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Optimization of the double-span purlins design sketch in a framework with portal frames through the rafter stays application

Hudz Serhii[1]*, Ichanska Natalia[2], Rendyuk Serhii[3], Molchanov Petro[4]

[1] National University «Yuri Kondratyuk Poltava Polytechnic»; https://orcid.org/0000-0002-4764-8635
[2] National University «Yuri Kondratyuk Poltava Polytechnic»; https://orcid.org/0000-0001-5963-9288
[3] National University «Yuri Kondratyuk Poltava Polytechnic»; https://orcid.org/0000-0003-1593-7632
[4] National University «Yuri Kondratyuk Poltava Polytechnic»; https://orcid.org/0000-0001-5335-4281
*Corresponding author E-mail: goods.sergiy@gmail.com

Secondary structures, such as rafter stays, attached to the load-bearing element increase its stiffness, change its sketch and lead to a redistribution of internal forces. The influence of rafter stays on the bearing capacity of the frame elements was analyzed. A number of measures are considered to ensure the spatial stability of the linear elements of frame buildings, which lead to a decrease in the metal consumption of steel purlins. Based on the analysis of internal forces, the peculiarities of the working conditions of the beam were identified and described. It is proposed to increase material savings through detailed calculation. A comparison of design results is presented in software for calculating building models with portal frames

Keywords: design sketch, framework, internal forces redistribution, metal consumption, purlin, rafter stay

Оптимізація розрахункової схеми двопролітних прогонів у каркасній системі з портальними рамами із застосуванням в’язевих підкосів

Гудзь С.А.[1]*, Ічанська Н.В.[2], Рендюк С.П.[3], Молчанов П.О.[4]

[1, 2, 3, 4] Національний університет «Полтавська політехніка імені Юрія Кондратюка»
*Адреса для листування E-mail: goods.sergiy@gmail.com

Приєднані до несучого елемента (основної балки або колони рамного каркасу) різного роду другорядні конструкції, такі як встановлені планомірно в’язеві підкоси, збільшують його жорсткість, змінюють його розрахункову схему і призводять до перерозподілу внутрішніх зусиль. Проаналізовано вплив розкріплення в’язевими підкосами на несучу здатність елементів каркасу із використанням математичних методів. Розглянуто ряд заходів для забезпечення просторової стійкості лінійних несучих елементів каркасних будівель, що призводять до зменшення металоємності сталевих прогонів покриття для типового каркасу. На основі аналізу внутрішніх зусиль було виявлено та описано особливості умов роботи балки. Розкрито переваги та недоліки конструктивних рішень відповідно до принципів простоти та ефективності. Запропоновано збільшити економію матеріалу за допомогою детального розрахунку. Вирішення задачі в системі комп’ютерної алгебри wxMaxima за розрахунковою моделлю показало суттєве зменшення внутрішніх зусиль, яке має високу збіжність із результатами розрахунку моделі методом скінченних елементів. Варто звернути увагу на значний перерозподіл зусиль, а саме згинальних моментів, у прогоні. Зменшення максимального розрахункового згинального моменту в ньому може досягати 20% при середній жорсткості опор, що суттєво впливає на загальну металоємність несучих елементів каркасу (до 15%). Наведено порівняння результатів розрахунку із використанням спеціалізованого програмного забезпечення для аналізу моделей будівель із портальними рамами (Consteel). Урахування факторів, що характеризують особливості роботи прогонів покриття позитивно відображається на рівні використання і запасу міцності матеріалу. В’язеві підкоси можуть не тільки виконувати свою безпосередню функцію, але й ефективно використовуватися для підкріплення прогонів, таким чином змінюючи їх розрахункову схему, зменшуючи витрати сталі, що є позитивним чинником

Ключові слова: розрахункова схема, рамний каркас, перерозподіл зусиль, металоємність, прогін, в’язевий підкіс

Introduction

One of the major structural components in the system of the building with a steel framework is the purlin. Purlins make a considerable part of the mass of load-carrying structures (Fig. 1). Roofing purlins of solid cross-section make up an average of 10 - 15% of the mass of the building, walls - 15 - 25% depending on the building ratio of dimensions.

Figure 1 – General view of a steel framework and purlins

As structural steel for frame buildings is a significant share in the total energy-intensive production of steel, the solution of the problem of reducing the metal consumption of load-bearing elements through optimization of the design sketch is an urgent scientific and technical problem that requires the construction of an analytical model.

Conventional mathematical models in general do not characterize all the behaviour features of the structure in the building framework, especially with significant stiffness of the connected elements. In such cases, the modelling often has little in common with the actual processes, does not correspond to the real picture of the stress-strain state, and needs refinement to adequately reflect the using ratio of the cross-section by stress required to ensure the reliability and stability of the structure as a whole.

We will analyse and identify ways to solve the problem of determining the bearing capacity of elements that are prone to lateral-torsional buckling. The tendency of the element to instability occurs due to insufficient restraining of the attached structures. These structures reduce the design length of the element and increase its overall stability. Stabilization can also take place through the arrangement of structural parts: stiffeners, rafter stays, the protrusions of the beams in the supporting areas, and the adjacency of the columns in the supports. Effective restraint should be provided for members carrying either a bending moment or bending moment and axial force by lateral restraint to both flanges. This may be provided by lateral restraint to one flange and a stiff torsional restraint to the cross-section preventing the lateral displacement of the compression flange relative to the tension flange, see Figure 2.

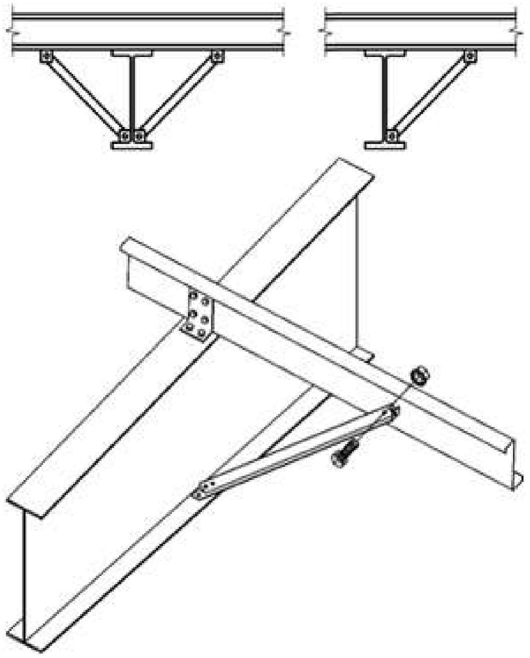

Figure 2 – The rafter-purlin-rafter stay system with double side rafter stays and one-side rafter stay; 3D-view of the typical stiff torsional restraint

Review of the research sources and publications

The articles [1 – 4] are devoted to the advanced design and geometrical optimization of steel portal frames. The papers [5, 6] present the influence of the diaphragm effect on the behaviour of pitched roof portal frames. The purpose of the research is to make a comparison between the simplified design model of a portal frame, where the supports simulating the purlins are considered with infinite axial stiffness, and a portal frame design model where the calculated stiffness of the cladding for the lateral supports is introduced manually.

Despite the leading role of purlin in the structure of roofing, conditions of its work and behaviour under loading remain low-investigated owing to the trouble of their description at difficult resistance, which the publication [7] confirms and demands careful research.

Rational designing, the detailed analysis of boundary conditions of restraining, internal forces and stresses with additional taking note of functioning features with reliably attached enclosing structures allows to reach a significant reduction of steel costs both for purlins in particular, and on cross frames, in general, that is described in the article [8]. A mass of purlins depends mainly on their bearing capacity and also with an increase in their span - on rigidity. With an increase in the step of frames steel expenses on them decrease, however, a mass of purlins grows that has a negative effect on the total mass of the framework. The solution to the problem of high metal consumption of the building is an optimization of the static purlins sketch. Application of effective cross-sections and design sketches of roofing elements leads to a decrease in design internal forces and deformations and consequently, to a reduction of material consumptions by designs in general.

In the article [9] we can find the answer to the question of how efficient is the lateral support of rafters by stays. In single-storey steel buildings, the problem of how to stabilize long-span carrying structures occurs very often. Economic aspects are decisive for the choice of the possible bracing system. The arrangement of stays along the girder has a high degree of effectiveness. The paper [10] presents the influence of purlin-to-beam connection stiffness, allowing the development of the stress skin action, in the case of roof shear panels attached to pitched portal frames. The main purpose of this work is to separate the purlin-to-beam connection flexibility from the equivalent model and to evaluate the impact on frame lateral deformation and stability expressed by the load multiplication factor α_{cr} when purlin to rafter connection stiffness varies from flexible to a stiff one.

In combined steel structures and cross frames of the solid constant or variable cross-section for the purpose of ensuring effective work of purlins, it is possible to use the additional effect of supporting rigid rafter stays between them and the bottom flange of the frame rafter [11, 12]. They are established for flange restraining out of the frame plane from the lateral displacement and torsion and also play the role of vertical bracings, which due to the small height of the solid rafter are almost impossible to arrange. At the same time, the design sketch of purlin changes and reduction of bending moment in it can reach 20%.

Definition of unsolved aspects of the problem

In the case of evenly distributed load on the supported by rafter stays purlin, reduction of bending moment due to the presence of additional intermediate flexible supports is taken into account approximately by the introduction of a reduction factor that depends only on the ratio of edge minor spans and the total span of the single-span purlin and does not depend on the level of supports stiffness, so the development of a universal accurate method of determining the bending moment is an unsolved problem.

Problem statement

Optimization of the load-bearing elements design sketch through using a clarification analytical model of their boundary conditions is one way of solving the urgent scientific and technical problem of reducing metal consumption for frame buildings. The use of welded, built-up, spatial cross-sections of purlins or roof systems without purlins solves this problem partially too, however, complicates the process of production and mounting that by a significant number of structural components plays an essential role in their choice. In the case of the use of easy thin-walled cold-formed profiles, the material is distributed on cross-section not absolutely rationally; some of its parts are conditionally excluded from work because of excessive sensitivity to loss of local stability, forming effective design geometrical properties of the incomplete cross-section. Besides, their efficiency decreases due to the impossibility of accounting for the plastic stage of material work. The need for the creation of additional stiffeners by means of bending of the sheet makes cross-section not so simple, and its identical thickness in flanges and web of profile causes lowering of the geometrical properties important at bending. The last one partially is eliminated thanks to providing continuity of purlins which is reached by blousing one purlin of Z-shaped form on another on a certain area on both sides from intermediate support or installation of pads, similar in form. In spite of the fact that in this sketch the bigger basic bending moment is perceived by the doubled cross-section, and the span bending moment decreases, the single-span sketch of such profiles is simpler and cheaper in mounting. Therefore, we will concentrate attention further on the purlins made of rolled profiles (channels and I-beams).

Basic material and results

For a decrease in a mass of purlins and avoidance of rather a difficult assembly joint, it is reasonable to use the continuous sketch of work in both planes without a local increase in the cross-section over support, which, usually, is carried out in the form of the double-span sketch on 6 meters that is caused by dimensions of cargo transport in Ukraine for delivery of structures to a construction site. The maximum transport length is 15 meters in the European Union, thus the maximum span is 7.5 meters. The positive property of double-span purlins in comparison with single-span is their deformability lowered by 2.5 times that on condition of the defining serviceability limit state at rather a small loading can lead to preservation of about 35% of the metal mass. However, the bending moment over average support in the double-span sketch exceeds the maximum span bending moment by almost 80%. That does it determinative and predetermines excess stocks of purlins bearing capacity in the span. When using channels there is a possibility of the creation of a compound I-section in middle support areas at different spatial orientations of profiles that leads to an increase in bearing capacity, torsional stiffness, and stability of purlins.

A double-span system of purlins arrangement in a framework (Fig. 3) is a relatively stiff system intended for use as brickwork restraints. The advantage of its use is a reduced number of erection components.

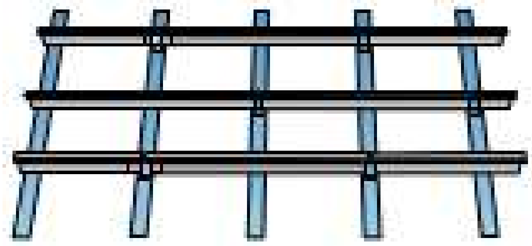

Figure 3 – Double-span system of purlins arrangement in a framework

However, at double-span purlins, there is big unevenness of edge and middle support reactions and, respectively, a difference of load on cross frames. At their consecutive arrangement, the frames located under the middle support of double-span purlins perceive loading nearly 70% more than adjacent frames. It leads to excessive consumption of steel on frames or the need for the production of cross frames with different cross-sections depending on the load on them. At alternate order of purlins arrangement, loads on each frame approximately are levelled to allow designing easier identical frames, but there is a need for developing separately edge single-span purlins of identical height. It is reached due to reduction of edge steps of frames and, respectively, the span of edge purlin approximately for 20%; reduction at the possibility of edge purlins step; selection of the cross-section of edge purlin with the following profile number of identical height according to assortment; for it is used increased strength steel or regulation of rafter stays stiffness.

On condition of the establishment of the rigid rafter, working for compression and tension in two places from frame cross-section, they can be executed from rolled equal angles with stiffeners or with short local connectors (Fig. 4).

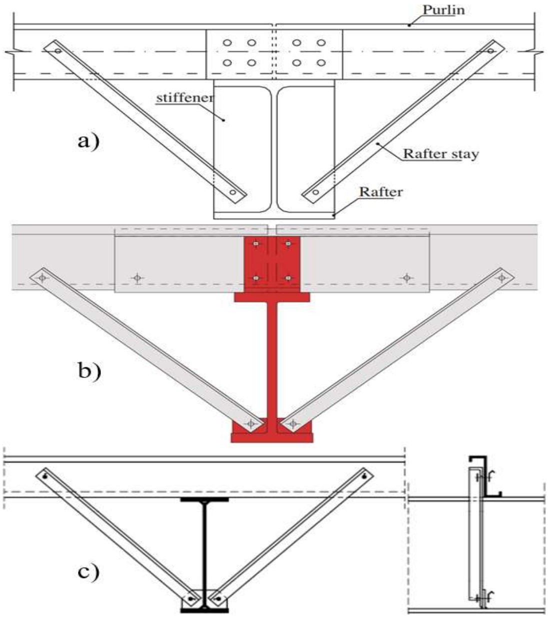

Figure 4 – Torsional restraint of a rafter with a lower flange in compression through rafter-stays
a) with stiffeners; b) with short local connectors

It is possible to execute the calculation of the purlins supported with rafter stays according to the rules of building mechanics [13]. But rafter stays need to be considered in that case as the flexible displaced supports. To do this, we must first determine the stiffness of the rafter according to the principle of virtual forces according to the scheme in Figure 5.

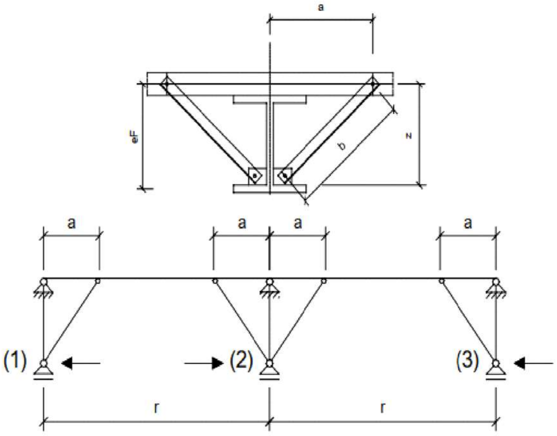

Figure 5 – Restraining of steel linear load-bearing elements by means of rafter stays and the simplified design sketch

Virtual forces are applied in the direction of the accepted displacements of the rafter lower flange, which are calculated, in this case, using the equations:

$$f_M = \int_0^l \frac{M_F M_V}{EI} dx \; ; \tag{1}$$

$$f_N = \int_0^l \frac{N_F N_V}{EA_S} dx \; ; \tag{2}$$

where M_F, N_F – bending moment and longitudinal force from the external load;
M_V, N_V – bending moment and longitudinal force from virtual forces;
EI, EA_S – stiffness of the purlin cross-section in bending and stiffness of the rafter stay cross-section in tension-compression.

Given the dimensions taken in Figure 5, after integration the displacements will be equal to:

$$f_M = \frac{\left(-\frac{4}{3}a + r\right)\frac{e_F^{\,2}}{4}}{EI} \; ; \tag{3}$$

$$f_N = \frac{2b\left(\frac{b}{z}\frac{e_F}{2a}\right)^2}{EA_S} \; . \tag{4}$$

The total stiffness of the rafter stays as pliable supports is inversely proportional to the total displacement of the rafter lower flange, and their pliability is the inverse value of the stiffness:

$$C = \frac{1}{f_M + f_N} \; ; \tag{5}$$

$$K = \frac{1}{C} \; . \tag{6}$$

Therefore, we will consider spans, geometrical properties of cross-sections, and elastic properties of supports (stiffness) known. For the unknowns, we will take the supporting bending moments. As the basic system, we will accept the beam divided by hinges into a row of single-span beams. In the basic system, it is possible to construct the plots of the bending moments caused by unit values of unknowns and the plot of the bending moments from the external load.

The canonical equation for the n-th (middle) support of double-span purlin will look like this:

$$\begin{gathered} M_{n-2}\delta_{n,n-2} + M_{n-1}\delta_{n,n-1} + M_n\delta_{n,n} + \\ +M_{n+1}\delta_{n,n+1} + M_{n+2}\delta_{n,n+2} + \Delta_{nF} = 0 \end{gathered} \tag{7}$$

where M_{n-2}, M_{n-1}, M_n, M_{n+1}, M_{n+2} – unknown bending moments;

$\delta_{n,n-2}$, $\delta_{n,n-1}$, $\delta_{n,n}$, $\delta_{n,n+1}$, $\delta_{n,n+2}$ – coefficients of the canonical equation;

Δ_{nF} – a free member of the canonical equation.

The number of equations (7) will be equal to the number of unknowns. In the matrix notation, the system of canonical equations for all five intermediate supports will look like $M\delta = \Delta$, where the vector M, the matrix δ and vector Δ will be written in the form:

$$M = \begin{pmatrix} M_{n-2} \\ M_{n-1} \\ M_n \\ M_{n+1} \\ M_{n+2} \end{pmatrix}; \tag{8}$$

$$\delta = \begin{pmatrix} \delta_{n-2,n-2} & \delta_{n-2,n-1} & \delta_{n-2,n} & \delta_{n-2,n+1} & \delta_{n-2,n+2} \\ \delta_{n-1,n-2} & \delta_{n-1,n-1} & \delta_{n-1,n} & \delta_{n-1,n+1} & \delta_{n-1,n+2} \\ \delta_{n,n-2} & \delta_{n,n-1} & \delta_{n,n} & \delta_{n,n+1} & \delta_{n,n+2} \\ \delta_{n+1,n-2} & \delta_{n+1,n-1} & \delta_{n+1,n} & \delta_{n+1,n+1} & \delta_{n+1,n+2} \\ \delta_{n+2,n-2} & \delta_{n+2,n-1} & \delta_{n+2,n} & \delta_{n+2,n+1} & \delta_{n+2,n+2} \end{pmatrix} \tag{9}$$

$$\Delta = \begin{pmatrix} -\Delta_{n-2,F} \\ -\Delta_{n-1,F} \\ -\Delta_{n,F} \\ -\Delta_{n+1,F} \\ -\Delta_{n+2,F} \end{pmatrix}. \tag{10}$$

The solution of the system can be obtained by the matrix method using the inverse matrix. To express the coefficients, it is necessary to consider the bending moments and the reactions in the supports:

$$\delta_{n,n-2} = \delta_{n-2,n} = \frac{K_{n-1}}{l_{n-1}l_n}; \tag{11}$$

$$\delta_{n,n-1} = \delta_{n-1,n} = \frac{l_n}{6EI} - \frac{K_{n-1}}{l_n}\left(\frac{1}{l_{n-1}} + \frac{1}{l_n}\right); \tag{12}$$

$$\delta_{n,n} = \frac{l_n}{3EI} + \frac{l_{n+1}}{3EI} + \frac{K_{n-1}}{l_n^2} + \frac{K_{n+1}}{l_{n+1}^2}; \tag{13}$$

$$\delta_{n,n+1} = \delta_{n+1,n} = \frac{l_{n+1}}{6EI} - \frac{K_{n+1}}{l_{n+1}}\left(\frac{1}{l_{n+1}} + \frac{1}{l_{n+2}}\right); \tag{14}$$

$$\delta_{n,n+2} = \delta_{n+2,n} = \frac{K_{n+1}}{l_{n+1}l_{n+2}}; \tag{15}$$

$$\begin{gathered} \delta_{n-1,n-2} = \delta_{n-2,n-1} = \frac{l_{n-1}}{6EI} - \\ -\frac{K_{n-2}}{l_{n-1}}\left(\frac{1}{l_{n-2}} + \frac{1}{l_{n-1}}\right) - \frac{K_{n-1}}{l_{n-1}}\left(\frac{1}{l_{n-1}} + \frac{1}{l_n}\right) \end{gathered} \tag{16}$$

$$\delta_{n-1,n-1} = \frac{l_{n-1}}{3EI} + \frac{l_n}{3EI} + \frac{K_{n-2}}{l_{n-1}^2} + K_{n-1}\left(\frac{1}{l_{n-1}} + \frac{1}{l_n}\right)^2 \tag{17}$$

$$\begin{gathered} \delta_{n-1,n+1} = \delta_{n-1,n+2} = \delta_{n-2,n+1} = \delta_{n-2,n+2} = \\ = \delta_{n+1,n-2} = \delta_{n+1,n-1} = \delta_{n+2,n-2} = \delta_{n+2,n-1} = 0 \end{gathered}; \tag{18}$$

$$\delta_{n-2,n-2} = \frac{l_{n-2}}{3EI} + \frac{l_{n-1}}{3EI} + \frac{K_{n-1}}{l_{n-1}^2} + K_{n-2}\left(\frac{1}{l_{n-2}} + \frac{1}{l_{n-1}}\right)^2 \tag{19}$$

$$\delta_{n+1,n+1} = \frac{l_{n+1}}{3EI} + \frac{l_{n+2}}{3EI} + \frac{K_{n+2}}{l_{n+2}^2} + K_{n+1}\left(\frac{1}{l_{n+1}} + \frac{1}{l_{n+2}}\right)^2 \tag{20}$$

$$\begin{gathered} \delta_{n+1,n+2} = \delta_{n+2,n+1} = \frac{l_{n+2}}{6EI} - \\ -\frac{K_{n+1}}{l_{n+2}}\left(\frac{1}{l_{n+1}} + \frac{1}{l_{n+2}}\right) - \frac{K_{n+2}}{l_{n+2}}\left(\frac{1}{l_{n+2}} + \frac{1}{l_{n+3}}\right) \end{gathered} \tag{21}$$

$$\delta_{n+2,n+2} = \frac{l_{n+2}}{3EI} + \frac{l_{n+3}}{3EI} + \frac{K_{n+1}}{l_{n+2}^2} + K_{n+2}\left(\frac{1}{l_{n+2}} + \frac{1}{l_{n+3}}\right)^2 \tag{22}$$

The free members of the canonical equations system will be equal to the following expressions:

$$\Delta_{n,F} = \frac{B_{n,F}}{EI} + \frac{A_{n+1,F}}{EI} + \frac{K_{n-1}}{l_n}R_{n-1} + \frac{K_{n+1}}{l_{n+1}}R_{n+1}; \tag{23}$$

$$\begin{gathered} \Delta_{n-2,F} = \frac{B_{n-2,F}}{EI} + \frac{A_{n-1,F}}{EI} - \\ -K_{n-2}\left(\frac{1}{l_{n-2}} + \frac{1}{l_{n-1}}\right)R_{n-2} + \frac{K_{n-1}}{l_{n-1}}R_{n-1} \end{gathered} \tag{24}$$

$$\begin{gathered} \Delta_{n+2,F} = \frac{B_{n+2,F}}{EI} + \frac{A_{n+3,F}}{EI} + \\ +\frac{K_{n+1}}{l_{n+2}}R_{n+1} - K_{n+2}\left(\frac{1}{l_{n+2}} + \frac{1}{l_{n+3}}\right)R_{n+2} \end{gathered} \tag{25}$$

$$\begin{gathered} \Delta_{n-1,F} = \frac{B_{n-1,F}}{EI} + \frac{A_{n,F}}{EI} + \\ +\frac{K_{n-2}}{l_{n-1}}R_{n-2} - K_{n-1}\left(\frac{1}{l_{n-1}} + \frac{1}{l_n}\right)R_{n-1} \end{gathered} \tag{26}$$

$$\begin{gathered} \Delta_{n+1,F} = \frac{B_{n+1,F}}{EI} + \frac{A_{n+2,F}}{EI} - \\ -K_{n+1}\left(\frac{1}{l_{n+1}} + \frac{1}{l_{n+2}}\right)R_{n+1} + \frac{K_{n+2}}{l_{n+2}}R_{n+2} \end{gathered} \tag{27}$$

To find fictitious reactions, it is necessary to build a plot of bending moments in the basic system from a given load. The constructed plot should be taken as a fictitious load. From this load, fictitious reactions are found by the formulas (we can use ready formulas to determine fictitious reactions [13, p. 208]):

$$B_{n,F} = A_{n,F} = \frac{ql_n^3}{24}; \tag{28}$$

$$B_{n-2,F} = A_{n-2,F} = \frac{ql_{n-2}^3}{24}; \tag{29}$$

$$B_{n-1,F} = A_{n-1,F} = \frac{ql_{n-1}^3}{24}; \tag{30}$$

$$B_{n+1,F} = A_{n+1,F} = \frac{ql_{n+1}^3}{24}; \tag{31}$$

$$B_{n+2,F} = A_{n+2,F} = \frac{ql_{n+2}^3}{24}; \tag{32}$$

$$A_{n+3,F} = \frac{ql_{n+3}^3}{24}; \tag{33}$$

$$R_{n-1} = \frac{ql_{n-1}}{2} + \frac{ql_n}{2}; \tag{34}$$

$$R_n = \frac{ql_n}{2} + \frac{ql_{n+1}}{2}; \tag{35}$$

$$R_{n+1} = \frac{ql_{n+1}}{2} + \frac{ql_{n+2}}{2}; \tag{36}$$

$$R_{n-2} = \frac{ql_{n-2}}{2} + \frac{ql_{n-1}}{2}; \tag{37}$$

$$R_{n+2} = \frac{ql_{n+2}}{2} + \frac{ql_{n+3}}{2}. \tag{38}$$

Taking the supports at the location of the rafters absolutely rigid, the pliability of them will be equal to zero:

$$K_{n-3} = K_n = K_{n+3} = 0. \tag{39}$$

The wxMaxima computer algebra system was used to find the solutions of the system. Solving the problem in this software according to the calculation model showed a significant reduction of the maximum design bending moment in the double-span purlin. It can reach 20% at standard span (6 m = 1 m + 4 m + 1 m) and average stiffness of supports with rafter stays (1.0 – 1.2 kN/mm) that essentially influences the general metal consumption of load-bearing elements of a framework (to 15%). The calculation of the model by the finite element method in software Consteel showed high convergence of the results, which does not exceed 0.3% in the direction of increasing reliability (Fig. 6). It was also found the optimal angle of rafter stay inclination relative to the vertical, which is about 30°. When it increases to 45°, a slightly smaller bending moment is obtained, but the rafter stays length increases, which significantly affects the total cost of metal.

Conclusions

Rafter stays can not only perform the direct function but also be used effectively for the restraining of purlins, thus changing their design sketch and reducing the use ratio of cross-section and steel costs, which is a positive factor in reducing the metal consumption of load-bearing elements in a framework with portal frames.

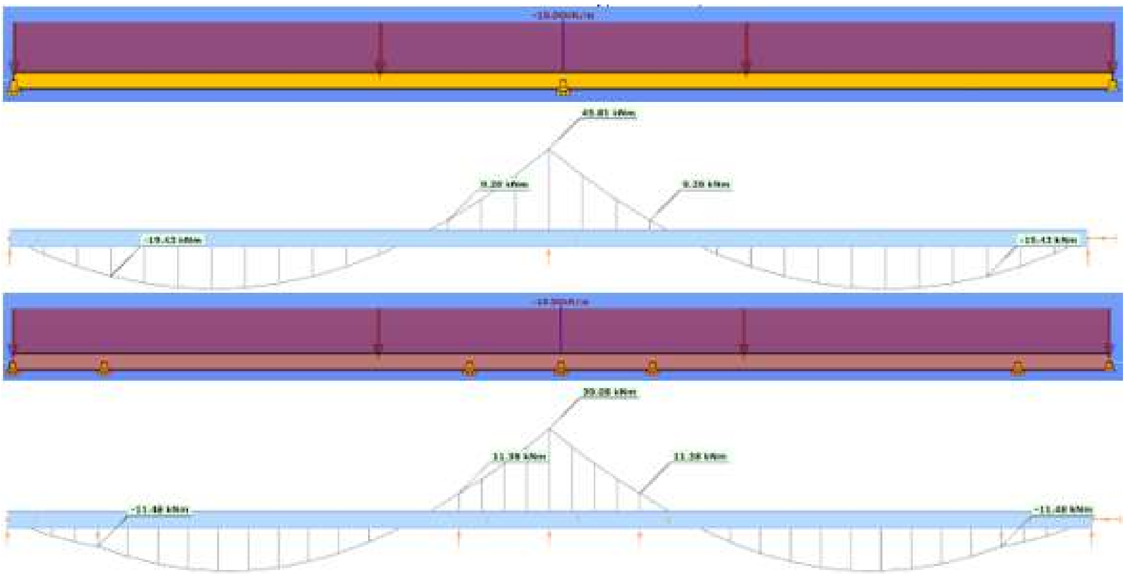

Figure 6 – Design sketches of double-span purlin without rafter stays and with them; comparison of bending moment values on plots in software Consteel

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