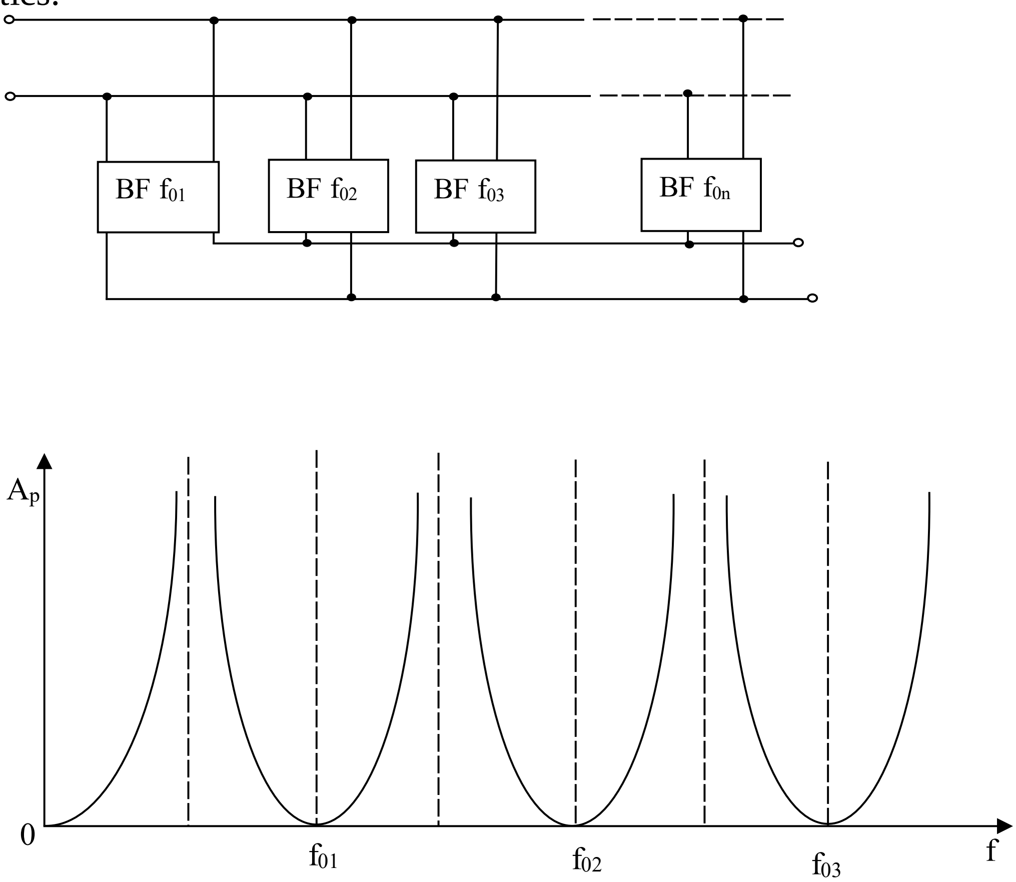


Fig. 2.26. Comb filter

We will consider the influence of load resistance on filter characteristics using the example of the simplest low-pass filter.

When the filter is loaded to the resistance $Z_H = Z_O$, i.e. to the characteristic, this mode is called the matching mode (Fig. 2.27).

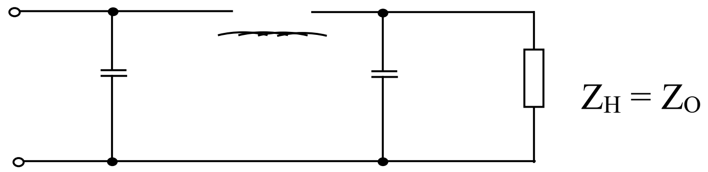


Fig. 2.27. Load of the filter on the characteristic resistance

In this case, the attenuation characteristic of the filter will have the form shown in Fig. 2.28.

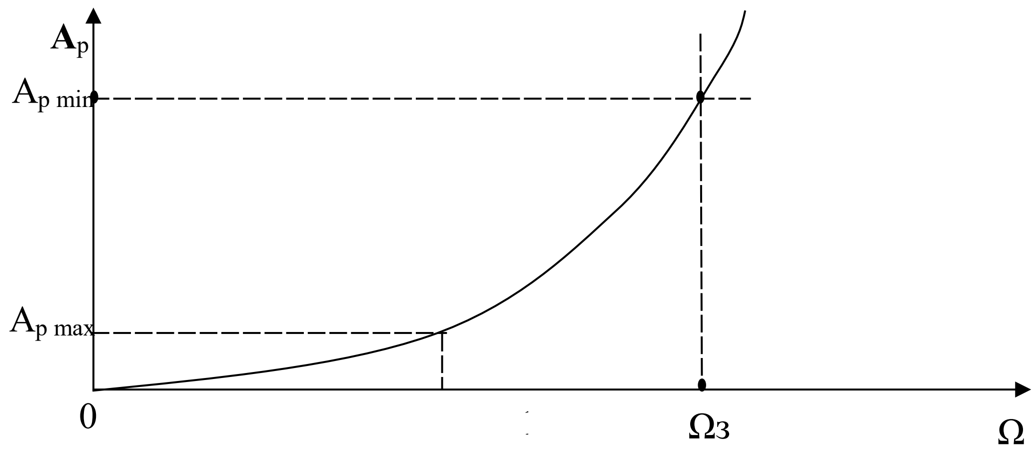


Fig. 2.28. Filter attenuation characteristic

It should be emphasized that a filter of this type will have a characteristic only if it is loaded with a resistance equal to the characteristic one. To find out whether the existence of such a regime is possible, it is necessary to find the characteristic resistance of the filter under consideration.

Using the theory of quadripoles, it is possible to write a formula for the U-shaped filter circuit that expresses the dependence of the characteristic resistance on the frequency $|z_o| = \rho / \sqrt{1-\Omega^2}$ where $\rho = \sqrt{\frac{L}{C}}$ is the wave resistance. The graph of such dependence is shown in fig. 2.29.

As can be seen from fig. 2.29, the impedance Z_0 is very frequency dependent both in character and magnitude. This is typical of any filter scheme. Accordingly, in order to match the load, it would be necessary to select its own load resistance for each frequency (active in the passband, reactive in the delay stopband). However, under actual operating conditions, the load resistance is usually a practically frequency-independent active resistance $R_н$. It follows from this that, in general, the filter operates on an unbalanced load and the matching mode can only be approached. Using the same theory of quadripoles, it is possible to determine the dependence of the filter attenuation on the load resistance.

The graph of such dependence for LPF is shown in Fig. 2.30.

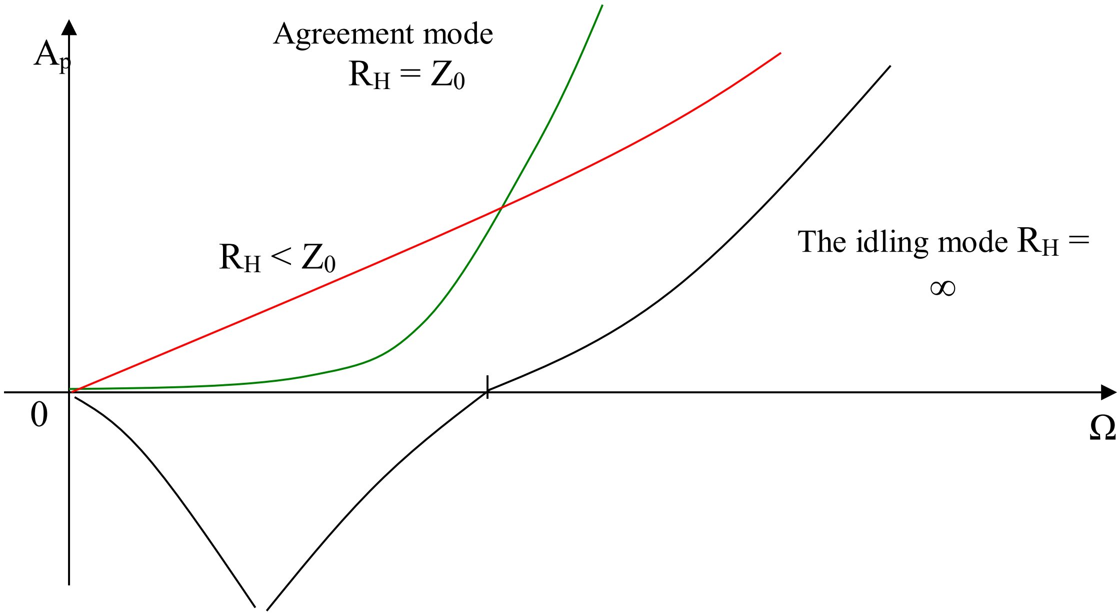


Fig. 2.30. Dependence of low-pass filter attenuation on load resistance

As can be seen from the graph, when the filter is idling ($R_н = \infty$) in the passband, the attenuation becomes negative, i.e. the transmission coefficient of the filter is greater than 1. It is clear that this does not mean signal amplification in terms of power, because the filter is a passive EC. The reason for this is the resonance in the series circuit $\frac{LC}{2}$, in which the output voltage of the filter can be several times greater than the input voltage.

It can also be seen from the graph that at low R_H values, the attenuation in the passband increases significantly and decreases in the stopband. Thus, it will be appropriate to choose such a mode when $R_н = Z_0$.

2.6. NON-INDUCTIVE FILTERS

Recently, in connection with the problem of microminiaturization of ECs and the construction of linear microelectronic ECs, first of all, design methods are intensively developed and the technology of manufacturing circuits that do not have inductance, i.e., so-called non-inductive ECs, is being improved. The fact is that the possibilities of reducing the weight and size of inductors, which have a fairly high quality factor, are practically exhausted today.

Non-inductive filters include piezoelectric, magnetostrictive, electromechanical filters, RC-filters, and others.

2.6.1. Piezoelectric filters

A piezoelectric resonator is an electromechanical oscillating system consisting of a piezoelectric element and conductive electrodes.

The peculiarity of such an oscillating system is its ability to transform electrical energy into the energy of mechanical vibrations and vice versa.

The operation of the piezoelectric resonator is based on the phenomenon of the piezo effect, which manifests itself in the fact that when bodies made of certain crystals are deformed, an electric voltage is formed between their individual surfaces, the polarity of which changes with the change in the sign of the deformation. If an alternating voltage is applied to opposite faces of a piezoelectric plate, the plate will perform mechanical oscillations with a frequency equal to the frequency of the applied voltage. This phenomenon is called the opposite piezo effect. Crystals of quartz, ferruginous salt, tourmaline, barium titanium, etc. have a piezo effect.

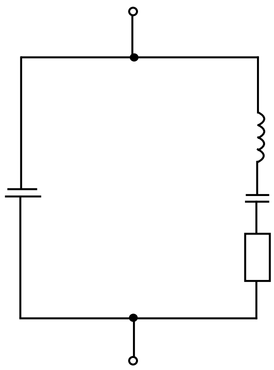
In the simplest case, a piezoelectric resonator is a plate that has the shape of a parallelepiped and is cut in a certain way relative to the crystallographic axes from a piezoelectric crystal. On two opposite sides of this plate, metal electrodes with current-conducting terminals are applied in one way or another.

Piezoelectric resonators made of quartz crystals have received the greatest use and distribution in communication technology. Such piezo resonators are called quartz. The high stability of characteristics and high quality factor of quartz resonators led to their wide use in communication technology and, in particular, in military radio-receiving and radio-transmitting equipment; firstly, to stabilize the frequency of the generators of radio stations and, secondly, to create quartz filters.

As a piezoelectric current occurs in the quartz plate, the plate behaves like an electrical circuit. Let's clarify the nature of this chain.

A quartz plate is an elastic body, so it has a resonant frequency of mechanical oscillations (f_0). If the frequency of the applied voltage is f_u, then the piezoelectric current is maximal in magnitude, and the plate resistance is purely active. When $f_u < f_0$, the current leads the voltage in phase, that is, the plate behaves like an RC-circuit. When $f_u > f_0$, the current lags in phase with the voltage, that is, the plate behaves like an RL-circuit. When $f_u \neq f_0$, the amplitude of the piezoelectric current and mechanical oscillations is smaller, the greater the difference between f_u and f_0. That is, the piezoelectric element has the properties of a continuous oscillating circuit. The complete piezo-electric resonator replacement scheme (except for L_q, C_q, R_q) contains the capacity of the crystal holder C_0. In fig. 2.31 shows the diagram of resonator replacement (a) and the graph of the frequency dependence of its resistance (b).

The most stable parameter of a piezo resonator is its resonance frequency ω_q. In quartz resonators, the quality factor value of the successive oscillating circuit in the substitution scheme reaches 10^6 and higher. Therefore, when calculating circuits with quartz resonators, you can consider a quartz resonator as a purely reactive bipolar device.



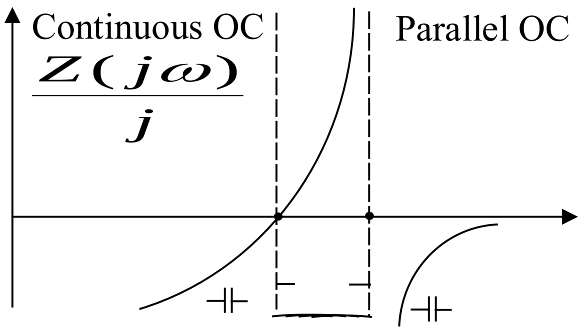


Fig. 2.31. Resonator replacement scheme

A narrowband step filter can be obtained by connecting several simple links (Fig. 2.32).

Fig. 2.32. The simplest filter links

If quartz filters of step schemes are formed by including quartz resonators in longitudinal and transverse branches, it is possible to obtain bursts of attenuation in the upper and lower stopbands.

Quartz resonators, due to their high quality factor and stability of electrical parameters, are widely used in the construction of bandpass filters with a small relative passband and high requirements for the amount of guaranteed attenuation and the width of the transition band. The implementation of such requirements by LC-filters turns out to be impossible, as a rule, even with the use of inductors with the maximum achievable quality factor.

In addition to quartz resonators, quartz filters have one or another number of capacitors and possibly transformers.

Sometimes, in order to increase the width of the transmission band, one or two inductance coils, which are called expansion coils, are introduced into the composition of quartz filters. These coils are connected to each of the filter branches, most often in parallel with quartz resonators.

Within the passband of the filter, its operating attenuation characteristic can be maximally flat, equal-wave or close to equal-wave, which is determined by both the calculation method and the precision of manufacturing quartz resonators. Depending on the passband of the filter, its attenuation characteristic can grow monotonically or have a certain number of attenuation bursts.

Depending on the requirements placed on the quartz filter, in particular on the relative width of its bandwidth, one or another typical structures are used: bridge without expansion coils, bridge with expansion coils, stepped or monolithic quartz filters. Most quartz filters used in communication equipment have a bridge structure.

The diagram of the four-pole bridge is shown in Fig. 2.33, a. In order to save resonators, quadrupoles with a differential transformer are widely used in practice, that is, differential-bridge quadrupoles, which are equivalent to bridge ones (Fig. 2.33, b).

Branches of a bridge (differential-bridge) quartz filter without expanding inductance coils are formed as a result of the parallel connection of one or another number of quartz resonators, the more complex the requirements for the attenuation characteristics of the filter.

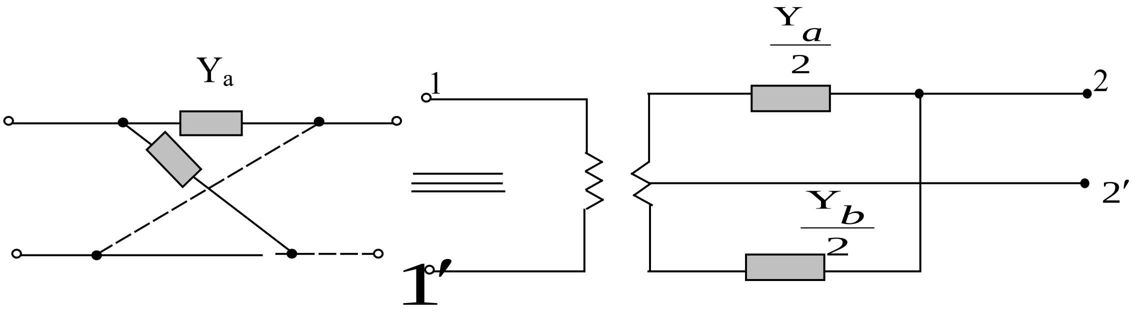


Fig. 2.33. Scheme of a bridge quadrupole

In fig. 2.34 shows a diagram of a differential bridge filter, which contains two quartz resonators in one branch, and one resonator and one capacitor in the other.

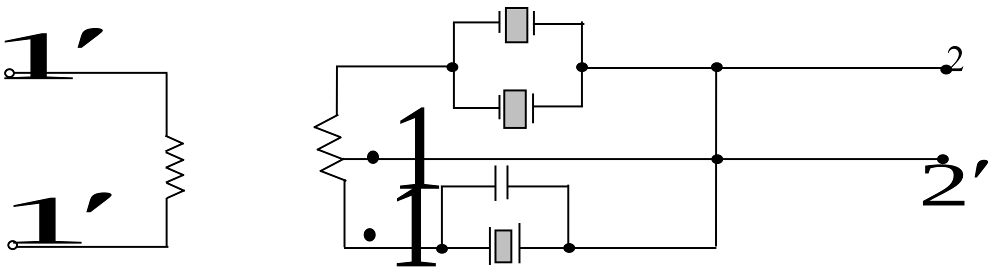


Fig. 2.34. Differential bridge filter scheme

In the considered schemes of differential-bridge quartz filters, the transformation coefficient of the differential transformer in relation to its half-winding was taken as unity. Of course, it can be chosen different from unity, which allows you to change the internal resistance of the generator within wide limits depending on the resistance value or filter load.

Recently, considerable attention has been paid to the development of methods for calculating and designing monolithic quartz filters. A monolithic quartz filter is a set of several frequency monolithic resonators made on the same basis (quartz substrate). Due to their close location, they form a single electromechanical oscillating system (Fig. 2.35).

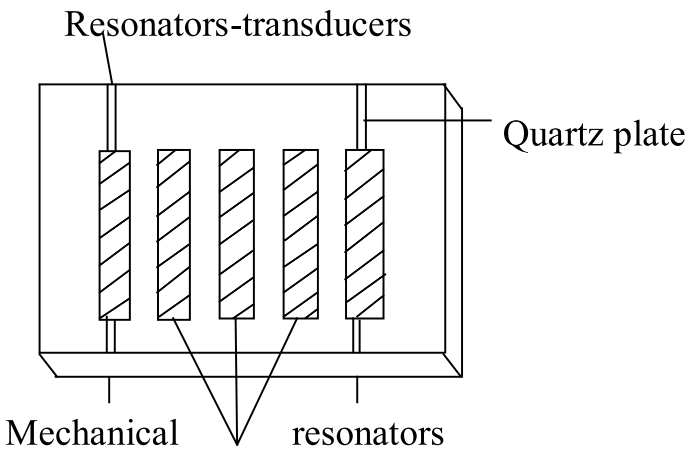


Fig. 2.35. Monolithic quartz filter

Quartz filters are widely used in the equipment of the Ministry of Communications. The use of quartz filters of new types, which were called monolithic, is one of the real ways of microminiaturization of selective circuits of analog radio electronics.

2.6.2. Electromechanical and magnetostrictive filters

Electromechanical filters, which use mechanical resonators made of steel or other alloys as resonators, are highly selective devices that allow a passband of several tens of hertz in the frequency range of several hundred kilohertz. They can be made in the form of a compact structure using different types of vibrations: longitudinal, transverse, torsional, bending vibrations, etc.

Modern electric filters usually contain a large number of resonators, which are connected to each other in a chain using special connections. Such filters are called electromechanical chain filters.

Electromechanical filters contain three components:

1) input electromechanical converter, which converts electrical oscillations into mechanical oscillations of one or another type;

2) a multi-resonator mechanical oscillating system, that is, a mechanical filter, which consists of mechanical resonators and connections;

3) an output electromechanical converter that converts filtered mechanical vibrations of a certain frequency band into electrical vibrations.

The structural scheme of the electromechanical filter is shown in fig. 2.36.

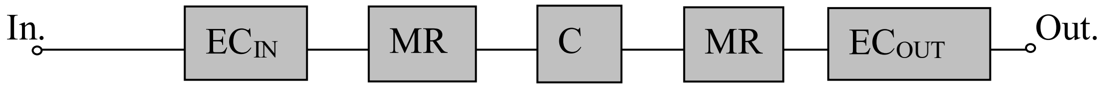


Fig. 2.36. Structural scheme of an electromechanical filter

EC_{in}, EC_{out} are input and output electromechanical converters;

MR are mechanical resonators;

C are connections.

Mechanical oscillations that occur in the input converter are transmitted to the mechanical oscillatory system consisting of a number of links. These oscillations are filtered by a multi-resonator oscillation system. At the output of the mechanical oscillating system, which ensures the allocation of a certain frequency band, the oscillations are perceived by the output converter, which converts mechanical oscillations into electrical ones.

When calculating electromechanical filters, a technique based on the calculation of equivalent LC-filters is widely used.

With the help of chain electromechanical filters, it is possible to obtain bandpass filters with a small width of the transition band.

The principle of operation of the magnetostrictive filter consists in the transformation of electrical vibrations into mechanical ones, filtering of mechanical vibrations and their reverse transformation into electrical ones. The conversion of oscillations is carried out by EC_{IN} and EC_{OUT} elements, the operation of which is based on the magnetostriction effect. A direct effect of magnetostriction is that if a ferromagnetic rod is placed in an alternating magnetic field, its length will change periodically. If you mechanically act on this rod, its magnetic permeability will change, that is, the reverse effect of magnetostriction will appear.

An electromechanical converter consists of an inductor, inside which is a nickel core, and permanent magnets that create the magnetization of the core. Under the action of the external magnetic field created by the input current, the length of the core periodically changes, that is, mechanical oscillations occur in it. These vibrations are transmitted to mechanical resonators, which are a chain of metal plates, discs or balls connected to each other by nickel rods-links.

Each resonator is equivalent to an oscillating circuit, and the coupling is the coupling capacitance between the circuits. The last resonator excites oscillations in the core of the coil of the output electromechanical converter. At the same time, due to the reverse effect of magnetostriction, the EMF of the output signal is induced in the coil winding.

The bandwidth of a magnetostrictive filter is usually several kilohertz.

The advantages of magnetostrictive filters:

– high stability of characteristics;

– resistance to impact loads.

Disadvantages:

– complexity of manufacturing and regulation;

– relatively high cost;

– limitation of the frequency range to several units of megahertz.

2.6.3. Active RC-filters

inductive filters, which are discussed above, should be used at frequencies higher than several tens of kilohertz, at lower frequencies the inductance of the coils should be greater, that leads to a significant increase in weight and size indicators and cost.

In this regard, non-inductive RC and ARC filters have become widespread in radio engineering equipment. The low-pass RC-filter consists of elements R_1 and C_1 (Fig. 2.37)

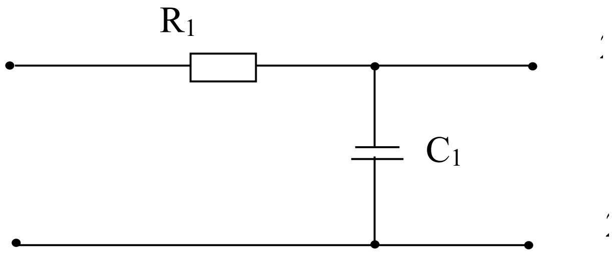


Fig. 2.37. RC-filter of lower frequencies

Since the resistor R_1 is in the longitudinal branch, such a filter passes direct current and low-frequency oscillations. As the frequency increases, the resistance of the capacitor decreases and it shunts the output signal, that is, the attenuation will increase with increasing frequency. The attenuation characteristic of such a filter is shown in Fig. 2.38.

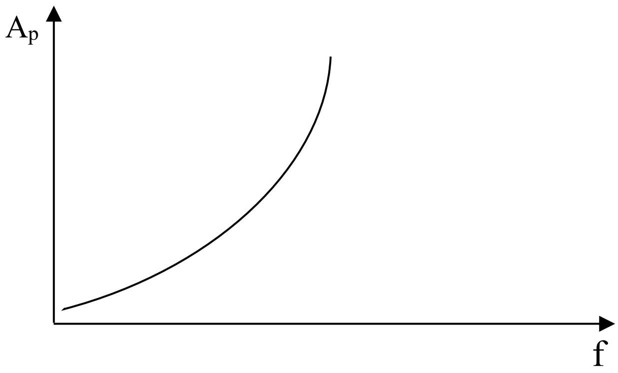


Fig. 2.38. Attenuation characteristics of the RC-filter of low frequencies

The RC-filter of upper frequencies contains a capacitor in the longitudinal branch, and a resistor in the transverse branch. Its scheme and characteristics are shown in fig. 2.39.

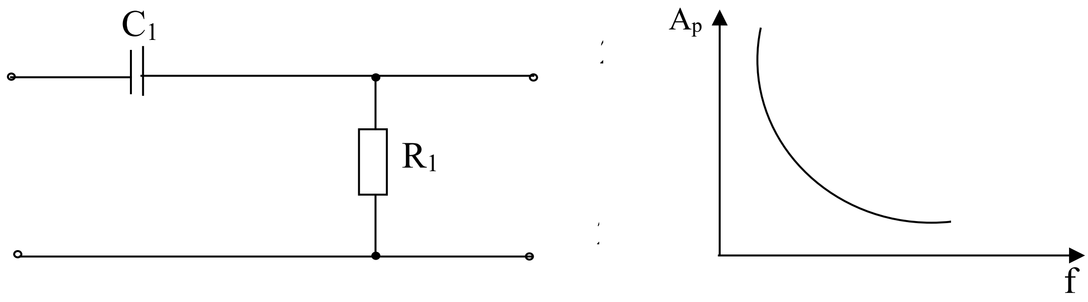


Fig. 2.39. Characteristics of attenuation of the RC-filter of upper frequencies

A band-pass RC filter can be obtained by connecting in series the links of the low-pass RC filter and the high-pass RC filter. With the appropriate selection of parameters R and C, a bandpass filter can be obtained. Its scheme and attenuation characteristics are shown in fig. 2.40.

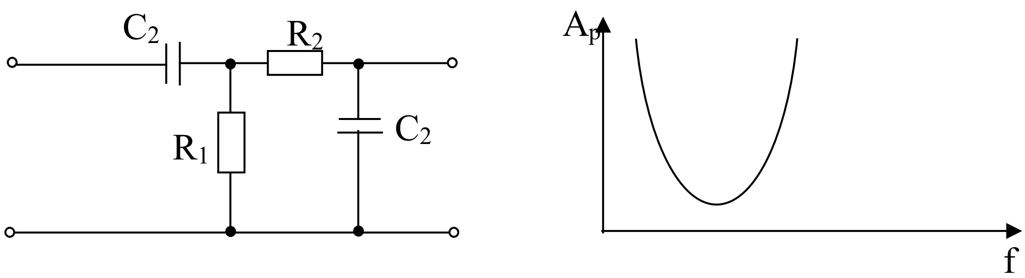


Fig. 2.40. Attenuation characteristics of the bandpass filter

In a similar way, it is possible to obtain a rejection RC filter (Fig. 2.41).

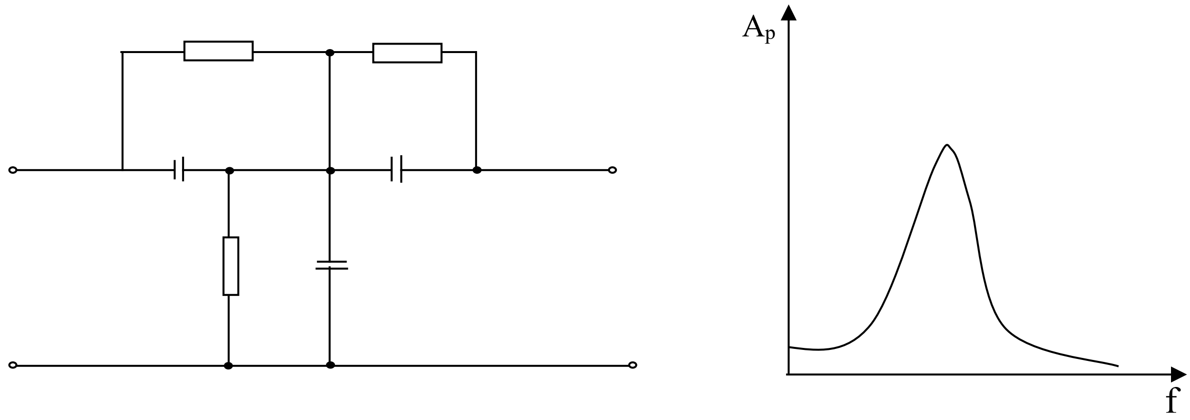


Fig. 2.41. Attenuation characteristic of the rejection filter

The disadvantage of RC filters is a rather large attenuation in the transmission band, which is explained by the presence of active resistances. Therefore, RC filters are often used in combination with signal amplifiers. A device consisting of a passive RC filter and a signal amplifier is called an active RC filter (ARC filter). We will consider the principle of operation of the ARC filter using the example of the high-frequency filter. It consists of a signal amplifier, in the negative feedback circuit (NFC) of which a LF RC filter is included (Fig. 2.42).

In the presence of a NFC circuit, the gain of the amplifier decreases the more the feedback voltage transmitted from the output of the amplifier to its input through the low-pass RC filter. Since the voltage at the output of the filter is maximum at lower frequencies and decreases with increasing frequency, we get that the gain of the amplifier changes according to the inverse law, that is, it will be maximum at upper frequencies and minimum at lower ones.

In this way, we will obtain the HPF. The main feature of ARC filters is amplification of oscillations in the passband. Thanks to this, it is possible to include a multi-link RC filter in the NFC circuit of the ARC-filter and form a given shape of the AFC of the active filter.

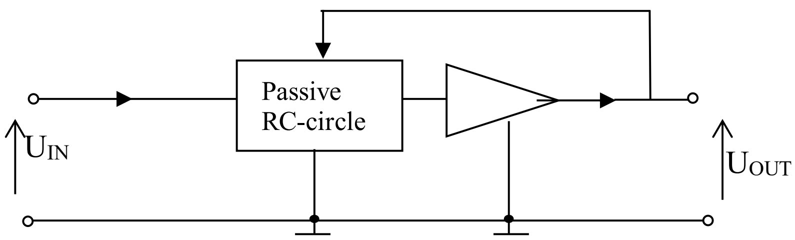


Fig. 2.42. Structural scheme of an active RC filter

Let's consider in more detail the implementation of an ARC-filter based on an operational amplifier.

Let's say we have an L-shaped link of the LPF (Fig. 2.43).

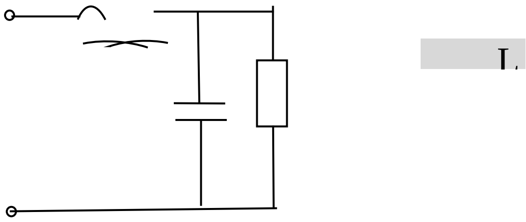


Fig. 2.43. L-shaped link of the LPF

Let's define the transfer function $K_{(p)} = \dfrac{U_{out(p)}}{U_{in(p)}}$ in terms of voltage

$$U_{out}(p) = I_p Z_{CR}(p),\ Z_{CR}(p) = \frac{1}{\left(pC + \left(\frac{1}{R}\right)\right)} = \frac{R}{(pRC+1)},\ U_{out}(p) = I(p) Z_{CR}(p),$$

$$Z_{CR}(p) = \frac{1}{\left(pC + \left(\frac{1}{R}\right)\right)} = \frac{R}{(pRC+1)},\ I(p) = \frac{U_{in}(p)}{\left(pL + \left(\frac{R}{pRC+1}\right)\right)}.$$

Hence

$$K_{(p)} = \frac{1}{\left(pL + \left(\frac{R}{pRC+1}\right)\right)} \cdot \frac{R}{(pRC+1)} = \frac{R}{\left(p^2 LCR + pL + R\right)} = \frac{1}{LCp^2} + \frac{pL}{R+1} =$$

$$= \frac{1}{\left(LC\left(p^2 + \frac{p}{RC}\right) + \frac{1}{LC}\right)} = \frac{\omega_0^2}{p^2} + 2\alpha p + \omega_0^2, \text{ where } \alpha = \frac{1}{2RC}.$$

Analyzing the transfer function, we see that it has the second order, as a result, the link is called a link of the 2nd order.

Consider the implementation of the 2nd-order link of the low-pass filter in the form of an active RC filter on the operational amplifier. First, consider the transmission characteristic of an operational amplifier with two-loop feedback (Fig. 2.44).

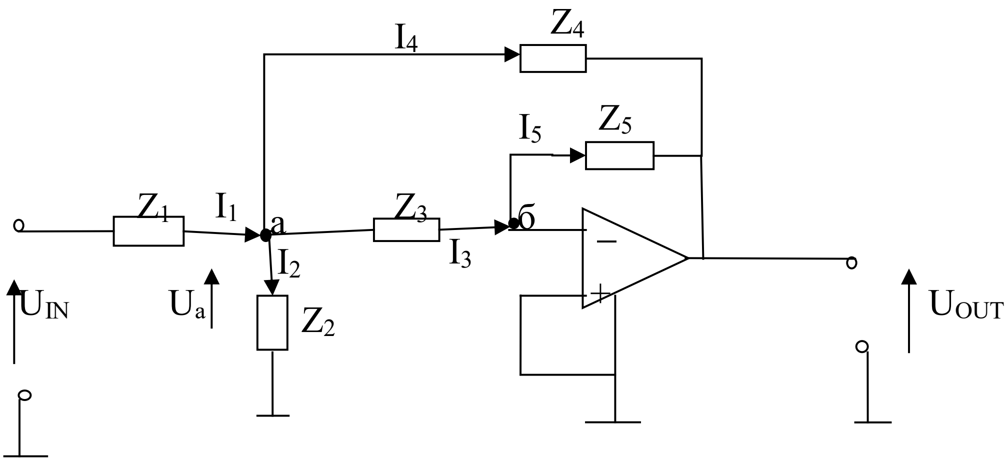


Fig. 2.44. Implementation of the 2nd-order link of the low-pass filter in the form of an active RC-filter on the operational amplifier

We have for node *a* according to Kirchhoff's first law

$$I_1 = \frac{U_{out} - U_a}{Z_2} = I_2 + I_3 + I_4, \qquad I_2 = \frac{U_a}{Z_2}, \qquad I_3 = \frac{U_a - U_b}{Z_3} \approx \frac{U_a}{Z_3}, \qquad I_4 = \frac{U_a - U_{out}}{Z_4},$$

$$I_5 = \frac{U_б - U_{out}}{Z_5} \approx \frac{-U_{вих}}{Z_5}, \; I_5 = I_3.$$

Then $YU_{in} = U_a(Y_1 + Y_2 + Y_3 + Y_4) - Y_4 U_{out}$.

From the equation $I_5 = I_3$ determine $U_a = \frac{-U_{out} Y}{Y_3}$ and obtain the expression for $K_{(p)}$

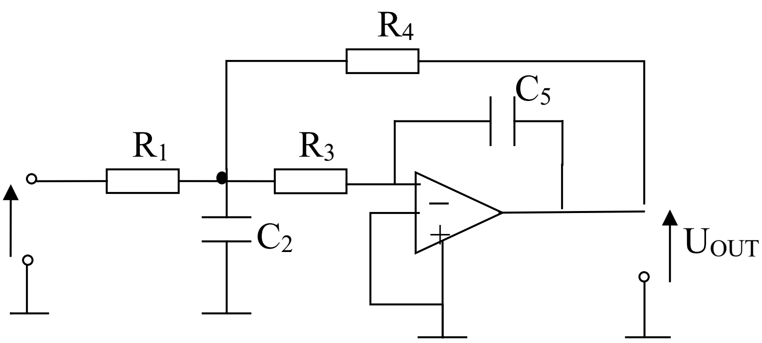
$$K_{(p)} = \frac{-Y_1 Y_2}{Y_5 \left(Y_1 + Y_2 + Y_3 + Y_4\right)} + Y_3 Y_4.$$

Let's select the conductances $Y_1 - Y_5$, which would provide the typical transfer function of the 2nd-order low-pass filter obtained earlier. Turning to the transfer function of the low-pass filter, we see that for its implementation it is necessary that the elements Y_1, Y_3 and Y_4 are resistors, and the elements Y_2 and Y_5 are capacitors. In this case, the transfer function will have the form

$$K_{(p)} = \frac{-G_1 G_2}{p^2 C_2 C_5} + p\left(G_1 + G_3 + G_4\right) \cdot C_5 + G_4 G_3,$$

and the scheme of the low-frequency RC-filter of the 2nd order can be presented in the form of fig. 2.45.

Fig. 2.45. Scheme of a low-frequency RC filter of the 2nd order



The scheme of a second-order high-frequency active RC filter can be obtained from the second-order low-pass filter circuit by rearranging capacitors and resistors (Fig. 2.46).

Thus, we considered one of the possible principles of construction of active RC-filters, which is based on the use of operational amplifiers covered by feedback circuits.

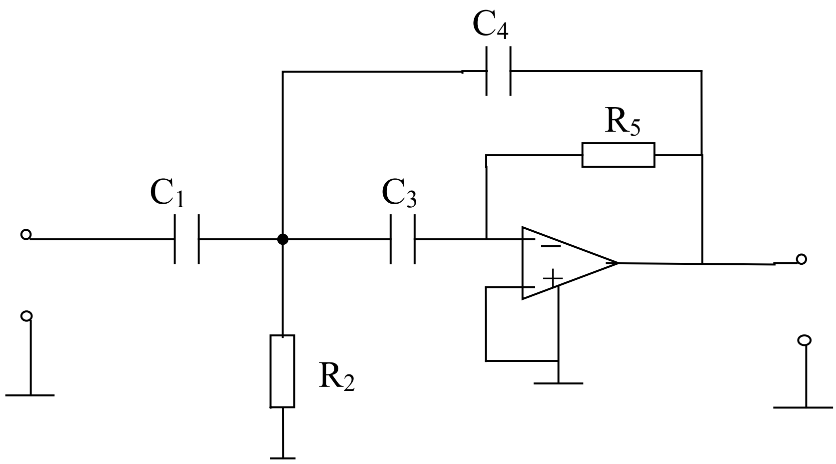


Fig. 2.46. Scheme of a high-frequency RC filter of the 2nd order

Consider the implementation of the ARC filter based on simulated inductances.

When constructing active RC-filters with simulated inductances, the inductive elements in conventional LC-filters are replaced with simulated ones. Gyrators are used to simulate inductances. A gyrator is a three-pole device whose input and output currents and voltages are related by the following ratios:

$$\begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \end{pmatrix} = \begin{pmatrix} 0 & G \\ -G & 0 \end{pmatrix} \times \begin{pmatrix} \dot{U}_1 \\ \dot{U}_2 \end{pmatrix}.$$

Conventional graphic designation of the gyrator is shown in fig. 2.47.

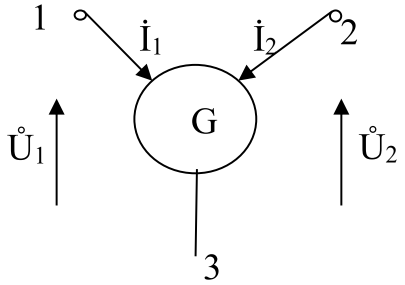


Fig. 2.47. Conventional graphic designation of the gyrator

Consider using a gyrator to simulate an inductance. Suppose that a capacitor with a capacity of C is turned on at the output of the gyrator (Fig. 2.48).

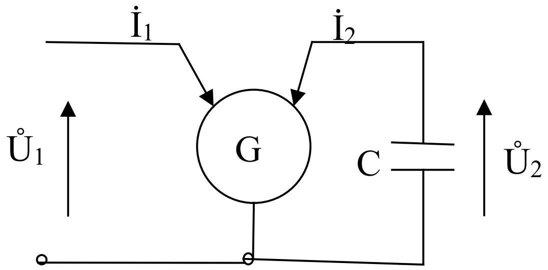


Fig. 2.48. Turning on the gyrator to simulate inductance

Let’s determine the input resistance of the scheme (Fig. 2.48). Based on the transmission equations in G-parameters, we can write for the gyrator

$$\dot{I}_1 = G\mathring{U}_2;\ \dot{I}_2 = -G\mathring{U}_1;\ \dot{I}_2 = (-j\omega C)\mathring{U}_2 = -G\mathring{U}_1,$$

i.e $\mathring{U}_2 = G\mathring{U}_1 / j\omega C$, and accordingly

$$Z_{IN} = \mathring{U}_1 / \dot{I}_1 = \mathring{U}_1 / G\mathring{U}_2 = j\omega C\mathring{U}_1 / G^2\mathring{U}_1 = j\omega C / G^2.$$

Thus, the gyrator allows you to simulate inductance with the help of a capacitor.

To solve the problem of constructing a gyrator, let's display its conductance matrix as the sum of two matrices

$$\begin{pmatrix} 0 & -G_1 \\ G_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ G_2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -G_1 \\ 0 & 0 \end{pmatrix}$$

The first of these matrices corresponds to a non-inverting current amplifier, and the second to an inverting one connected in the reverse direction. As is known, the sum of the conductance matrices corresponds to the parallel connection of these amplifiers, that is, the functional scheme of the gyrator is shown in fig. 2.49.

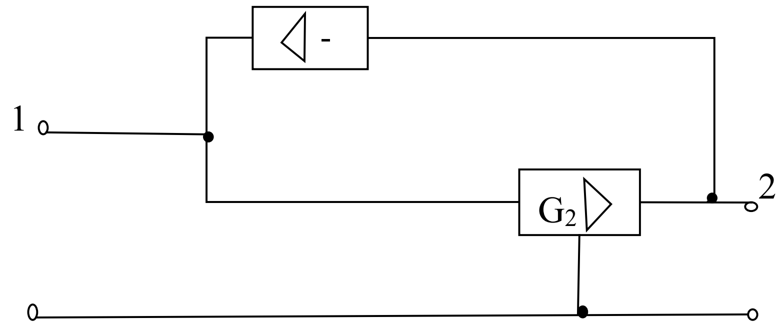


Fig. 2.49. Functional scheme of the gyrator

The schematic scheme of the gyrator is shown in fig. 2.50.

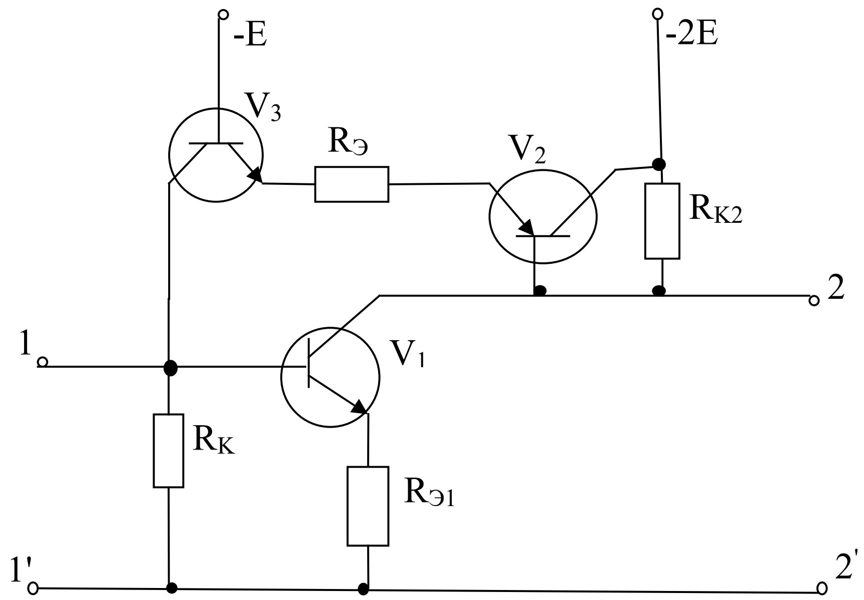


Fig. *2.50. Schematic scheme of the gyrator*

In this scheme, the non-inverting current amplifier is made on transistor V_1, and the inverting one is made on transistors V_2 and V_3.

2.7. DIGITAL FILTERS

2.7.1. The general understanding of the principle of digital signal processing

A general understanding of the principle of digital processing of a continuous signal can be obtained from the scheme shown in fig. 2.51.

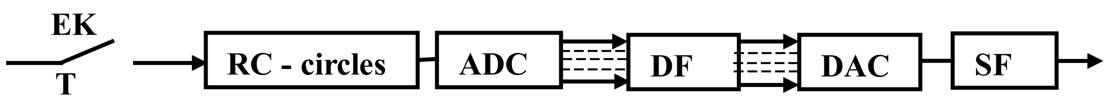


Fig. 2.51. Structural scheme of digital processing of a continuous signal

The input signal $S_{(t)}$ is first discretized in time using an electronic key (EK) operating with a step T. The signal $S_{(t)}$ at the output of the EK has the form of a sequence of equal short pulses, which are counts of the signal $S_{(t)}$.

Each count is stored in the integrating RC circuit for the time required to trigger the analog-to-digital converter (ADC). This time should not exceed the discretization step T.

In an ADC, each count is quantized by level and converted into a binary number, which is composed of *n*-digits, each of which is represented by a “0” or a “1”. Quantization consists in the fact that the count is measured and assigned one level out of the total number of possible ones. This number is equal to 2^n. For example, with $n=10$, we get $2^n=1024$ levels.

Thus, a digital count takes place at the output of the ADC in the form of a binary *n*-bit number.

The sequence of digital readings is fed to a digital filter (DF).

The DF is a computing device in which certain mathematical operations (addition, multiplication, as well as time delay) corresponding to a given algorithm are performed on digital readings. As a result of these operations, new digital readings corresponding to the filtered signal appear at the output of the DF. Counts are reproduced in analog form in the digital-to-analog converter.

In a quadrupole, which can be called a synthesizing filter (SF), readings of an analog form are converted into a continuous output signal.

It should be noted that when considering the principle of operation of the digital signal processing scheme, analog-to-digital and digital-to-analog conversion is not of decisive importance. That is why in the future it is possible to proceed from the assumption that non-quantized counts are introduced in the DF, that is, we will consider the principles of operation of discrete systems.

2.7.2. Models of discrete signals

Discrete signals are determined at discrete moments of time and are represented by sequences of numbers.

The sequence of numbers *x*, in which the *n*th member of the sequence is defined as $x(n)$, can be written in the form

$$x=\{x(n)\},\ -\infty<\nu<\infty.$$

For convenience, *x*(n) will be called the *n*th sample (count) of the sequence.

Discrete signals are often represented graphically (fig. 2.52)

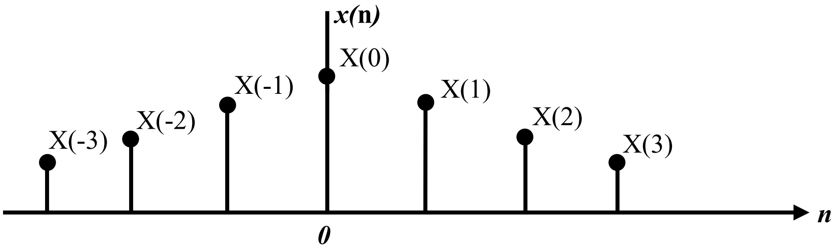


Fig. 2.52. Graphical representation of a discrete signal

In discrete signal processing, typical sequences play an important role (for analog systems, typical actions are delta function, single-step function):

a) unit pulse (Fig. 2.53)

$$\delta(n) = \begin{cases} 0 & n \neq 0; \\ 1 & n = 0 \end{cases}$$

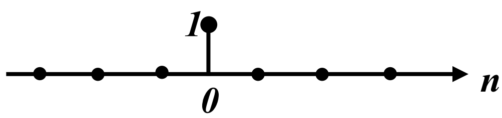


Fig. 2.53. A graphical representation of a discrete unit pulse

b) single step sequence (Fig. 2.54)

$$U(n) = \left\{ \begin{matrix} 1 & n \geq 0 \\ 0 & n \langle 0 \end{matrix} \right\}.$$

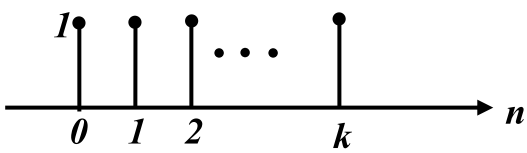


Fig. 2.54. A graphical representation of a discrete unit step sequence

A single step sequence is related to a single pulse ratio $U(n) = \sum_{k=-\infty}^{n} \delta(k)$.

Accordingly, a unit impulse is related to a unit step sequence by the formula $\delta(n) = U(n) - U(n-1)$.

The sequence *x*(n) is called periodic with period ***N*** if

$x(n) = x(n+N)$ for all *n*.

The product and sum of two sequences *x* and *y* are defined as the product and sum of samples, respectively:

$$xy = \{x(n)y(n)\};$$

$$x + y = \{x(n) + y(n)\}$$

The multiplication of sequences x by the number a is defined as $xa = \{ax(n)\}$

.

A sequence of y is a delayed or shifted sequence of x if y has a value

$y(n) = x(n - n_0)$, where n_0 is an integer.

Any sequence can be represented as a sum of weighted and delayed unit pulses

$$P(n) = a_{-3}\delta(n+3) + a_1\delta(n-1) + a_2\delta(n-2) + a_7\delta(n-7).$$

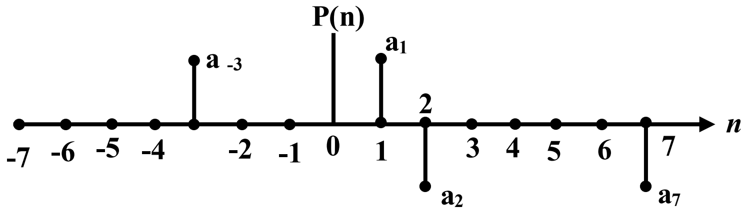


Fig. 2.55. Representation of a discrete sequence

Thus, for the general case, an arbitrary sequence x(n) can be written in the form

$$x(n) = \sum_{k=-\infty}^{\infty} X(k)\delta(n-k).$$

2.7.3. The concept of the impulse characteristic of a discrete circuit

In order to extract information, signals must be processed. The technique of signal processing is to transform a signal into another signal that is more appropriate.

Signal processing systems can be classified like the signals themselves.

Yes, discrete systems are systems in which there are discrete signals (number sequences) at the input and at the output.

A discrete processing system is defined mathematically as a single-valued transformation or operator T, which transforms the input sequence x(n) into the output sequence y(n), which is mathematically written in the form

$$y(n) = T[X(n)],$$

and it is graphically depicted as shown in Fig. 2.56.

Fig. 2.56. Discrete processing system

The classes of discrete systems are determined by imposing restrictions on the transformation T[ ].

Therefore, in the future, we will consider only the class of linear systems invariant with respect to displacement, because they are relatively simple in mathematical terms.

The class of linear discrete systems is determined by the principle of superposition, which is as follows: if $y_1(n)$ and $y_2(n)$ are responses (output sequences) to input sequences x_1(n) and x_2(n), then the discrete system is linear if and only if when

$$T\left[ax_1(n)+bx_2(n)\right]=aT\left[x_1(n)\right]+bT\left[x_2(n)\right]=ay_1(n)+by_2(n),$$

where *a* and *b* are arbitrary constants.

The class of shift-invariant discrete systems is characterized by the following property: if $y(n)$ is the response (output sequence) to the input sequence $x(n)$, then for the shift-invariant discrete system for the output sequence shifted by k counts $y(n-k)$ will correspond to the input a sequence also shifted by k, i.e. $x(n-k)$, where k is a positive or negative integer.

Based on the introduced notions of linear and shift-invariant discrete systems, it is possible, as for analog linear electric circuits, to introduce the concept of the impulse characteristic of a discrete linear shift-invariant system, as well as the analytical expression of a discrete convolution, which allows, based on the known impulse characteristic of a discrete-invariant system, to displacement of the linear system to determine the response in the form of the initial numerical sequence.

Earlier we showed that an arbitrary sequence x(n) can be represented as a delayed and weighted sum of unit impulses

$$x(n)=\sum_{k=-\infty}^{\infty}x(k)\delta(n-k).$$

It was determined above that the discrete processing system performs an unambiguous transformation of the input sequence x(n) into the output sequence $y(n)$. That is,

$$y(n)=T\left[\sum_{k=-\infty}^{\infty}x(k)\delta(n-k)\right].$$

Using the principle of linearity (that is, we assume that a linear discrete system exists), we will have

$$y(n)=\sum_{k=-\infty}^{\infty}x(k)T\left[\delta(n-k)\right].$$

The second term of the right-hand side of the given expression $T\left[\delta(n-k)\right]$ is the response of a discrete linear system to a unit impulse $\delta(n-k)$, i.e.

$$T\left[\delta(n-k)\right]=h_k(n)^*,$$

where $\delta(n-k)$ is the unit impulse shifted by k.

But for the invariant shift of the system, we can write

$$T\left[\delta(n-k)\right]=h(n-k).$$

The shifted numerical sequence (response) $h(n-k)$ is a response to a single pulse $\delta(n-k)$ and is called the impulse response of a linear invariant shift by a discrete system (for analog linear electrical circuits, the response of a linear electrical circuit to the delta function $\delta(t)$ is called the impulse response $g(t)$).

Thus, the expression * is transformed taking into account the introduced notion of impulse characteristic into the form

$$y(n)=\sum_{k=-\infty}^{\infty}x(k)h(n-k)\ ^{**}.$$

and is called a discrete convolution, which allows, based on the known impulse characteristic of the linear invariant shift of a discrete system, to determine the response (output numerical sequence) of the system to the action of the input numerical sequence *x*(n) on it. For analog linear electrical circuits, the response to arbitrary action of impulses is calculated using the Duhamel integral

$$U_{out}(t)=\int_0^t g(t-T)U_{in}(\tau)\,d\tau.$$

If we replace the variable ** in the expression, we get another expression for discrete convolution

$$y(n)=\sum_{k=-\infty}^{\infty}x(n-k)h(k).$$

For analog systems

$$U_{out}(t)=\int_0^{\infty}U_{in}(t-\tau)g(\tau)\,d\tau.$$

For two discrete systems, cascading and parallel switching can take place. Two linear invariant shift systems connected in cascade form a linear invariant shift system with an impulse response equal to the convolution of the impulse responses of the original systems.

Two linear invariant shift systems, switched on in parallel, form a linear invariant shift system with an impulse response equal to the sum of the impulse responses of the output systems.

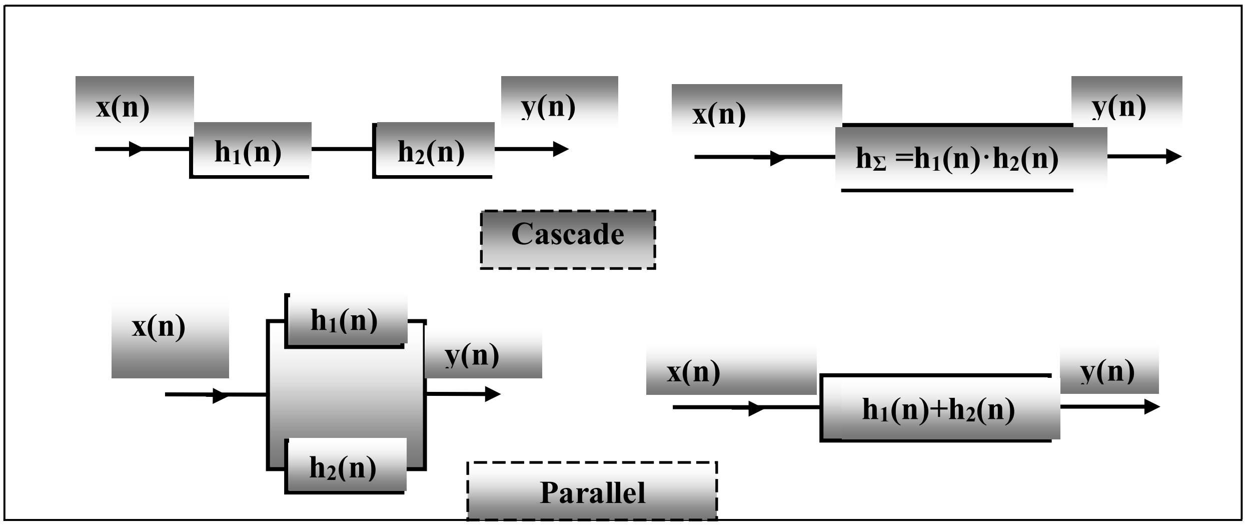


Fig. 2.57. Cascading and parallel inclusion of discrete processing systems

2.7.4. Algorithms of digital filtering in the time domain

Elements of the digital filter

A linear discrete system can be defined by a composition of such three elements.

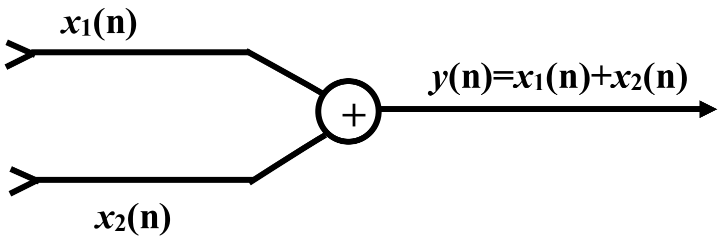


Fig. 2.58. Adder of sequences

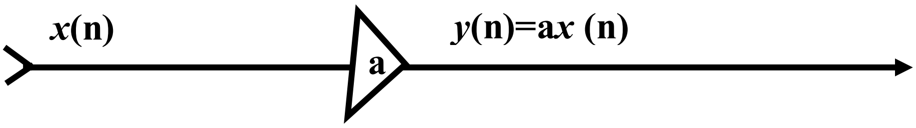


Fig. 2.59. Multiplier by constant coefficient ***a***

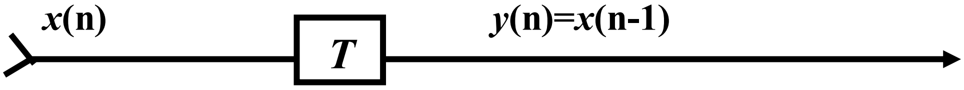


Fig. 2.60. Delay element (T-step discretization)

The difference equation

As a rule, during digital signal processing, there are periodic sequences of $x(n)$ and $h(n)$ with periods of n counts. In this case, the concept of a circular discrete convolution is introduced, which is represented by relations

$$y(n)=\sum_{k=0}^{n-1}x(n-k)h(n-k), \text{ or}$$

$$y(n)=\sum_{k=0}^{n-1}x(n-k)h(k).$$

The sequence $y(n)$ will also be periodic with a period of n counts, so it is enough to calculate it on one period.

In the expression of the circular discrete convolution, the discrete impulse response $h(k)$ can be interpreted as the result of discretization with a step T of the continuous impulse response of the corresponding analog filter.

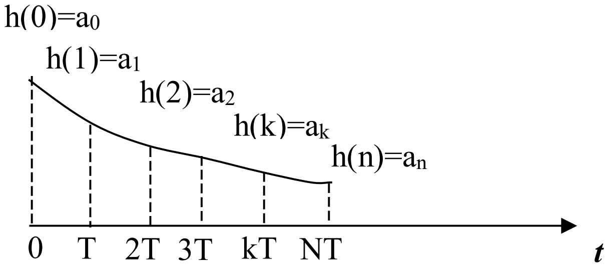


Fig. 2.61. Discrete impulse characteristic h(k)

That is, the discrete impulse response is represented by a numerical sequence $a(k)$.

In this way, the circular discrete convolution can be presented in the form

$$y(n)=\sum_{k=0}^{n-1}a(k)x(n-k).$$

From the point of view of mathematics, the given expression of circular discrete convolution is a difference equation.

In the general case, a shear-invariant linear discrete system can be described by a difference equation of the general form

$$y(n)=\sum_{k=0}^{n-1}a(k)x(n-k)-\sum_{k=1}^{m-1}b(k)y(n-k).$$

The second term of this equation is characterized by the presence of inverse relations. In the presence of inverse connections, the value of the output numerical sequence (output signal) depends not only on n readings of the input sequence $x(n)$, but also on a certain number of m readings of the output sequence at previous moments.

2.7.5. Z-transformation of frequency characteristics of digital filtering

For a discrete sequence $x(n)$, one can introduce the concept of the discrete Laplace transform in relation to the expression

$$x\left(e^{pT}\right)=\sum_{n=0}^{\infty}x(n)e^{-pTn}$$

Here, as in the case of the continuous Laplace transform, the complex variable p is equal to $j\omega$.

During the analysis and synthesis of discrete signal processing systems, the Z-transformation, which is related to the discrete Laplace transform and derives from it, has received considerable spread.

The direct Z-transformation $X(Z)$ of the sequence $X(N)$ is given by the formula

$$X(Z)=\sum_{n=0}^{\infty}x(n)z^{-n},$$

where $z=e^{pT}$.

The Z-transform practically coincides with the discrete Laplace transform and differs only in the image argument. With such a replacement, the transcendental functions from the argument p will turn into rational functions from the argument z. Thanks to this, the analysis is shortened.

Let us consider the Z-image of some discrete signals.

So:

a) unit impulse $\delta(n)$

$$X(z)=Z[\delta(n)]=1;$$

b) single step sequence $U(n)$

$$X(z) \ = \ Z[U(n)]=\frac{1}{1-2^{-1}};$$

c) exponential sequence $x(n)=a^n$

$$X(z) \ = \ Z\left[x(n)\right]=\frac{1}{1-a2^{-1}}.$$

The inverse Z-transform is given by the formula

$$x(n)=Z^{-1}\left[X(z)\right]=\frac{1}{2\pi j}\int_c X(z)Z^{n-1}dz.$$

The Z-transform is characterized by the following properties:

1) linearity if $X(n)=a_1x_1(n)+a_2x_2(n)$, then $X(Z)=a_1x_1(z)+a_2x(z)$;

2) offset if $X(n-m)$, then $Z\left[x(n-m)\right] \ =Z^{-m}x(z)$;

3) Z-transform convolution. If $Y(n)=\sum_{k=0}^{\infty}x(k)h(n-k)$, then $Y(z)=x(z)\cdot y(z)$;

A digital device that operates on the basis of a general difference equation

$$Y(n)=\sum_{k=0}^{n-1}a(k)x(n-k)-\sum_{k=1}^{m-1}b(k)y(n-k)$$

is called a digital filter.

An important characteristic of a digital filter is the transfer function $H(z)$, which represents the ratio of the Z-images of the output $y(z)$ and the input sequence $x(z)$ of the filter under zero initial conditions $H(z)=\dfrac{y(z)}{x(z)}$.

Performing the Z-transformation on the left and right sides of the difference equation of the general form, we obtain the transfer function of the digital filter

$$H(z)=\frac{\sum_{n=0}^{n-1}a(k)z^{-k}}{1+\sum_{k=1}^{m-1}b(k)z^{-k}}.$$

For the difference equation with $b(k)=0$, we have

$$y(n)=\sum_{k=0}^{n-1}a(k)x(n-k).$$

Then

$$H(z)=\sum_{k=0}^{n-1}a(k)z^{-k}.$$

Using the transfer functions, it is possible to obtain the frequency characteristics of the DF. For this, a substitution is made $Z=e^{j\omega T}$. The modulus of the complex transfer function is called AFC, and the argument is PFC.

2.7.6. Schematic implementations of digital filtering in the time domain

In analog filters, depending on the way the filter is assigned (impulse characteristic or transfer function), two approaches are possible: in the time domain or in the frequency domain.

Let's consider a time approach.

The time approach is based on the calculation of the difference equations (discrete convolution), which were given earlier $y(n)=\sum_{k=0}^{n-1}a(k)x(n-k)$ and $y(n)=\sum_{k=0}^{n-1}a(k)x(n-k)-\sum_{k=1}^{m-1}b(k)y(n-k)$.

The filter operating on the basis of the first equation is called a non-recursive filter and has a finite duration of the impulse response. Such filters are also called finite impulse response filters (FIR filters).

The filter, which functions on the basis of the 2nd equation, is called a recursive filter, it is characterized by an infinite impulse response and can be called a FIR filter.

Non-recursive filters can be implemented in various forms. The direct form corresponds to the realization of a filter according to the first difference equation.

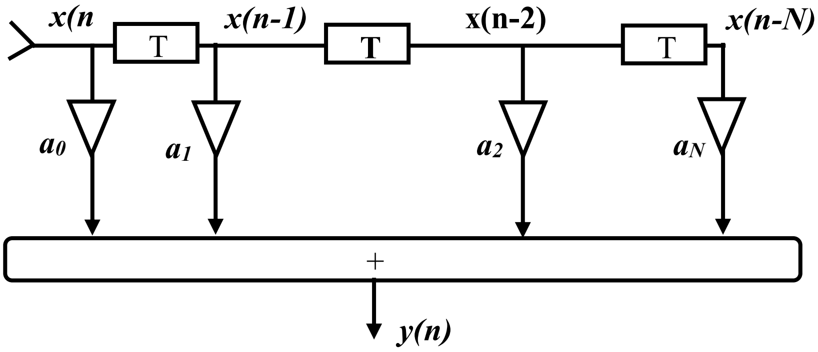


Fig. 2.62. Structural scheme of the FIR filter

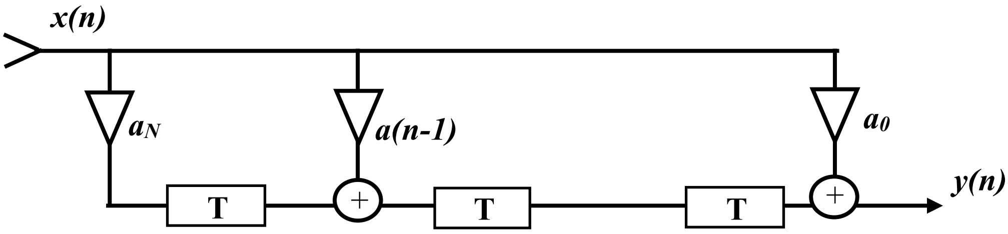


Fig. 2.63. The equivalent form of the structural scheme of the FIR filter

There are also several different forms of recursive filter implementation.

The direct form of realization corresponds to the direct use of the second equation.

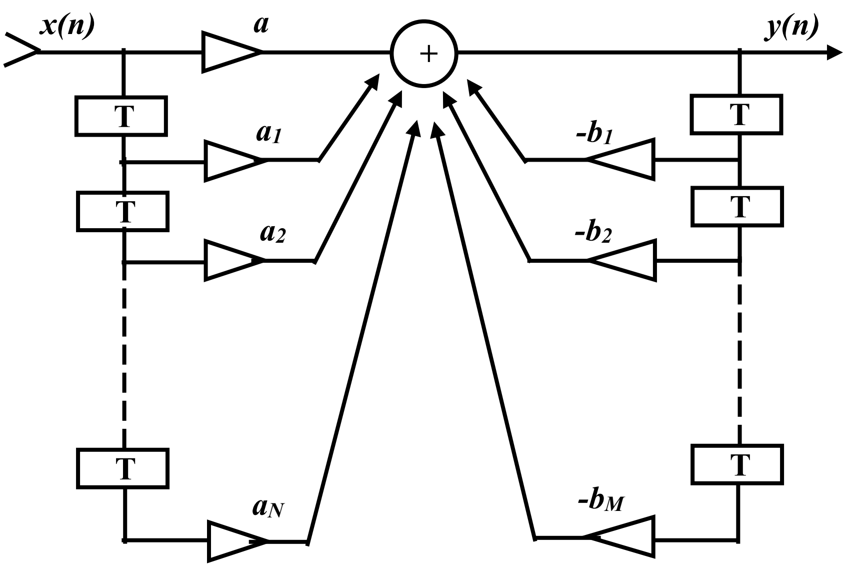


Fig. 2.64. Direct structural scheme of the IIR filter

The canonical form $(N = M)$ is derived from the transfer function

$$H(z)=\frac{\sum_{n=0}^{N-1}a(k)z^{-k}}{1+\sum_{k=1}^{M-1}b(k)z^{-k}}=H_1(z)H_2(z),$$

where

$H_1(z)=\dfrac{1}{1+\sum_{n=1}^{M-1}b(k)z^{-k}}$ – is the transfer function of the recursive filter, and

$H_2(z)=\sum_{n=0}^{N-1}a(k)z^{-k}$ – is the transfer function of the non-recursive filter.

Difference equations correspond to such transfer functions

$$U(n)=x(n)-\sum_{k=1}^{M-1}b(k)u(n-k),\ Y(n)=\sum_{n=0}^{N-1}a(k)u(n-k).$$

These equations can be implemented according to such a scheme (Fig. 2.65).

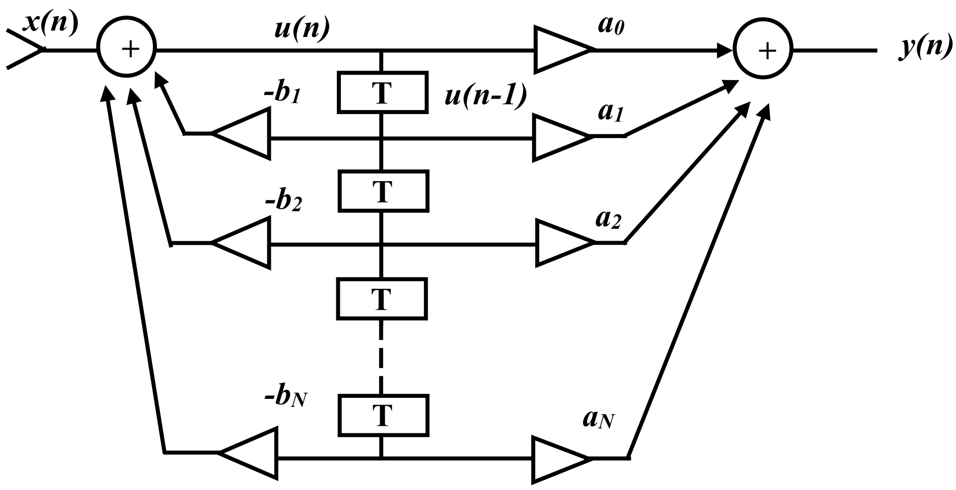


Fig. 2.65. Canonical structural scheme of the IIR filter

The part of the circuit located to the left of the delay elements corresponds to the first difference equation. The implementation of the 2nd equation is based on the fact that the counts $u(n)$, $u(n-1)$, etc. of the auxiliary signal are already received and stored at the outputs of the delay elements. Therefore, additional delay elements are not required to implement the second equation. This made it possible to reduce the number of delay elements by half compared to the direct form of implementation.

As a rule, the implementation of high-order digital DFs in a direct or canonical form is impractical due to calculation errors due to the finite bit rate. In this case, it is better to implement filters using a second-order link

$$H(z)=\frac{a(0)+a(1)z^{-1}+a(2)z^{-2}}{1+b(1)z^{-1}+b(2)z^{-2}}.$$

At the same time, serial or parallel forms of implementation can be used. For the serial form, the transfer function of the DF is written in the form

$$H(z)=\prod_{L=1}^{L} H_i(z).$$

Accordingly, the structural scheme is shown as in Fig. 2.66.

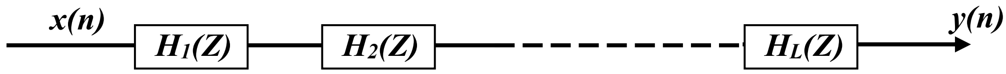


Fig. 2.66. Structural scheme of sequential implementation of DF

For the parallel form, the transfer function of the DF is written in the form $H(z)=\sum_{i=1}^{L} H_i(z)$ at $b(z)=0$.

Accordingly, the structural scheme looks like in Fig. 2.67.

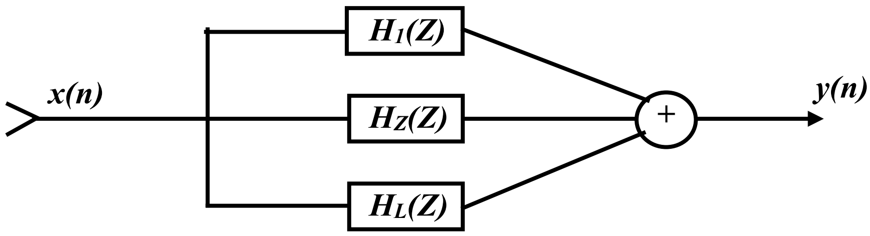


Fig. 2.67. Structural diagram of parallel implementation of DF

2.7.7. Algorithm and scheme of digital filtering in the frequency domain

2.7.7.1. Algorithm of digital filtering in the frequency domain

As is known, with the spectral method of analysis, the transfer function $T(j\omega)$ allows you to determine the spectral density of the output signal based on the known spectral density of the input signal

$$\Phi_2(j\omega)=\Phi_1(j\omega)T(j\omega).$$

Knowing the complex spectral density of the output signal, using the inverse Fourier transform, it is possible to determine the output signal

$$U_{out}(f)=\frac{1}{\pi}\int_0^{\infty}\Phi_2(j\omega)e^{j\omega t}d\omega.$$

This spectral analysis algorithm is used for continuous (analog) signals.

The algorithm is digital, similar to the one discussed above, except that instead of the spectral densities and the complex transfer function, there are their spectral readings, which are determined by time readings based on the discrete Fourier transform

$$X_n(\omega)=\sum_{k=0}^{N-1}X(k)e^{-j\frac{2\pi}{N}nk},$$

where $x(k)$ are the time counts of the input signal.

As for the spectral readings of the transfer function, they are also determined on the basis of the discrete Fourier transform from the time readings of the impulse response $n(k)$

$$H_n=\sum_{k=0}^{N-1}n(k)e^{-j\frac{2\pi}{N}nk}.$$

Thus, the digital filtering algorithm in the frequency domain contains four stages.

1) based on the DFT, there are spectral readings of the input time discretized signal $x(k)$

$$X_n(\omega)=\sum_{k=0}^{N-1}X(k)e^{-j\frac{2\pi}{N}nk};$$

2) spectral readings of the transfer function are based on the DFT

$$H_n(\omega)=\sum_{k=0}^{N-1}h(k)e^{-j\frac{2\pi}{N}nk};$$

3) the convolution of the spectral readings of the signal and the spectral readings of the transfer function is carried out

$$X_n(\omega)\cdot H_H(\omega);$$

4) the inverse discrete Fourier transform from the convolution of two sequences is performed

$$g(k)=\frac{1}{N}\sum_{n=0}^{N-1}x_n(n)H_n(\omega)e^{-j\frac{2\pi}{N}nk}.$$

The result of the 4th stage will be the output numerical sequence $y(k)$ (output signal).

The structural diagram of digital filtering in the frequency domain is shown in Fig. 2.68.

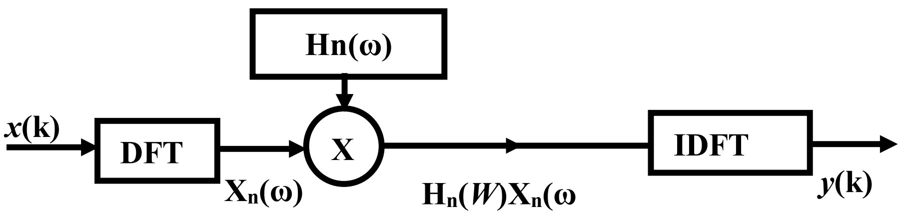


Fig. 2.68. The structural scheme of the parallel implementation of the DF

2.7.7.2. The Fast Fourier Transform

Analyzing the given scheme, we see that the digital filtering algorithm in the frequency domain is reduced to direct and inverse DFT.

According to the formula of direct DFT

$$X_n(\omega)=\sum_{k=0}^{N-1} X(k) e^{-j\frac{2\pi}{N}nk}$$

to calculate N values of spectral readings $X_k(\omega)$, approximately N^2 multiplications and N^2 additions must be performed. As N increases, the time to calculate $X_k(\omega)$ increases significantly. The same situation is observed when calculating the inverse DFT.

Fast Fourier transform (FFT) algorithms have been developed to speed up the DFT calculation.

Let us consider the base-2 FFT algorithms that are applied to sequences of length $N=2^k$.

The main idea of FFT is as follows. The sequence $X(k)$ is divided into two $\frac{N}{2}$-point sequences $X_1(k)$ and $X_2(k)$, for which the sequences of spectral readings $\{X_{1n}(\omega)\}$ and $\{X_{2n}(\omega)\}$ are found. Then, based on the spectral readings of these sequences, the required *N*-point FFT $\{X_n(\omega)\}$ is determined.

If the last operation will be performed simply and will not require complex calculations, then to determine $N=2^k$ spectral readings, it is necessary to perform $\left(\frac{N}{2}\right)^2+\left(\frac{N}{2}\right)^2=\frac{N^2}{2}$ operation.

If you continue the process of splitting $\{X_1(k)\}$ and $\{X_2(k)\}$ sequences into two parts and finding their own FFTs for each of them, you can significantly reduce the number of operations.

Let us consider the DFT of the sequence $X(k), X_n(\omega)=\sum_{k=0}^{n-1} X(k) e^{-j\frac{2\pi}{N}nk}$.

Let's enter a value $W_N=e^{-j\frac{2\pi}{N}}$. Then $X_n(\omega)=\sum_{k=0}^{n-1} X(k) W_N^{+kn}$.

Let's divide $\{X(k)\}$ into two parts $\{X_1(k)\}$ and $\{X_2(k)\}$, containing, respectively, even and odd terms $\{X(k)\}$,

$$\{X_1(k)\}=\{X(2k)\},\ k=0,1,2...\frac{N}{2-1};$$

$$\{X_2(k)\}=\{X(2k+1)\},\ k=0,1,2...\frac{N}{2-1}.$$

Then, since $e^{j\frac{2\pi}{N}2}=e^{j\frac{2\pi}{\frac{N}{2}}}$ then $W_{N^2}=W_{N/2}$.

Thus,

$$X_n(\omega)=\left\{\sum_{k=0}^{N/2-1} x(2k) W_{N/2}^{nk}\right\}+\left\{\sum_{k=0}^{N-1} x(2k+1) W_{N/2}^{nk}\right\} W_k^n$$

Hence $X_n(\omega) = X_{1n}(\omega) + X_{2n}(\omega) W_N^n$, for $0 \le n \le \frac{N}{2} - 1$, i.e., this expression allows you to determine the spectral readings from 0 to $\frac{N}{2} - 1$.

The spectral readings from $\frac{N}{2}$ to N are calculated based on the second expression

$$X_{n+\frac{N}{2}}(\omega) = X_{1n}(\omega) - X_{2n}(\omega) W_N^n .$$

These formulas represent the basic operation of the FFT (the so-called "butterfly").

A schematic image of the "butterfly" can be shown in fig. 2.69.

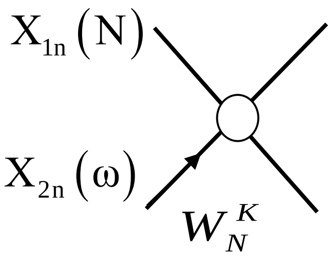
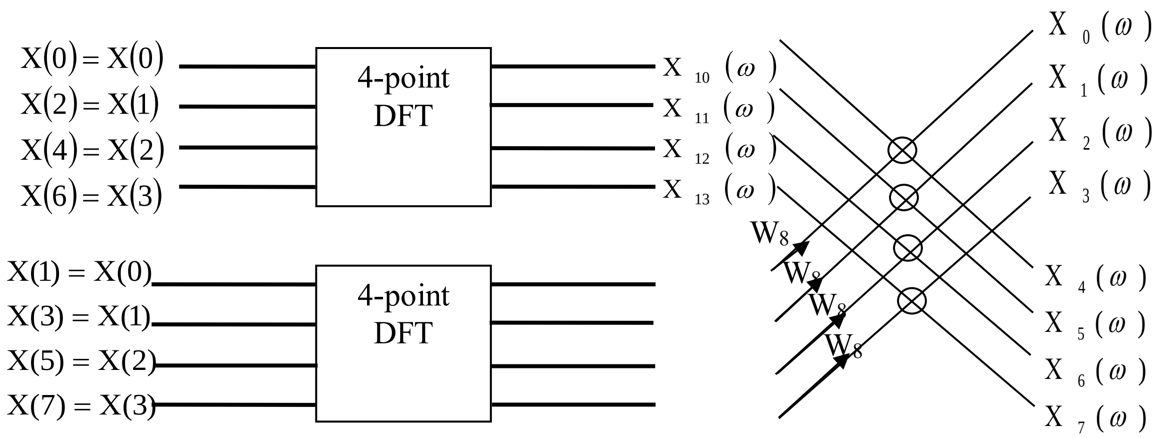


Fig. 2.69. Schematic representation of the operation "butterfly"



On the other hand, 4-point transformations can be determined through 2-point transformations, which are calculated according to the formulas:

$$X_n(\omega) = \sum_{k=0}^{N-1} X(k) W_N^{+nk} = \sum_{k=0}^{\frac{N}{2}-1} X(2k) W_N^{2nk} + \sum_{k=0}^{\frac{N}{2}-1} X(2k+1) W_N^{(2k+1)^n} ;$$

$$X_n(\omega) = X(0) + W_{N/4}^0 ; \; X(1) = X(0) + X(1) ;$$

$$X_1(\omega) = X(0) + + W_{N/4}^1 ; \; X(1) = X(1) - X(2) .$$

It was previously shown that N^2 multiplications are required for direct FFT calculation, and $N\log_2 N$ multiplication operations for N-point FFT. So, the FFT algorithm reduces the number of operations by $\frac{N^2}{N\log_2 N}$ times. So, with $N = 1024$, 100 times.

2.7.8. Synthesis of digital filters

We will consider the issue of DF synthesis on the example of solving the RF synthesis problem. Two methods are possible:

– a direct direct determination of filter parameters by time or frequency characteristics;

– an indirect definition based on the discretization of an analog filter that satisfies the specified requirements.

At the same time, it is proposed to perform two stages.

At the first stage, a suitable analog filter prototype is selected. At the second stage, the transition from the prototype analog filter to the digital filter is carried out.

We have a number of methods of implementing analog filters. Let's consider one of the simplest methods of bilinear transformation.

According to this method, the variable *p* of the transfer function of the analog filter $H(p)$ is replaced by $p = e^{j\omega}$, which is the argument of the transfer function of the digital filter $H(z)$.

This replacement is performed using different formulas for different types of filters.

For example, for LPF $p = y\frac{1-z^{-1}}{1+z^{-1}}$, i.e $H(z) = H(p)_p = y\frac{1-2^{-1}}{1+2^{-1}}$.

In the process of this transformation, a nonlinear transformation of frequencies is performed. The ratio between the frequencies of the p-plane (analog frequency Ω) and the frequencies of the z-plane (digital frequencies ω) is determined by the expression $\Omega = y \cdot tg(\pi\omega)$, where y is the conversion coefficient, which is equal to $y = ctg\pi\omega$.

Control questions and tasks

1. What is the filter cutoff frequency and under what condition is it determined ?

2. Why does LPF pass low-frequency signals and not high-frequency signals ?

3. How is it possible to change the cut-off frequency of the HPF ?

4. Why does the phase characteristic change sign in the RF transparency band (Fig. 3b) ?

5. Under what conditions is the quasi-resonant frequency of the SF determined ?

6. Why does RF pass signals of both low and high frequencies ?

7. Explain the principle of action LPF and its AFC.
8. Explain the principle of action of HPF and its AFC.
9. Explain the principle of action of BF and its AFC.
10. How to experimentally determine the limit frequencies of the filter ?
11. How does the loss resistance of inductive coils affect AFC and PFC of filters ?
12. Show that LPF (Fig. 2) and HPF (Fig. 4) are K-type filters.
13. What are the conditions for matching filters ?
14. How is the transmission constant of the filter determined ?
15. What transformations take place during digital signal processing ?
16. What is a discrete signal and a discrete sequence ?
17. What are the relationships and differences between the spectra of discrete and analog signals ?
18. How to determine the spectrum of the corresponding discrete signal from the known spectrum of an analog signal ?
19. What is the phenomenon of overlapping spectra when discretizing signals ?
20. How is digital signal coding carried out ?
21. How is the autocorrelation function and spectral density of ADC quantization noise determined ?
22. It is known that in order to obtain intelligible human speech, it is sufficient to sample it with a frequency of 8 kHz. What frequency range can be correctly transmitted by such a digital recording ? What must be done in the case of sampling to correctly transmit this range ?
23. The signal x[n], which is different from zero on the segment [A,B], is convolved with the signal h[n], which is different from zero on the segment [C,D]. Find the segment on which the resulting signal can be different from zero.
24. Calculate how many multiplications need to be done to calculate the convolution of a signal of length N with a kernel of length M.
25. The signal sampling frequency is 44100 Hz. The size of the FFT is 4096. What is the length of the analyzed block in seconds? At what frequencies (in hertz) will the signal be decomposed?
26. What frequency resolution of the spectrum will we get in the previous example? What size FFT should be used to get a frequency resolution of about 4 Hz?
27. Implement finding and displaying the spectrum of a given signal section. Enter the possibility of choosing the length of the signal, the size of the FFT, the type of the weight window.
28. Implement fast convolution of two signals through the frequency domain.
29. Implement sectional convolution of two signals through the frequency domain.

30. Implement the filter design algorithm according to the given frequency response. Design a low-pass filter with arbitrary parameters.

31. Show analytically that the inverse DFT can be performed using the relation (1.12).

32. Numerically perform DFT according to formulas (1.7), (1.8), (1.12) and compare the reconstructed signals. Calculate their frequency response and frequency response.

33. Calculate the inverse DFT in accordance with (1.13), using only information about the FFC of the signal.

34. What is the difference between a recursive filter and a non-recursive one? Write the equations for both filters.

35. What is the task of designing digital filters? What approaches are used in this case?

36. What does the filtering algorithm using DPF look like?

37. What technical limitations affect the characteristics of designed filters?

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